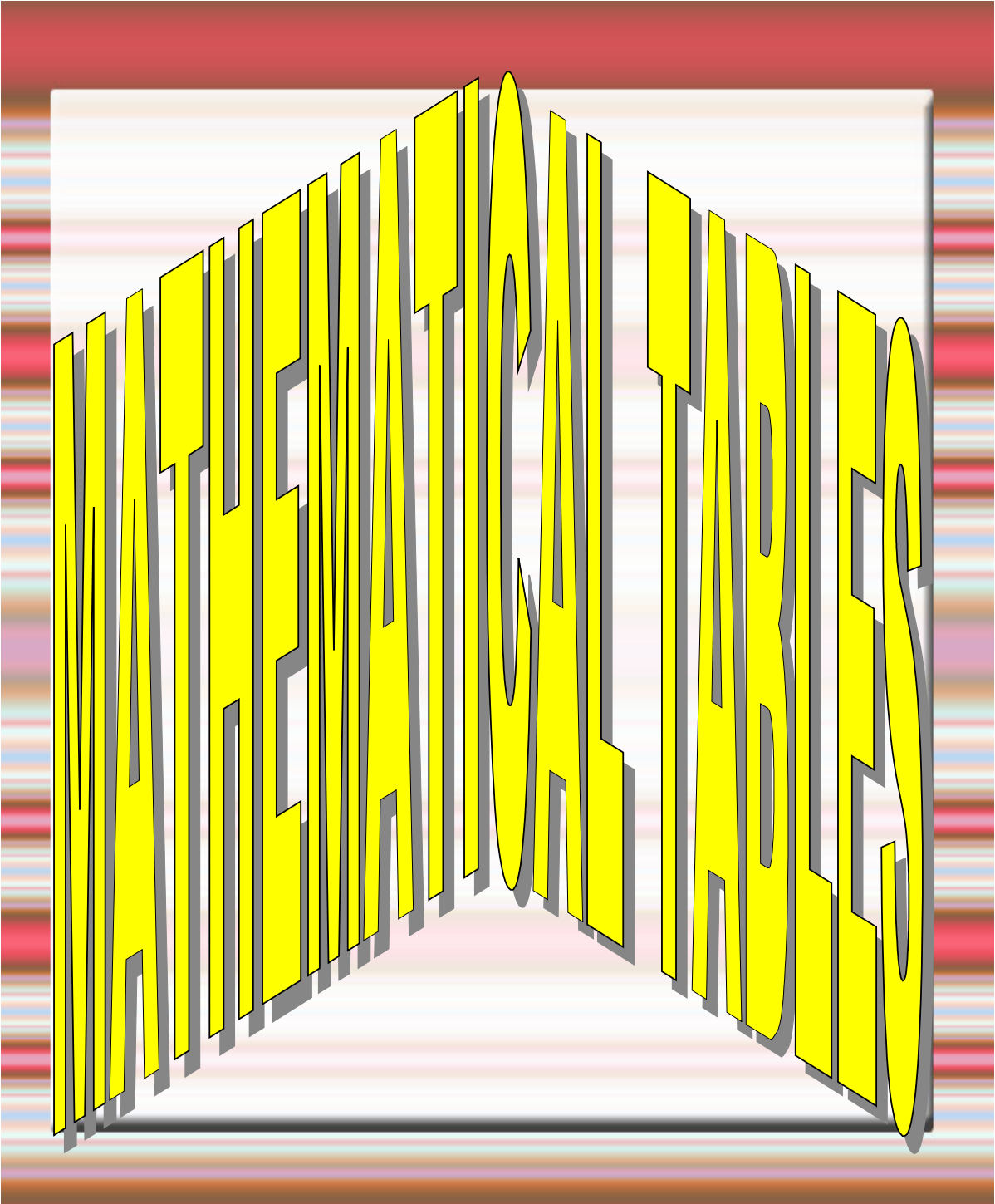


**MATHEMTAICAL TABLES**



# MATHEMATICAL TABLES

## **Table Of Derivatives**

$y = f(x)$	$\bar{y}$
<b>constant</b>	<b>0</b>
<b>x</b>	<b>1</b>
$x^n$	$n x^{(n-1)}$
$u \pm v$	$\bar{u} \pm \bar{v}$
$cu$	$cu$
$uv$	$u \bar{v} + \bar{u} v$
$\frac{u}{v}$	$\bar{u} v - u \frac{\bar{v}}{(v^2)}, v \neq 0$
$\frac{c}{v}$	$\frac{-c}{v^2} * \bar{v}, v \neq 0$
$u^v$	$v u^{(v-1)} \bar{u} + u^v \ln u . v$
$y = f(u), u = \phi(x)$	$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$
$\sin(x)$	$\cos(x)$
$\cos(x)$	$-\sin(x)$
$\tan(x)$	$\sec^2(x)$
$\cot(x)$	$-cosec^2(x)$
$\sec(x)$	$\sec(x) \tan(x)$
$cosec(x)$	$-cosec(x) . \cot(x)$
$e^x$	$e^x$
$\ln(x)$	$\frac{1}{x}$
$a^x$	$a^x \ln(a)$
$\log_a x$	$\frac{1}{x} \log_a e$

## MATHEMTAICAL TABLES

$y = f(x), x = \phi(y)$	$\frac{dy}{dx} = 1 / \left( \frac{dx}{dy} \right)$
$\sin^{-1}(x)$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1}(x)$	$\frac{-1}{\sqrt{1-x^2}}$
$\tan^{-1}(x)$	$\frac{1}{(1+x^2)}$
$\cot^{-1}(x)$	$\frac{-1}{(1+x^2)}$
$\sec^{-1}(x)$	$\frac{1}{(x\sqrt{x^2-1})}$
$\operatorname{cosec}^{-1}(x)$	$\frac{-1}{(x\sqrt{x^2-1})}$
$\sinh(x)$	$\cosh(x)$
$\cosh(x)$	$\sinh(x)$
$\tanh(x)$	$\operatorname{sech}^2(x)$
$\operatorname{coth}(x)$	$-\operatorname{cosech}^2(x)$
$\operatorname{sech}(x)$	$-\operatorname{sech}(x) \tanh(x)$
$\operatorname{cosech}(x)$	$-\operatorname{cosech}(x) \operatorname{coth}(x)$

$\sinh^{-1}(x)$	$\frac{1}{\sqrt{x^2+1}}$
$\cosh^{-1}(x)$	$\frac{1}{\sqrt{x^2-1}}, (x > 1)$
$\tanh^{-1}(x)$	$\frac{1}{(1-x^2)}, ( x  < 1)$
$\operatorname{coth}^{-1}(x)$	$\frac{-1}{(x^2-1)}, ( x  > 1)$
$\operatorname{sech}^{-1}(x)$	$\frac{-1}{(x\sqrt{1-x^2})}, ( x  < 1)$
$\operatorname{cosech}^{-1}(x)$	$\frac{-1}{(x\sqrt{1+x^2})}$
$x = \phi(t), y = \Psi(t)$	$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$

**Table Of Integration**

تعريف التكامل كعملية عكسية للتفاضل نستطيع كتابة الجدول التالي  
للتكاملات القياسية التي تستخدم أساسا عند حساب أى تكامل :

لاحظ أنه فى كل القوانين الآتية قد تكون  $u$  هى المتغير المستقل وقد تكون أية  
دالة فى متغير ما

## MATHEMATICAL TABLES

$\int u^n du = \frac{u^{(n+1)}}{(n+1)} + c, (n \neq -1)$
$\int \frac{du}{u} = \ln u + c$
$\int \sin(u) du = -\cos(u) + c$
$\int \cos(u) du = \sin(u) + c$
$\int \sec^2(u) du = \tan(u) + c$
$\int \operatorname{cosec}^2(u) du = -\cot(u) + c$
$\int \sec(u) \tan(u) du = \sec(u) + c$
$\int \operatorname{cosec}(u) \cot(u) du = -\operatorname{cosec}(u) + c$
$\int e^u du = e^u + c$
$\int a^u du = \frac{a^u}{(\ln a)} + c$
$\int \frac{du}{(\sqrt{a^2 - u^2})} = \sin^{-1}\left(\frac{u}{a}\right) + c$
$\int \frac{du}{(a^2 + u^2)} = \frac{1}{a} \tan^{-1}\left(\frac{u}{a}\right) + c$
$\int \frac{du}{(u\sqrt{u^2 - a^2})} = \frac{1}{a} \sec^{-1}\left(\frac{u}{a}\right) + c$
$\int \sinh(u) du = \cosh(u) + c$
$\int \cosh(u) du = \sinh(u) + c$
$\int \operatorname{sech}^2(u) du = \tanh(u) + c$
$\int \operatorname{sech}(u) \tanh(u) du = -\operatorname{sech}(u) + c$
$\int \operatorname{cosech}(u) \cdot \operatorname{coth}(u) du = -\operatorname{cosech}(u) + c$
$\int \operatorname{cosech}^2(u) du = -\operatorname{coth}(u) + c$
$\int \frac{du}{(\sqrt{a^2 + u^2})} = \sinh^{-1}\left(\frac{u}{a}\right) + c = \ln(u + \sqrt{u^2 + a^2}) + c$
$\int \frac{du}{(\sqrt{u^2 - a^2})} = \cosh^{-1}\left(\frac{u}{a}\right) + c = \ln(u + \sqrt{u^2 - a^2}) + c$
$\int \frac{du}{(a^2 - u^2)} = \frac{1}{a} \tanh^{-1}\left(\frac{u}{a}\right) + c = \frac{1}{2a} \ln\left(\frac{a+u}{a-u}\right) + c$
$\int \frac{du}{(u\sqrt{a^2 - u^2})} = \frac{1}{a} \operatorname{sech}^{-1}(u) + c$
$\int \frac{du}{(u\sqrt{u^2 + a^2})} = \frac{1}{a} \operatorname{cosech}^{-1}\left(\frac{u}{a}\right) + c$

## MATHEMTAICAL TABLES

$\int \tan(u) du = \ln(\sec(u)) + c$
$\int \cot(u) du = \ln(\sin(u)) + c$
$\int \sec(u) du = \ln(\sec(u) + \tan(u)) + c$
$\int \operatorname{cosec}(u) du = -\ln(\operatorname{cosec}(u) + \cot(u)) + c$
$\int \tanh(u) du = \ln(\cosh(u)) + c$
$\int \operatorname{coth}(u) du = \ln(\sinh(u)) + c$
$\int \operatorname{sech}(u) du = 2 \tan^{-1}(e^u) + c$
$\int \operatorname{cosech}(u) du = -2 \operatorname{coth}^{-1}(e^u) + c$

### Quadratic Equation

#### Equation Of The Second Degree

حلول معادلة من الدرجة الثانية : -

صورة المعادلة :

$$ax^2 + bx + c = 0$$

حيث  $a, b, c$  اى ثوابت

حلول المعادلة : -

$$x = -b \pm \frac{\sqrt{(b^2 - 4ac)}}{2a}$$

بعض القوانين الجبرية : -

$$(a \pm b)^2 = a^2 \pm 2ab + b^2$$

$$(a \pm b)^3 = a^3 \pm 3a^2 * b + 3a * b^2 \pm b^3$$

$$a^2 - b^2 = (a - b)(a + b)$$

## MATHEMATAICAL TABLES

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$a^n - b^n = (a-b)(a^{(n-1)} + a^{(n-2)}b + a^{(n-3)}b^2 + \dots + ab^{(n-2)} + b^{(n-1)})$$

نظرية ذات الحدين :-

$$(a+b)^n = {}^n c_0 a^n + {}^n c_1 a^{(n-1)} b + {}^n c_2 a^{(n-2)} b^2 + {}^n c_3 a^{(n-3)} b^3 + \dots + {}^n c_n b^n$$

حيث  $n$  عدد صحيح موجب

$$(1 \pm x)^n = 1 \pm {}^n c_1 x + {}^n c_2 x^2 \pm {}^n c_3 x^3 + \dots, |x| < 1$$

حيث  $n$  هنا عدد صحيح سالب أو كسر موجب أو كسر سالب حيث

$${}^n c_r = \frac{n!}{[r!(n-r)!]} = \frac{[n(n-1)(n-2)\dots(n-r+1)]}{r!}$$

$$r! = r(r-1)(r-2)\dots 3*2*1$$

$${}^n p_r = \frac{n!}{(n-r)!}$$

متسلسلة تيلور (مفكوك تيلور) **Taylor Series** :

$$f(x) = f(a) + \frac{(x-a)}{1!} f'(a) + \frac{(x-a)^2}{2!} f''(a) + \dots$$

لو وضعنا  $a=0$  فى المفكوك السابق

## MATHEMTAICAL TABLES

$$f(x) = f(0) + \frac{x}{1!} f'(0) + \frac{x^2}{2!} f''(0) + \dots$$

حالات خاصة :-

$$\frac{1}{(1 \pm x)} = (1 \pm x)^{-1} = 1 \mp x + x^2 \mp x^3 + \dots, |x| < 1$$

$$\sqrt{(1 \pm x)} = (1 \pm x)^{\left(\frac{1}{2}\right)} = 1 \pm \frac{1}{2}x - \frac{1}{8}x^2 \pm \frac{1}{16}x^3 - \dots, |x| < 1$$

$$\frac{1}{\sqrt{(1 \pm x)}} = (1 \pm x)^{\left(\frac{1}{2}\right)} = 1 \mp \frac{1}{2}x + \frac{3}{8}x^2 \mp \frac{5}{16}x^3 + \dots, |x| < 1$$

متسلسلة فوريير **Fourier Series** :-

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos(nx) + b_n \sin(nx)], (-\pi \leq x \leq \pi)$$

حيث **f** دالة دورية **Periodic Function** :

حيث **a, b** هي معاملات فوريير

$$a_m = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(mx) dx$$

$$b_m = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(mx) dx, m=0, 1, 2, \dots$$

\*\* إذا كانت **Even Function**  $f(x) = -f(x)$  دوال زوجية :

$$a_m = \frac{2}{\pi} \int_0^{\pi} f(x) \cos(mx) dx, b_m = 0$$



## MATHEMTAICAL TABLES

**Odd Function**  $f(x) = -f(x)$  إذا كانت **دوال فردية** :

$$a_m = 0, b_m = \frac{2}{\pi} \int_0^{\pi} f(x) \sin(mx) dx$$

<b>Even - Harmonic Function</b>	<b>Odd - Harmonic Function</b>
$f(x) = f(-x), f(x + \frac{\pi}{2}) = -f(\frac{\pi}{2} - x)$  $a_m = \frac{4}{\pi} \int_0^{\frac{\pi}{2}} f(x) \cos(mx) dx$  <i>for</i> $m = 1, 2, 3, 5, 7, \dots$ $a_m = 0$ <i>for</i> $m = 0, 2, 4, 6, \dots$ $b_m = 0$ <i>for</i> $m = 1, 2, 3, 4, \dots$	$f(x) = -f(-x), f(x + \frac{\pi}{2}) = -f(\frac{\pi}{2} - x)$  $b_m = \frac{4}{\pi} \int_0^{\frac{\pi}{2}} f(x) \sin(mx) dx$  <i>for</i> $m = 1, 3, 5, 7$ $a_m = 0$ <i>for</i> $m = 0, 1, 2, 3, \dots$ $b_m = 0$ <i>for</i> $m = 2, 4, 6, \dots$

### **Integrals Containing Sin Function**

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## MATHEMTAICAL TABLES

$\int \sin(ax) dx = -\left(\frac{1}{a}\right) \cos(ax) + c$
$\int \sin^2(ax) dx = \left(\frac{1}{2}\right)x - \left(\frac{1}{4a}\right)\sin(2ax) + c$
$\int \sin^3(ax) dx = -\left(\frac{1}{a}\right)\cos(ax) + \left(\frac{1}{3a}\right)\cos^3(ax) + c$
$\int \sin^4(ax) dx = \left(\frac{3}{8}\right)x - \left(\frac{1}{4a}\right)\sin(2ax) + \left(\frac{1}{32a}\right)\sin(4ax) + c$
$\int \sin^n(ax) dx = \frac{-(\sin^{(n-1)}(ax)\cos(ax))}{na} + \frac{(n-1)}{n} \int \sin^{(n-2)}(ax) dx, n = \text{integer} > 0$
$\int x \sin ax dx = \frac{(\sin ax)}{a^2} - \frac{(x \cos ax)}{a} + c$
$\int x^2 \sin(ax) dx = \frac{2x}{(a^2)} \sin(ax) - \left[\frac{x^2}{a} - \frac{2}{a^3}\right] \cos(ax) + c$
$\int x^3 \sin(ax) dx = \left[\frac{3x^2}{a^2} - \frac{6}{a^4}\right] \sin(ax) - \left[\frac{x^3}{a} - \frac{6x}{a^3}\right] \cos(ax) + c$
$\int x^n \sin(ax) dx = \frac{-x^n}{a} \cos(ax) + \frac{n}{a} \int x^{(n-1)} \cos(ax) dx, (n > 0)$
$\int \frac{(\sin(ax))}{a} dx = ax - \frac{(ax)^3}{3.3!} + \frac{(ax)^5}{5.5!} - \frac{(ax)^7}{7.7!} + \dots + c$
$\int \frac{(\sin(ax))}{x^2} dx = \frac{-(\sin(ax))}{x} + a \int \frac{(\cos(ax))}{x} dx$
$\int \frac{(\sin(ax))}{x^n} dx = \frac{-1}{(n-1)} * \left[\frac{(\sin(ax))}{x^{(n-1)}}\right] + \left(\frac{a}{(n-1)}\right) * \int \frac{(\cos(ax))}{x^{(n-1)}} dx$
$\int \frac{dx}{(\sin ax)} = \frac{1}{a} \ln\left(\tan\left(\frac{ax}{2}\right)\right) + c$
$\int \frac{dx}{(\sin^2(ax))} = \frac{-1}{a} \cot(ax) + c$
$\int \frac{dx}{(\sin^3(ax))} = \frac{-(\cos(ax))}{(2a\sin^2(ax))} + \frac{1}{(2a)} \ln\left(\tan\left(\frac{ax}{2}\right)\right) + c$
$\int \frac{dx}{(\sin^n(ax))} = \frac{-1}{(a(n-1))} \frac{(\cos(ax))}{(\sin^{(n-1)}(ax))} + \frac{(n-2)}{(n-1)} \int \frac{dx}{(\sin^{(n-2)}(ax))}, n > 1$
$\int \frac{xdx}{(\sin ax)} = \frac{1}{(a^2)} \left(ax + \frac{(ax)^3}{3.3!} + 7 \frac{(ax)^5}{3.5.5!} + 31 \frac{(ax)^7}{3.7.7!} + 127 \frac{(ax)^9}{3.5.9!} + \dots\right) + c$
$\int \frac{xdx}{(\sin^2 ax)} = \frac{-x}{a} \cot ax + \frac{1}{a^2} \ln(\sin ax) + c$

## MATHEMATICAL TABLES

$\int \frac{xdx}{(\sin^n ax)} = \frac{-(x \cos ax)}{[(n-1)a \sin^{(n-1)} ax]} - \left[ \frac{1}{((n-1)(n-2)a^2 \sin^{(n-2)} ax)} \right] + \left[ \frac{(n-2)}{(n-1)} \int \frac{xdx}{(\sin^{(n-2)} ax)} \right], (n > 2)$
$\int \frac{dx}{(1 + \sin ax)} = \frac{-1}{a} \tan \left[ \frac{\pi}{4} - \frac{ax}{2} \right] + c$
$\int \frac{dx}{(1 - \sin ax)} = \frac{1}{a} \tan \left[ \frac{\pi}{4} + \frac{ax}{2} \right] + c$
$\int \frac{xdx}{(1 + \sin ax)} = \frac{-x}{a} \tan \left[ \frac{\pi}{4} - \frac{ax}{2} \right] + \frac{2}{a^2} \ln \cos \left[ \frac{\pi}{4} - \frac{ax}{2} \right] + c$
$\int \frac{xdx}{(1 - \sin ax)} = \frac{x}{a} \cot \left[ \frac{\pi}{4} - \frac{ax}{2} \right] + \frac{2}{a^2} \ln \left( \sin \left[ \frac{\pi}{4} - \frac{ax}{2} \right] \right) + c$
$\int \frac{(\sin ax)}{(1 \pm \sin ax)} dx = \pm x + \frac{1}{a} \tan \left[ \frac{\pi}{4} \mp \frac{ax}{2} \right] + c$
$\int \frac{dx}{[\sin ax (1 \pm \sin ax)]} = \frac{1}{a} \tan \left[ \frac{\pi}{4} \mp \frac{ax}{2} \right] + \frac{1}{a} \ln \left( \tan \frac{ax}{2} \right) + c$
$\int \frac{dx}{(1 + \sin ax)^2} = \frac{-1}{2a} \tan \left[ \frac{\pi}{4} - \frac{ax}{2} \right] - \frac{1}{6a} \tan^3 \left[ \frac{\pi}{4} - \frac{ax}{2} \right] + c$
$\int \frac{dx}{(1 - \sin ax)^2} = \frac{1}{2a} \cot \left[ \frac{\pi}{4} - \frac{ax}{2} \right] + \frac{1}{6a} \cot^3 \left[ \frac{\pi}{4} - \frac{ax}{2} \right] + c$
$\int \sin ax \frac{dx}{(1 + \sin ax)^2} = \frac{-1}{2a} \tan \left[ \frac{\pi}{4} - \frac{ax}{2} \right] + \frac{1}{6a} \tan^3 \left[ \frac{\pi}{4} - \frac{ax}{2} \right] + c$
$\int \frac{(\sin ax)}{(1 - \sin ax)^2} dx = \frac{-1}{2a} \cot \left[ \frac{\pi}{4} - \frac{ax}{2} \right] + \frac{1}{6a} \cot^3 \left[ \frac{\pi}{4} - \frac{ax}{2} \right] + c$
$\int \frac{dx}{(1 + \sin^2 ax)} = \frac{1}{(2\sqrt{2a})} \sin^{-1} \left[ \frac{(3\sin^2 ax - 1)}{(\sin^2 ax + 1)} \right] + c$
$\int \frac{dx}{(1 - \sin^2 ax)} = \frac{1}{a} \tan ax + c$
$\int \sin ax \sin bx dx = \frac{[\sin(a-b)x]}{[2(a-b)]} - \frac{[\sin(a+b)x]}{[2(a+b)]} + c, \text{ for }  a  \neq  b $
$\int \frac{dx}{(b + c \sin ax)} = \frac{2}{(a\sqrt{(b^2 - c^2)})} \tan^{-1} \left[ \frac{(b \tan(\frac{ax}{2}) + c)}{(\sqrt{(b^2 - c^2)})} \right] + k$
$\text{for } : b^2 > c^2$
$\frac{1}{(a\sqrt{(c^2 - b^2)})} \ln \left[ \frac{(b \tan(\frac{ax}{2}) + c - \sqrt{(c^2 - b^2)})}{(b \tan(\frac{ax}{2}) + c + \sqrt{(c^2 - b^2)})} \right] + k$

## MATHEMTAICAL TABLES

$\int \frac{(\sin ax)}{(b+c \sin ax)} dx = \frac{x}{c} - \frac{b}{c} \sqrt{\left(\frac{dx}{(b+c \sin ax)}\right)}$
$\int \frac{dx}{(b+c \sin ax)^2} = \frac{(c \cos ax)}{(a(b^2-c^2)(b+c \sin ax))} + \frac{b}{(b^2-c^2)} \int \left(\frac{dx}{(b+c \sin ax)}\right)$
$\int \frac{(\sin ax)}{(b+c \sin ax)^2} dx = \frac{(b \cos ax)}{(a(c^2-b^2)(b+c \sin ax))} + \frac{c}{(c^2-b^2)} \int \frac{dx}{(b+c \sin ax)} + c$
$\int \sin px \sin^n x dx = \frac{-(\sin^n x \cos px)}{p} + \frac{n}{2p} \int \sin^{(n-1)} x \cos(p-1)x dx + \frac{n}{2p} \int \sin^{(n-1)} x \cos(p+1)x dx$
$\int \frac{(\sin x)}{(\sqrt{(a^2+b^2 \sin^2 x)})} dx = \frac{-1}{b} \sin^{-1} \left( \frac{b \cos x}{\sqrt{(a^2+b^2)}} \right) + c$
$\int \frac{(\sin x)}{(\sqrt{(a^2-b^2) \sin^2 x})} dx = \frac{-1}{b} \ln  (b \cos x + \sqrt{(a^2-b^2 \sin^2 x)})  + c$
$\int \sin x \sqrt{(a^2+b^2 \sin^2 x)} dx = -\cos \frac{x}{2} \sqrt{(a^2+b^2 \sin^2 x)} - \frac{(a^2+b^2)}{2b} \sin^{-1} \left( \frac{b \cos x}{(\sqrt{(a^2+b^2)})} \right) + c$
$\int \sin x \sqrt{(a^2-b^2 \sin^2 x)} dx = \frac{-(\cos x)}{2} \sqrt{(a^2-b^2 \sin^2 x)} - \frac{(a^2-b^2)}{2b} \ln  (b \cos x + \sqrt{(a^2-b^2 \sin^2 x)})  + c$
$\int \frac{(\sin 2x)}{(\sin x)} dx = 2 \sin x + c$
$\int \frac{(\sin 2x)}{(\sin^2 x)} dx = 2 \ln \sin x + c$
$\int \frac{(\sin 2x)}{(\sin^3 x)} dx = \frac{-2}{(\sin x)} + c$
$\int \frac{(\sin 2x)}{(\sin^n x)} dx = \frac{-2}{((n-2) \sin^{(n-2)} x)} + c, n \geq 3$
$\int \frac{(\sin x)}{(\sin 2x)} dx = \frac{1}{2} \ln \left  \left( \cot \left( \frac{x}{2} - \frac{\pi}{4} \right) \right) \right  + c$
$\int \frac{(\sin^2 x)}{(\sin 2x)} dx = \frac{-1}{2} \ln  ( \cos x )  + c$
$\int \frac{(\sin^3 x)}{(\sin 2x)} dx = \frac{-1}{2} \ln \left  \left( \cot \left( \frac{x}{2} - \frac{\pi}{4} \right) \right) \right  - \frac{1}{2} \sin x + c$
$\int \frac{(\sin 3x)}{(\sin x)} dx = x + \sin 2x + c$
$\int \frac{(\sin 3x)}{(\sin^2)} dx = 3 \ln \left  \left( \tan \left( \frac{x}{2} \right) \right) \right  + 4 \cos x + c$
$\int \frac{(\sin 3x)}{(\sin^3)} dx = -3 \cot x - 4x + c$

# MATHEMATICAL TABLES

## **Integrals Containing Cos Function**

$\int \cos ax \, dx = \frac{1}{a} \sin ax + c$
$\int \cos^3 ax \, dx = \frac{1}{a} \sin ax - \frac{1}{3a} \sin^3 ax + c$
$\int \cos^3 ax \, dx = \frac{1}{a} \sin ax - \frac{1}{3a} \sin^3 ax + c$
$\int \cos^4 ax \, dx = \frac{3}{8} x + \frac{1}{4a} \sin 2ax + \frac{1}{32a} \sin 4ax + c$
$\int \cos^n ax \, dx = \frac{(\cos^{(n-1)} ax \sin ax)}{na} + \frac{(n-1)}{n} \int \cos^{(n-2)} ax \, dx$
$\int x \cos ax \, dx = \frac{(\cos ax)}{a^2} + \frac{(x \sin ax)}{a} + c$
$\int x^2 \cos ax \, dx = \frac{2x}{a^2} \cos ax + \left[ \frac{x^2}{a} - \frac{2}{a^2} \right] \sin ax + c$
$\int x^3 \cos ax \, dx = \left[ \frac{3x^2}{a^2} - \frac{6}{a^2} \right] \cos ax + \left[ \frac{x^3}{a} - \frac{6x}{a^2} \right] \sin ax + c$
$\int x^n \cos ax \, dx = \frac{(x^n \sin ax)}{a} - \frac{n}{a} \int x^{(n-1)} \sin ax \, dx$
$\int \frac{\cos ax}{x} \, dx = \ln(ax) - \frac{(ax)^2}{2.2!} + \frac{(ax)^4}{2.2!} - \frac{(ax)^6}{6.6!} + \dots + c$
$\int \frac{(\cos ax)}{x^2} \, dx = \frac{-(\cos ax)}{x} - a \int \frac{(\sin ax)}{x} \, dx$
$\int \frac{(\cos ax)}{x^n} \, dx = \frac{-(\cos ax)}{[(n-1)x^{(n-1)}]} - \frac{a}{(n-1)} \int \frac{(\sin ax)}{x^{(n-1)}} \, dx, \text{ for } n \neq 1$
$\int \frac{dx}{(\cos ax)} = \frac{1}{a} \ln \left[ \tan \left( \frac{ax}{2} + \frac{\pi}{4} \right) \right] + c$
$\int \frac{dx}{(\cos^2 ax)} = \frac{1}{a} \tan ax + c$
$\int \frac{dx}{(\cos^{3ax})} = \frac{(\sin ax)}{(2a \cos^{2ax})} + \frac{1}{2a} \ln \left[ \tan \left( \frac{\pi}{4} + \frac{ax}{2} \right) \right] + c$
$\int \frac{dx}{\cos^n ax} = \frac{1}{(a(n-1))} * \left( \frac{(\sin ax)}{(\cos^{(n-1)} ax)} \right) + \frac{(n-2)}{(n-1)} \int \frac{dx}{(\cos^{(n-2)} ax)} \text{ for } n > 1$
$\int \frac{xdx}{(\cos ax)} = \frac{1}{a^2} * \left[ \frac{(ax)^2}{2} + \frac{(ax)^4}{4.2!} + 5 \frac{(ax)^6}{6.4!} + 61 \frac{(ax)^8}{8.6!} + 1385 \frac{(ax)^{10}}{10.8!} + \dots \right] + c$
$\int \frac{xdx}{(\cos^2 ax)} = \frac{x}{a} \tan ax + \frac{1}{a^2} \ln \cos ax + c$
$\int \frac{dx}{(1 + \cos ax)} = \frac{1}{a} \tan \left( \frac{ax}{2} \right) + c$
$\int \frac{dx}{(1 - \cos ax)} = \frac{-1}{a} \cot \left( \frac{ax}{2} \right) + c$
$\int \frac{xdx}{(1 + \cos ax)} = \frac{x}{a} \tan \left( \frac{ax}{2} \right) + \frac{2}{a^2} \ln \left( \cos \frac{ax}{2} \right) + c$

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$\int \frac{(\cos ax)}{(1 + \cos ax)} = x - \frac{1}{a} \tan\left(\frac{ax}{2}\right) + c$
$\int \frac{(\cos ax)}{(1 - \cos ax)} dx = -x - \frac{1}{a} \cot \frac{ax}{2} + c$
$\int \frac{dx}{(\cos ax(1 + \cos ax))} = \frac{1}{a} \ln\left(\tan\left[\frac{\pi}{4} + \frac{ax}{2}\right]\right) - \frac{1}{a} \tan \frac{ax}{2} + c$
$\int \frac{dx}{(\cos ax(1 - \cos ax))} = \frac{1}{a} \ln\left(\tan\left[\frac{\pi}{4} + \frac{ax}{2}\right]\right) - \frac{1}{a} \cot\left(\frac{ax}{2}\right) + c$
$\int \frac{dx}{(1 + \cos ax)^2} = \frac{1}{2a} \tan \frac{ax}{2} - \frac{1}{6a} \tan^3 \frac{ax}{2} + c$
$\int \frac{dx}{(1 - \cos ax)^2} = \frac{-1}{2a} \cot \frac{ax}{2} - \frac{1}{6a} \cot^3 \frac{ax}{2} + c$
$\int \frac{(\cos ax)}{(1 + \cos ax)^2} dx = \frac{1}{2a} \tan \frac{ax}{2} - \frac{1}{6a} \tan^3 \frac{ax}{2} + c$
$\int \frac{dx}{(1 + \cos^2 ax)} = \frac{1}{(2\sqrt{2a})} \sin^{-1}\left[\frac{(1 - 3\cos^2 ax)}{(1 + \cos^2 ax)}\right] + c$
$\int \frac{dx}{(1 - \cos^2 ax)} = \frac{-1}{a} \cot ax + c$
$\int \cos ax \cos bx dx = \frac{(\sin(a-b)x)}{(2(a-b))} + \frac{(\sin(a+b)x)}{(2(a+b))} + k \text{ for } : a  \neq  b $
$\int \frac{dx}{(b+c \cos ax)} = \frac{2}{(a\sqrt{(b^2-c^2)})} \tan^{-1}\left[\frac{((b-c) \tan(\frac{ax}{2}))}{(\sqrt{(b^2-c^2)})}\right] + k$ $\text{for } : b^2 > c^2 = \frac{1}{(a\sqrt{(c^2-b^2)})} \ln\left[\frac{((c-b) \tan(\frac{ax}{2}) + \sqrt{(c^2-b^2)})}{((c-b) \tan(\frac{ax}{2}) - \sqrt{(c^2-b^2)})}\right] + k$
$\int \frac{(\cos ax)}{(b+c \cos ax)} dx = \frac{x}{c} - \frac{b}{c} \int \frac{dx}{(b+c \cos ax)}$
$\int \frac{dx}{(\cos ax(b+c \cos ax))} = \frac{1}{ab} \ln\left(\tan\left[\frac{ax}{2} + \frac{\pi}{4}\right]\right) - \frac{a}{b} \int \frac{dx}{(b+c \cos ax)}$
$\int \frac{dx}{(b+c \cos ax)^2} = \frac{(c \sin ax)}{[a(c^2-b^2)(b+c \cos ax)]} - \frac{b}{(c^2-b^2)} \int \frac{dx}{(b+c \cos ax)}$
$\int \frac{(\cos ax)}{(b^2+c \cos ax)^2} = \frac{(b \sin ax)}{[a(b^2-c^2)(b+c \cos ax)]} - \frac{c}{(b^2-c^2)} \int \frac{dx}{(b+c \cos ax)}$
$\int \frac{dx}{(b^2+c^2 \cos^2 ax)} = \frac{1}{(ab\sqrt{(b^2+c^2)})} \tan^{-1}\left(\frac{b \tan ax}{\sqrt{(b^2+c^2)}}\right) + k$
$\int \frac{dx}{(b^2-c^2 \cos^2 ax)} = \frac{1}{(ab\sqrt{(b^2-c^2)})} + k$ $\text{for } b^2 > c^2 = \frac{1}{(2ab\sqrt{(c^2-b^2)})} \ln\left[\frac{(b \tan ax - \sqrt{(c^2-b^2)})}{(b \tan ax + \sqrt{(c^2-b^2)})}\right] + k$
$\int \cos ax \cos^n x dx = \frac{(\cos^n x \sin ax)}{a} + \frac{n}{2a} \int \cos^{(n-1)} x \cos(a-1) x dx - \frac{n}{2a} \int \cos^{(n-1)} x \cos(a+1) x dx$
$\int \frac{(\cos x)}{(\sqrt{(a^2+b^2 \cos^2 x)})} dx = \left(\frac{1}{b}\right) \sin^{-1}\left(\frac{(b \sin x)}{(\sqrt{(a^2+b^2)})}\right) + k$

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$\int \frac{(\cos x dx)}{(\sqrt{(a^2 - b^2 \cos^2 x)})} = \frac{1}{b} \ln  (b \sin x + \sqrt{(a^2 - b^2 \cos^2 x)})  + k$
$\int \cos x \sqrt{(a^2 + b^2 \cos^2 x)} dx = \frac{\sin x}{2} \sqrt{(a^2 + b^2 \cos^2 x)} + \frac{(a^2 + b^2)}{2b} \sin^{-1} \frac{(b \sin x)}{(\sqrt{(a^2 + b^2)})} + k$
$\int \frac{(\cos 2x)}{(\cos x)} dx = 2 \sin x - \ln  (\tan (\frac{\pi}{4} + \frac{x}{2}))  + c$
$\int \frac{(\cos 2x)}{(\cos x)} dx = 2 \sin x - \ln  (\tan (\frac{\pi}{4} + \frac{x}{2}))  + c$
$\int \frac{(\cos 2x)}{(\cos^2 x)} dx = 2x - \tan x + c$
$\int \frac{(\cos 2x)}{(\cos^3 x)} dx = \frac{-(\sin x)}{(2 \cos^2 x)} + \frac{3}{2} \ln  (\tan (\frac{\pi}{4} + \frac{x}{2}))  + c$
$\int \frac{(\cos 2x)}{(\cos^n x)} dx = \frac{-(\sin x)}{((n-1) \cos^{n-1} x)} + \frac{n}{(n-1)} \int \frac{dx}{(\cos^{(n-2)} x)}$
$\int \frac{(\cos^2 x)}{(\cos 2x)} dx = \frac{x}{2} - \frac{1}{4} \ln  (\frac{(1 - \tan x)}{(1 + \tan x)})  + c$
$\int \frac{(\cos^3 x)}{(\cos 2x)} dx = \frac{1}{2} \sin x + \frac{1}{(4\sqrt{2})} \ln  (\frac{(1 - \sqrt{2}) \sin x}{(1 + \sqrt{2}) \sin x})  + c$
$\int \frac{(\cos^n x)}{(\cos 2x)} dx = \frac{1}{2} \int \cos^{(n-2)} x dx + \frac{1}{2} \int \frac{(\cos^{(n-2)} x)}{(\cos 2x)} dx$
$\int \frac{(\cos 3x)}{(\cos x)} dx = \sin 2x - x + c$
$\int \frac{(\cos 3x)}{(\cos^2 x)} dx = 4 \sin x - 3 \ln  (\tan (\frac{\pi}{4} + \frac{x}{2}))  + c$
$\int \frac{(\cos 3x)}{(\cos^3 x)} dx = 4x - 3 \tan x + c$
$\int \frac{(\cos 3x)}{(\cos^n x)} dx = 4 \int \frac{dx}{(\cos^{(n-3)} x)} - 3 \int \frac{dx}{(\cos^{(n-1)} x)}$

### **Integrals Containing Sin & Cos Function**

## MATHEMTAICAL TABLES

1. $\int \sin ax \cos ax \, dx = \frac{1}{2a} \sin^2 ax + c$
2. $\int \sin^2 ax \cos^2 ax \, dx = \frac{x}{8} - \frac{(\sin 4ax)}{32a} + c$
3. $\int \sin^n ax \cos ax \, dx = \frac{1}{(a(n+1))} \sin^{(n+1)} ax + c$ for : $n \neq -1$
4. $\int \sin^n ax \cos^n ax \, dx = -\left(\frac{1}{(a(n+1))}\right) \cos^{(n+1)} ax + c$ for : $n \neq -1$
5. $\int \sin^n ax \cos^m ax \, dx = \frac{-((\sin^{(n-1)} ax \cos^{(m+1)} ax))}{(a(n+m))} + \frac{(n-1)}{(n+m)} \int \sin^{(n-2)} ax \cos^m ax \, dx$ for : $m > 0, n > 0 = \frac{(\sin^{(n+1)} ax \cos^{(m-1)} ax)}{(a(n+m))} + \frac{(m-1)}{(n+m)} \int \sin^n ax \cos^{(m-2)} ax \, dx$ , for : $m > 0, n > 0$
$\int \frac{dx}{(\sin ax \cos ax)} = \frac{1}{a} \ln \tan ax + c$
$\int \frac{dx}{(\sin^2 ax \cos ax)} = \frac{1}{a} \left[ \ln \tan \left[ \frac{\pi}{4} + \frac{ax}{2} \right] - \frac{1}{(\sin ax)} \right] + c$
$\int \frac{dx}{(\sin ax \cos^2 ax)} = \frac{1}{a} \left( \ln \tan \left( \frac{ax}{2} \right) + \left( \frac{1}{\cos ax} \right) \right) + c$
$\int \frac{dx}{(\sin^3 ax \cos ax)} = \frac{1}{a} \left( \ln \tan ax - \left( \frac{1}{(2\sin^2 ax)} \right) \right) + c$
$\int \frac{dx}{(\sin ax \cos^3 ax)} = \frac{1}{a} \left( \ln \tan ax + \frac{1}{(2\cos^2 ax)} \right) + c$
$\int \frac{dx}{(\sin^2 ax \cos^2 ax)} = \frac{-2}{a} \cot 2ax + c$
$\int \frac{dx}{(\sin^2 ax \cos^3 ax)} = \frac{1}{a} \left\{ \frac{(\sin ax)}{(2\cos^2 ax)} - \frac{1}{(\sin ax)} + \frac{3}{2} \ln \tan \left[ \frac{\pi}{4} + \frac{ax}{2} \right] \right\} + c$
$\int \frac{dx}{(\sin^3 ax \cos^2 ax)} = \frac{1}{a} \left( \frac{1}{(\cos ax)} - \frac{(\cos ax)}{(2\sin^2 ax)} + \frac{3}{2} \ln \tan \frac{ax}{2} \right) + c$
$\int \frac{dx}{(\sin ax \cos^n ax)} = \frac{1}{(a(n-1) \cos^{(n-1)} ax)} + \int \frac{dx}{(\sin ax \cos^{(n-2)} ax)}$ for : $n \neq 1$
$\int \frac{dx}{(\sin^n ax \cos ax)} = -\left(\frac{1}{(a(n-1) \sin^{(n-1)} ax)}\right) + \int \frac{dx}{(\sin^{(n-2)} ax \cos ax)}$ for : $n \neq 1$
$\int \frac{dx}{(\sin^n ax \cos^m ax)} = -\left(\frac{1}{(a(n-1))} \cdot \frac{1}{(\sin^{(n-1)} ax \cos^{(m-1)} ax)}\right) + \frac{(n+m-2)}{(n-1)} \int \frac{dx}{(\sin^{(n-2)} ax \cos^m ax)}$ $\frac{1}{(a(m-1))} \cdot \frac{1}{(\sin^{(n-1)} ax \cos^{(m-1)} ax)} + \frac{(n+m-2)}{(m-1)} \int \frac{dx}{(\sin^n ax \cos^{(m-2)} ax)}$ for : $n > 0, m > 1$
$\int \frac{(\sin ax \, dx)}{(\cos^2 ax)} = \frac{1}{a} \sec ax + c$



## MATHEMATICAL TABLES

$\int \frac{(\sin ax)}{(\cos^n ax)} dx = \frac{1}{(a(n-1)\cos^{(n-1)} ax)} + c$
$\int \frac{(\sin^2 ax)}{(\cos ax)} dx = \frac{-1}{a} (\sin ax) + \frac{1}{a} \ln \tan \left[ \frac{\pi}{4} + \frac{ax}{2} \right] + c$
$\int \frac{(\sin^2 ax)}{(\cos^3 ax)} dx = \frac{1}{a} \left\{ \frac{(\sin ax)}{(2\cos^2 ax)} - \frac{1}{a} \ln \tan \left[ \frac{\pi}{4} + \frac{ax}{2} \right] \right\}$
$\int \frac{(\sin^2 ax)}{(\cos^n ax)} dx = \frac{(\sin ax)}{(a(n-1)\cos^{(n-1)} ax)} - \frac{1}{(n-1)} \int \frac{dx}{(\cos^{(n-2)} ax)}$
$\int \frac{(\sin^3 ax)}{(\cos ax)} dx = \frac{-1}{a} \left[ \frac{(\sin^2 ax)}{2} + \ln \cos ax \right] + c$
$\int \frac{(\sin^3 ax)}{(\cos^2 ax)} dx = \frac{1}{a} \left[ \cos ax + \frac{1}{(\cos ax)} \right] + c$
$\int \frac{(\sin^3 ax)}{(\cos^n ax)} dx = \frac{1}{a} \left\{ \frac{1}{((n-1)\cos^{(n-1)} ax)} - \frac{1}{((n-3)\cos^{(n-3)} ax)} \right\} + c$
$\int \frac{(\sin^n ax)}{(\cos ax)} dx = \frac{-(\sin^{(n-1)} ax)}{(a(n-1))} + \int \frac{(\sin^{(n-2)} ax)}{(\cos ax)} dx$
$\int \frac{(\cos ax)}{(\sin^n ax)} dx = \frac{-1}{(a(n-1)\sin^{(n-1)} ax)} + c$
$\int \frac{(\cos^2 ax)}{(\sin ax)} dx = \frac{1}{a} (\cos ax + \ln \tan \left( \frac{ax}{2} \right)) + c$
$\int \frac{(\cos^2 ax)}{(\sin^3 ax)} dx = \frac{-1}{2a} \left[ \frac{(\cos ax)}{(\sin^2 ax)} - \ln \tan \frac{ax}{2} \right] + c$
$\int \frac{(\cos^2 ax)}{(\sin^n ax)} dx = \frac{-1}{(n-1)} \left[ \frac{(\cos ax)}{(a\sin^{-1} ax)} + \int \frac{dx}{(\sin^{(n-2)} ax)} \right]$
$\int \frac{(\cos^3 ax)}{(\sin ax)} dx = \frac{1}{a} \left[ \frac{(\cos^2 ax)}{2} + \ln \sin ax \right] + c$
$\int \frac{(\cos^3 ax)}{(\sin^2 ax)} dx = \frac{-1}{a} \left[ \sin ax + \frac{1}{(\sin ax)} \right] + c$
$\int \frac{(\cos^3 ax)}{(\sin^n ax)} dx = \frac{1}{a} \left[ \frac{1}{((n-3)\sin^{(n-2)} ax)} - \frac{1}{((n-1)\sin^{(n-1)} ax)} \right] + c$
$\int \frac{(\cos^n ax)}{(\sin ax)} dx = \frac{(\cos^{(n-1)} ax)}{(a(n-1))} + \int \frac{(\cos^{(n-2)} ax)}{(\sin ax)} dx$

## MATHEMATICAL TABLES

### **Integrals Containing Tan & Cot Function**

$\int \tan ax \, dx = \frac{-1}{a} \ln \cos ax + c$
$\int \tan^3 ax \, dx = \frac{1}{2a} \tan^3 ax + \frac{1}{a} \ln \cos ax + c$
$\int \tan^3 ax \, dx = \frac{1}{2a} \tan^3 ax + \frac{1}{a} \ln \cos ax + c$
$\int \tan^n ax \, dx = \frac{1}{(a(n-1))} \tan^{(n-1)} ax - \int \tan^{(n-2)} ax \, dx$
$\int x \tan ax \, dx = \frac{(ax^3)}{3} + \frac{(a^3 x^5)}{15} + \frac{(2a^2 x^7)}{105} + \frac{(17a^7 x^9)}{2835} + \dots + c$
$\int \frac{(\tan ax)}{x} \, dx = ax + \frac{(ax)^3}{9} + 2 \frac{(ax)^5}{75} + 17 \frac{(ax)^7}{2205} + \dots + c$
$\int \frac{(\tan^n ax)}{(\cos^2 ax)} \, dx = \frac{1}{(a(n+1))} \tan^{(n+1)} ax + c$
$\int \frac{dx}{(\tan ax \pm 1)} = \pm \left(\frac{x}{2}\right) + \frac{1}{2a} \ln(\sin ax \pm \cos ax) + c$
$\int \frac{(\tan ax)}{(\tan ax \pm 1)} \, dx = \left(\frac{x}{2}\right) \mp \left(\frac{1}{2a}\right) \ln(\sin ax \pm \cos ax) + c$
$\int \frac{(\tan ax)}{(a + B \tan x)} = \frac{1}{(a^2 + B^2)} (Bx - a \ln  (a \cos x + B \sin x) ) + c$
$\int \frac{dx}{(1 + \tan^2 x)} = \frac{x}{2} + \frac{1}{4} \sin 2x + c$
$\int \frac{dx}{(a^2 + B^2 \tan^2 x)} = \frac{1}{(a^2 - B^2)} \left\{ \left(x - \left \frac{B}{a}\right  \tan^{-1} \left[\left \frac{B}{a}\right  \tan x\right]\right) \right\}$
$\int \frac{dx}{(a^2 - B^2 \tan^2 x)} = \frac{1}{(a^2 + B^2)} \left[ x + \frac{B}{2a} \ln \left  \frac{(a + B \tan x)}{(a - B \tan x)} \right  \right] + k$
$\int \frac{(\tan x)}{(1 + \tan^2 x)} \, dx = \frac{-(\cos^2 x)}{2} + c$
$\int \frac{(\tan x)}{(1 + a^2 \tan^2 x)} = \ln \frac{(\cos^2 x + a^2 \sin^2 x)}{(2(a^2 - 1))} + k$
$\int \cot ax \, dx = \frac{1}{a} \ln \sin ax + c$

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$\int \cot^2 ax \, dx = \frac{-(\cot ax)}{a} - x + c$
$\int \cot^3 ax \, dx = \frac{-1}{2a} \cot^2 ax - \frac{1}{a} \ln \sin ax + c$
$\int \cot^n ax \, dx = \frac{-1}{(a(n-1))} \cot^{(n-1)} ax - \int \cot^{(n-2)} ax \, dx$

## **Integrals Containing $\sin^{-1}$ & $\cos^{-1}$ Function**

$\int \sin^{-1} \frac{x}{a} \, dx = x \sin^{-1} \frac{x}{a} + \sqrt{(a^2 - x^2)} + c$
$\int x \sin^{-1} \frac{x}{a} \, dx = \left[ \frac{x^2}{2} - \frac{a^2}{4} \right] \sin^{-1} \frac{x}{a} + \frac{x}{4} \sqrt{(a^2 - x^2)} + c$
$\int x^2 \sin^{-1} \frac{x}{a} \, dx = \frac{x^3}{3} \sin^{-1} \frac{x}{a} + \frac{1}{9} (x^2 + 2a^2) \sqrt{(a^2 - x^2)} + c$
$\int \frac{(\sin^{-1} \frac{x}{a})}{x} \, dx = \frac{x}{a} + \frac{1}{2.3.3} * \left( \frac{x^3}{a^3} \right) + \frac{1.3}{2.4.5.5} * \left( \frac{x^5}{a^5} \right) + \frac{1.3.5}{2.4.6.7.7} * \left( \frac{x^7}{a^7} \right) + \dots + c$
$\int \frac{(\sin^{-1} \frac{x}{a})}{x^2} \, dx = \frac{-1}{x} \sin^{-1} \frac{x}{a} - \frac{1}{a} \ln \left( \frac{(a + \sqrt{(a^2 - x^2)})}{x} \right) + c$
$\int x^3 \sin^{-1} \frac{x}{a} \, dx = \frac{(8x^4 - 3a^4)}{32} \sin^{-1} \frac{x}{a} + \frac{(2x^6 + 3xa^2)}{32} \sqrt{(a^2 - x^2)} + c$
$\int x^4 \sin^{-1} \frac{x}{a} \, dx = \frac{(x^5)}{5} \sin^{-1} \frac{x}{a} + \left[ \frac{(3x^4 + 4x^2 a^2 + 8a^4)}{75} \right] \cdot \sqrt{(a^2 - x^2)} + c$
$\int x^n \sin^{-1} \frac{x}{a} \, dx = \frac{x^{(n+1)}}{(n+1)} \sin^{-1} \frac{x}{a} - \frac{1}{(n+1)} \int \left( \frac{x^{(n+1)}}{\sqrt{(a^2 - x^2)}} \right) dx$
$\int \frac{1}{x^n} \sin^{-1} \frac{x}{a} \, dx = \frac{-(\sin^{-1} \frac{x}{a})}{((n-1)x^{(n-1)})} + \frac{1}{(n-1)} \int \frac{dx}{[(x^{(n-1)})\sqrt{(a^2 - x^2)}]}$

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$\int \cos^{-1} \frac{x}{a} dx = \left[ \frac{x^2}{2} - \frac{a^2}{4} \right] \cos^{-1} \frac{x}{a} - \frac{x}{4} \sqrt{(a^2 - x^2)} + c$
$\int x \cos^{-1} \frac{x}{a} dx = \left[ \frac{x^2}{2} - \frac{a^2}{4} \right] \cos^{-1} \frac{x}{a} - \frac{x}{4} \sqrt{(a^2 - x^2)} + c$
$\int x^2 \cos^{-1} \frac{x}{a} dx = \frac{x^3}{3} \cos^{-1} \frac{x}{a} - \frac{1}{9} (x^2 + 2a^2) \sqrt{(a^2 - x^2)} + c$
$\int \frac{(\cos^{-1} \frac{x}{a})}{x} dx = \frac{\pi}{2} \ln x - \frac{x}{a} - \frac{1}{2.3.3} * \left( \frac{x^3}{a^3} \right) - \frac{1.3}{2.4.5.5} \left( \frac{x^5}{a^5} \right) - \frac{1.3.5}{2.4.6.7.7} \left( \frac{x^7}{a^7} \right) - \dots + c$
$\int \frac{(\cos^{-1} \frac{x}{a})}{x^2} dx = \frac{-1}{x} \cos^{-1} \frac{x}{a} + \frac{1}{a} \ln \frac{(a + \sqrt{(a^2 - x^2)})}{x} + c$

## **Integrals Containing $\tan^{-1}$ & $\cot^{-1}$ Function**

$\int \tan^{-1} \frac{x}{a} dx = x \tan^{-1} \frac{x}{a} - \frac{a}{2} \ln(a^2 + x^2) + c$
$\int x \tan^{-1} \frac{x}{a} dx = \frac{1}{2} (x^2 + a^2) \tan^{-1} \frac{x}{a} - \frac{ax}{2} + c$
$\int x^2 \tan^{-1} \frac{x}{a} dx = \frac{x^3}{3} \tan^{-1} \frac{x}{a} - a \frac{x^2}{6} + \frac{a^3}{6} \ln(x^2 + a^2) + c$
$\int x^n \tan^{-1} \frac{x}{a} dx = \frac{(x^{(n+1)})}{(n+1)} \tan^{-1} \frac{x}{a} - \frac{a}{(n+1)} \int \left( \frac{x^{(n+1)}}{(a^2 + x^2)} \right)$
$\int \frac{(\tan^{-1} \frac{x}{a})}{x} dx = \frac{x}{a} - \frac{x^3}{(3^2 a^5)} + \frac{x^5}{(5^5 a^5)} - \frac{x^7}{(7^2 a^2)} + \dots + c$
$\int \frac{(\tan^{-1} \frac{x}{a})}{x^2} dx = \frac{-1}{x} \tan^{-1} \frac{x}{a} - \frac{1}{2a} \ln \frac{(a^2 + x^2)}{x^2} + c$
$\int \frac{(\tan^{-1} \frac{x}{a})}{x^n} dx = \frac{-1}{((n-1)x^{(n-1)})} \tan^{-1} \frac{x}{a} + \frac{a}{(n-1)} \int \frac{dx}{(x^{(n-1)}(a^2 + x^2))}$

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$\int \frac{(x^2 \tan^{-1} x)}{(1+x^2)} dx = x \tan^{-1} x - \frac{1}{2} \ln(1+x^2) - \frac{1}{2} (\tan^{-1} x)^2 + c$
$\int \frac{(x^3 \tan^{-1} x)}{(1+x^2)} dx = \frac{-1}{2} x + \frac{1}{2} (1+x^2) \tan^{-1} x - \int \frac{(x \tan^{-1} x)}{(1+x^2)} dx$
$\int \frac{(x^4 \tan^{-1} x)}{(1+x^2)} dx = \frac{-1}{6} x^2 + \frac{2}{3} \ln(1+x^2) + \left(\frac{x^3}{6} - x\right) \tan^{-1} x + \frac{1}{2} (\tan^{-1} x)^2 + c$
$\int \frac{(x \tan^{-1} x)}{(\sqrt{1-x^2})} dx = -\sqrt{1-x^2} \tan^{-1} x + \sqrt{2} \tan^{-1} \left( \frac{x\sqrt{2}}{(\sqrt{1-x^2})} \right) - \sin^{-1} x + c$
$\int \frac{(\tan^{-1} x)}{(\alpha + \beta x)^2} dx = \frac{1}{(\alpha^2 + \beta^2)} \left[ \ln \left  \frac{(\alpha + \beta x)}{(\sqrt{1+x^2})} \right  - \frac{(\beta - \alpha x)}{(\alpha + \beta x)} \tan^{-1} x \right] + c$
$\int \cot^{-1} \frac{x}{a} dx = x \cot^{-1} \frac{x}{a} + \frac{a}{2} \ln(a^2 + x^2) + c$
$\int x \cot^{-1} \frac{x}{a} dx = \frac{1}{2} (x^2 + a^2) \cot^{-1} \frac{x}{a} + \frac{ax}{2} + c$
$\int x^2 \cot^{-1} \frac{x}{a} dx = \frac{(x^3)}{3} \cot^{-1} \frac{x}{a} + \frac{(ax^2)}{6} - \frac{(a^3)}{6} \ln(x^2 + a^2) + c$

## **Integrals Containing $\sec^{-1}$ & $\operatorname{cosec}^{-1}$ Function**

$\int \sec^{-1} \frac{x}{a} dx = x \sec^{-1} \frac{x}{a} - a \ln \left  (x + \sqrt{x^2 - a^2}) \right  + c \text{ for } : 0 < \sec^{-1} \frac{x}{a} < \frac{\pi}{2}$
$x \sec^{-1} \frac{x}{a} + a \ln \left  (x + \sqrt{x^2 - a^2}) \right  + c \text{ for } : \frac{\pi}{2} < \sec^{-1} \frac{x}{a} < \pi$
$\int x \sec^{-1} \frac{x}{a} dx = \frac{x^2}{2} \sec^{-1} \frac{x}{a} - \left(\frac{a}{2}\right) \sqrt{x^2 - a^2} + c \text{ for } : 0 < \sec^{-1} \frac{x}{a} < \frac{\pi}{2}$
$\frac{x^2}{2} \sec^{-1} \frac{x}{a} + \left(\frac{a}{2}\right) \sqrt{x^2 - a^2} + c \text{ for } : \frac{\pi}{2} < \sec^{-1} \frac{x}{a} < \pi$
$\int x^2 \sec^{-1} \frac{x}{a} dx = \frac{x^3}{3} \sec^{-1} \frac{x}{a} - \frac{ax}{6} \sqrt{x^2 - a^2} + \frac{a^3}{6} \ln \left  (x + \sqrt{x^2 - a^2}) \right  + c, 0 < \sec^{-1} \frac{x}{a} < \frac{\pi}{2}$
$\frac{x^3}{3} \sec^{-1} \frac{x}{a} + \frac{ax}{6} \sqrt{x^2 - a^2} + \frac{a^3}{6} \ln \left  (x + \sqrt{x^2 - a^2}) \right  + c, \frac{\pi}{2} < \sec^{-1} \frac{x}{a} < \pi$

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$\int x^n \sec^{-1} \frac{x}{a} dx = \frac{(x^{n+1})}{(n+1)} \sec^{-1} \frac{x}{a} - \frac{a}{(n+1)} \int \frac{(x^n)}{\sqrt{(x^2-a^2)}} dx, 0 < \sec^{-1} \frac{x}{a} < \frac{\pi}{2}$ $\frac{x^{(n+1)}}{(n+1)} \sec^{-1} \frac{x}{a} + \frac{a}{(n+1)} \int \frac{(x^n)}{\sqrt{(x^2-a^2)}} dx, \frac{\pi}{2} < \sec^{-1} \frac{x}{a} < \pi$
$\int \frac{1}{x} \sec^{-1} \frac{x}{a} dx = \frac{\pi}{2} \ln x  + \frac{a}{x} + \frac{(a^3)}{2.3.3.x^3} + \frac{(1.3.a^5)}{2.4.5.5.x^5} + \frac{(1.3.5.a^7)}{2.4.6.7.7.x^7} + \dots + c$
$\int \frac{1}{x^2} \sec^{-1} \frac{x}{a} dx = \frac{\sqrt{(x^2-a^2)}}{ax} - \frac{1}{x} \sec^{-1} \frac{x}{a} + c$
$\int \frac{1}{x^3} \sec^{-1} \frac{x}{a} dx = \frac{-1}{(2x^2)} \sec^{-1} \frac{x}{a} + \frac{\sqrt{(x^2-a^2)}}{(4ax^2)} + \frac{1}{(4a^2)} \cos^{-1} \left  \frac{x}{a} \right  + c$

### **Integrals Containing Hyperbolic Functions**

$\int \sinh ax dx = \frac{-1}{a} \cosh ax + c$
$\int \cosh ax dx = \frac{1}{a} \sinh ax + c$
$\int \sinh^2 ax dx = \frac{1}{2a} \sinh ax \cosh ax - \frac{1}{2} x + c$
$\int \cosh^2 ax dx = \frac{1}{2a} \sinh ax \cosh ax + \frac{1}{2} x + c$
$\int \sinh^n ax dx = \frac{1}{an} \sinh^{(n-1)} ax \cosh ax - \frac{(n-1)}{n} \int \sinh^{(n-2)} ax dx \text{ for } : n > 0$ $\frac{1}{(a(n+1))} \sinh^{(n+1)} ax \cosh ax - \frac{(n+2)}{(n+1)} \int \sinh^{(n+2)} ax dx \text{ for } : n < 0, n \neq -1$
$\int \frac{dx}{(\sinh ax)} = \frac{1}{a} \ln \tanh \frac{ax}{2} + c$
$\int \frac{dx}{(\cosh ax)} = \frac{2}{a} \tan^{-1} e^{ax} + c$
$\int x \sinh ax dx = \frac{1}{a} x \cosh ax - \frac{1}{a^2} \sinh ax + c$
$\int x \cosh ax dx = \frac{1}{a} x \sinh ax - \frac{1}{a^2} \cosh ax + c$
$\int \tanh ax dx = \frac{1}{a} \ln \cosh ax + c$

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$\int \coth ax \, dx = \frac{1}{a} \ln \sinh ax + c$
$\int \tanh^2 ax \, dx = x - \frac{(\tanh ax)}{(a)} + c$
$\int \coth^2 ax \, dx = x - \frac{(\coth ax)}{(a)} + c$
$\int \sinh ax \sinh Bx \, dx = \frac{1}{(a^2 - B^2)} (a \sinh Bx \cosh ax - B \cosh Bx \sinh ax) + k$
$\int \cosh ax \cosh Bx \, dx = \frac{1}{(a^2 - B^2)} (a \sinh ax \cosh Bx - B \sinh Bx \cosh ax) + k$
$\int \cosh ax \sinh Bx \, dx = \frac{1}{(a^2 - B^2)} (a \sinh Bx \sinh ax - B \cosh Bx \cosh ax) + k$

### **Integrals Containing Exponential Functions**

$\int A^{(ax+B)} \, dx = \frac{1}{(a \ln A)} A^{(ax+B)} + k \text{ for } A > 0, A \neq 1$
$\int F(e^{ax}) \, dx = \frac{1}{a} \int F(t) \frac{dt}{t}, \text{ where } t = e^{ax}$
$\int x e^{ax} \, dx = \frac{(ax-1)}{a^2} e^{ax} + k$
$\int x^2 e^{ax} \, dx = \frac{(a^2 x^2 - 2ax + 2)}{a^3} e^{ax} + k$
$\int x^3 e^{ax} \, dx = \frac{(a^3 x^3 - 3a^2 x^2 + 6ax - 6)}{a^4} e^{ax} + k$
$\int x^4 e^{ax} \, dx = \frac{(a^4 x^4 - 4a^3 x^3 + 12a^2 x^2 - 24ax + 24)}{a^5} e^{ax} + k$
$\int x^n e^{ax} \, dx = e^{ax} \left( \frac{x^n}{a} - \frac{(nx^{(n-1)})}{a^2} + \frac{(n(n-1)x^{(n-2)})}{a^3} - \dots + (-1)^{(n-1)} \frac{(n!x)}{a^n} + (-1)^n \frac{(n!)}{a^{(n+1)}} \right) + k$

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### **Integrals Containing Logarithmic Functions**

$\int \ln x \, dx = x \ln x - x + c$
$\int (\ln x)^2 \, dx = x (\ln x)^2 - 2x \ln x + 2x + c$
$\int (\ln x)^3 \, dx = x (\ln x)^3 - 3x (\ln x)^2 + 6x \ln x - 6x + c$
$\int (\ln x)^n \, dx = x (\ln x)^n - n \int (\ln x)^{(n-1)} \, dx, \text{ for } n \neq -1$
$\int \frac{dx}{\ln x} = \ln(\ln x) + \ln x + \frac{(\ln x)^2}{2.2!} + \frac{(\ln x)^3}{3.3!} + \dots + c$
$\int \frac{dx}{(\ln x)^n} = \frac{-x}{[(n-1)(\ln x)^{(n-1)}]} + \frac{1}{(n-1)} \int \frac{dx}{(\ln x)^{(n-1)}, \text{ for } n \neq 1$
$\int x^m \ln x \, dx = x^{(m+1)} \left[ \frac{(\ln x)}{(m+1)} - \frac{1}{(m+1)^2} \right] + c, \text{ for } m \neq -1$
$\int x^m (\ln x)^n \, dx = \frac{[x^{(m+1)} (\ln x)^n]}{(m+1)} - \frac{n}{(m+1)} \int x^m (\ln x)^{(n-1)} \, dx \text{ for } m \neq -1, n \neq -1$
$\int \frac{(\ln x)^n}{x} \, dx = \frac{(\ln x)^{(n+1)}}{(n+1)} + c$

### **Integrals Containing Inverse Hyperbolic Functions**

$\int \operatorname{sh}^{-1} \frac{x}{a} \, dx = x \operatorname{sh}^{-1} \frac{x}{a} - \sqrt{(x^2 + a^2)} + k$
$\int \operatorname{cosh}^{-1} \frac{x}{a} \, dx = x \operatorname{cosh}^{-1} \frac{x}{a} - \sqrt{(x^2 - a^2)} + k$
$\int \operatorname{tanh}^{-1} \frac{x}{a} \, dx = x \operatorname{tanh}^{-1} \frac{x}{a} + \frac{a}{2} (a^2 - x^2) + k$
$\int \operatorname{coth}^{-1} \frac{x}{a} \, dx = x \operatorname{coth}^{-1} \frac{x}{a} + \frac{a}{2} \ln(x^2 - a^2) + k$



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### **Some Definite Integrals**

$$\int_0^{\infty} e^{-a^2 x^2} dx = \frac{\sqrt{\pi}}{2a} \text{ at } a > 0$$

$$\int_0^{\infty} x^2 e^{-a^2 x^2} dx = \frac{\sqrt{\pi}}{4a^3} \text{ at } a > 0$$

$$\int_0^{\infty} e^{-x} \ln x dx \approx -0.5772$$

$$\int_0^{\infty} \frac{(\cos ax)}{x} dx = \infty, (\infty - \text{any number})$$

## MATHEMTAICAL TABLES

قد تم سرد بعض أهم التكاملات والتفاضلات الرئيسية فى هذا الموضوع  
وسيتم اكمالها فى العدد الثانى وبالإضافة الى بعض القوانين الجبرية الأخرى  
والتى تفيد الباحثين فى مجال الرياضيات الهندسية

**[memory code 84 @ yahoo .com](mailto:memory_code_84@yahoo.com)**