1. $\int x\sqrt{x}\,dx = \int x^{\frac{3}{2}}\,dx = \frac{2}{5}x^{\frac{5}{2}} + c$

2. $\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 dx = \left(x - 2 + \frac{1}{x}\right) dx = \frac{1}{2}x^2 - 2x + \ln|x| + c$

3. $\int \frac{2-\sqrt{1-x^2}}{\sqrt{1-x^2}}\,dx = \int \frac{2}{\sqrt{1-x^2}}\,dx - \int dx = 2\sin^{-1}x - x + c$

4. $\int \frac{x^2+5x-1}{\sqrt{x}}\,dx = \int x^{\frac{-1}{2}}\,(x^2 + 5x - 1)\,dx =$

$\int x^{\frac{3}{2}} + 5x^{\frac{1}{2}} - x^{\frac{-1}{2}}\,dx = \frac{2}{5}x^{\frac{5}{2}} + \frac{10}{3}x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + c$

5. $\int \frac{(x-1)^2}{x}\,dx = \int \frac{x^2}{x} - \frac{2x}{x} + \frac{1}{x}\,dx = \int x - 2 + \frac{1}{x}\,dx =$

$\frac{x^2}{2} - 2x + \ln|x| + c$

6. $\int (e^x + 1)^2\,dx = \int e^{2x} + 2e^x + 1\,dx =$

$\frac{1}{2}e^{2x} + 2e^x + x + c$

7. $\int (3x+5)^{20}\,dx$

$\frac{1}{3}\int 3(3x+5)^{20}\,dx$

$= \frac{1}{21}(3x+5)^{21}$

نساوي التكامل على صورة دالة « مشتقتها

نضرب » 3 ونقسم على 3

حل اخر / نفرض $y = 3x + 5$

نساوي التكامل على صورة دالة « مشتقتها

نضرب » 2 ونقسم على 2

$$8. \int x\,(1+x^2)^{\frac{1}{2}}\ dx = \frac{1}{2}\int 2x\,(1+x^2)^{\frac{1}{2}}\ dx$$

$$= \frac{1}{2}\times\frac{2}{3}\,(1+x^2)^{\frac{3}{2}} = \frac{1}{3}\,(1+x^2)^{\frac{3}{2}} + c$$

$$9. \int \sqrt{\sin x}\ \cos x\, dx$$

$$y = \sin x$$

$$dy = 1\ \cos x\ dx$$

$$\int y^{\frac{1}{2}}\quad dy = \frac{3}{2}\ y^{\frac{3}{2}} == \frac{3}{2}\sqrt{x^2} + c$$

$$10. \int x\ \cos x^2\ dx$$

$$y = x^2$$

$$dy = 2x\, dx \rightarrow \frac{1}{2}\ dy = x\, dx$$

$$\frac{1}{2}\int \cos y = \frac{1}{2}\sin y = \frac{1}{2}\sin x^2 + c$$

$$11. \int \cos(\sin x)\ \cos x\ dx$$

$$y = \sin x$$

$$dy = \cos x\ dx$$

$$\int \cos y\, dy = \sin y = \sin(\sin x) + c$$

$$12. \int \frac{dx}{x\ \ln x} = \int \frac{1}{\ln x}\times\frac{1}{x}\ dx = \ln\ |\ \ln x\ | + c$$ البسط مشتقة للمقام

13. $\int (x^2 - 3x + 1)^9 \ (2x - 3)dx$

$y = \ (x^2 - 3x + 1)$

$dy = (2x - 3)dx$

$\int y^9 \ dy = \frac{1}{10} y^{10} = \frac{1}{10} (x^2 - 3x + 1)^{10} + \ c$

14. $\int x^2 \ \sqrt{x^3 + 5} \ dx$

$y = \ x^3 + 5$

$dy = 3x^2 \ dx \ \ \rightarrow \frac{1}{3} dy = x^2 \ dx$

$\frac{1}{3} \int \sqrt{y} \, dy = \ \frac{2}{9} \sqrt{y^3} == \ \frac{2}{9} \ (x^2 + 5)^{\frac{3}{2}} + \ c$

15. $\int \frac{(2 \ ln \ x + 3)^3}{x} dx$

$y = \ 2 \ ln \, x + 3$

$dy = \frac{2dx}{x} \rightarrow \frac{1}{2} dy = \ \frac{dx}{x}$

$\frac{1}{2} \int y^3 \, dy = \ \frac{1}{8} y^4 \ = \frac{1}{8} (2 \ ln \, x + 3)^4 \ + c$

16. $\int \frac{sin \ \sqrt[3]{x}}{\sqrt[3]{x^2}} \ dx \ \ \ put \ \ y = \ (x)^{\frac{1}{3}} \qquad dy = \ \frac{1}{3} (x)^{\frac{-2}{3}} dx$

$3dy = \frac{dx}{\sqrt[3]{x^2}} \ \rightarrow 3 \int sin \, y \ dy = -cos \, y = \ - \ cos \sqrt[3]{x} + c$

$$17. \int \frac{dx}{\cos x}\, dx = \int \sec x \, dx = \int \sec x \left(\frac{\sec x+\tan x}{\sec x+\tan x}\right)$$

أولاً :- نضرب ونقسم في $\sec x + \tan x$

ثانياً : نلاحظ البسط مشتقة المقام

$$\int \frac{\sec^2 x+\sec x \tan x}{\sec x+\tan x}\, dx$$

$$\ln | \sec x + \tan x \ | + c$$

$$18. \int \tan x \, dx = \int \frac{\sin x}{\cos x}\, dx = -\int -\frac{\sin x}{\cos x}\, dx$$

نلاحظ البسط مشتقة المقام

$$= -\ln | \cos x \ | + c$$

$$19. \int \cot x \, dx = \int \frac{\cos x}{\sin x} dx$$

نلاحظ البسط مشتقة المقام

$$\ln | \sin x \ | + c$$

$$20. \int \frac{\sin 2x}{\sqrt{3-\cos^4 x}} dx = \int \frac{\sin 2x}{\sqrt{3-(\cos^2 x)^2}} dx = \int \frac{2 \sin x \cos x}{\sqrt{3-(\cos^2 x)^2}} dx$$

$$y = \left(\frac{\cos^2 x}{\sqrt{3}}\right)$$

$$put \ \cos^2 x = \sqrt{3} \sin y$$

$$-2 \cos x \sin x \, dx = \sqrt{3} \ \cos y \, dy \ \rightarrow$$

$$-\int \frac{\sqrt{3} \ \cos y \ dy}{\sqrt{3-(\sqrt{3} \sin y)^2}} = -\frac{\sqrt{3}}{\sqrt{3}} \int \frac{\cos y \ dy}{\sqrt{1-\sin^2 y}} = -\int \frac{\cos y \ dy}{\cos y}$$

$$-\int dy = -y + c = - \sin^{-1}\left(\frac{\cos^2 x}{\sqrt{3}}\right) + c$$

$$22. \int \frac{e^{2x}}{e^{4x}+5} dx = \int \frac{e^{2x}}{(e^{2x})^2+5}\, dx$$

قاعدة $\int \frac{dy}{y^2+a^2} = \frac{1}{a} \tan^{-1} \frac{y}{a}$

$$y = e^{2x} \rightarrow dy = 2e^{2x} dx$$

$$\int \frac{dy}{y^2+5} = \frac{1}{\sqrt{5}} \tan^{-1} \frac{y}{\sqrt{5}} + \ c = \frac{1}{\sqrt{5}} \tan^{-1} \frac{e^{2x}}{\sqrt{5}} + \ c$$

$$21. \int \frac{e^{2x}}{e^{4x}-5} dx = \int \frac{e^{2x}}{(e^{2x}-\sqrt{5})\ (e^{2x}+\sqrt{5})} dx$$

Put $y = e^{2x}$

$$dy = 2e^{2x} \quad \rightarrow \frac{1}{2} dy = e^{2x} dx$$

$$\frac{1}{2}\int \frac{dy}{(y-\sqrt{5})\ (y+\sqrt{5})} = \quad \frac{A}{(y-\sqrt{5})} + \frac{B}{(y+\sqrt{5})}$$

$$\frac{A(y+\sqrt{5})+B(y-\sqrt{5})}{(y-\sqrt{5})\ (y+\sqrt{5})}$$

$$y = \sqrt{5} \rightarrow 1 = A2\sqrt{5} \quad \rightarrow \quad A = \frac{1}{2\sqrt{5}}$$

$$y = -\sqrt{5} \rightarrow 1 = -B2\sqrt{5} \rightarrow B = -\frac{1}{2\sqrt{5}}$$

$$\frac{1}{2} \int \frac{\frac{1}{2\sqrt{5}}}{(y-\sqrt{5})} + \frac{-\frac{1}{2\sqrt{5}}}{(y+\sqrt{5})}$$

$$\frac{1}{4\sqrt{5}} \int \frac{1}{(y-\sqrt{5})} - \frac{1}{4\sqrt{5}} \int \frac{1}{(y+\sqrt{5})}$$

$$\frac{1}{4\sqrt{5}} ln \mid (y-\sqrt{5}) \mid - \frac{1}{4\sqrt{5}} ln \mid (y+\sqrt{5}) + c$$

$$\frac{1}{4\sqrt{5}} (ln \mid (y-\sqrt{5}) \mid - ln \mid (y+\sqrt{5}) \mid)$$

$$\frac{1}{4\sqrt{5}} ln \mid \frac{(y-\sqrt{5})}{(y+\sqrt{5})} \mid + c$$

$$\frac{1}{4\sqrt{5}} ln \mid \frac{(e^{2x}-\sqrt{5})}{(e^{2x}+\sqrt{5})} \mid + c$$

ملاحظة / من خواص
اللوغاريتم الطرح يرجع
قسمة والجمع يرجع ضرب

23. $\int \frac{e^{\sqrt{2x-1}}}{\sqrt{2x-1}}\ dx$

$put\ \ y = \sqrt{2x-1}$

$dy = \frac{2}{2\sqrt{2x-1}}dx \quad \rightarrow \quad dy = \frac{dx}{\sqrt{2x-1}}$

$\int e^y\ \ dy = e^y + c \quad \rightarrow \ e^{\sqrt{2x-1}} + \ c$

---

24. $\int x^3\ \ (1-2x^4)^3 dx$

$y = \ 1 - 2x^4$

$dy = \ -8x^3 dx \rightarrow -\frac{1}{8}dy = x^3 dx$

$-\frac{1}{8}\int y^3\, dy = \ -\frac{1}{32}y^4 + c \ \rightarrow = \ -\frac{1}{32}(1-2x^4)^4 + c$

---

25. $\int sin(2-3x)\, dx$

$y = (2-3x)$

$dy = \ -3dx \quad \rightarrow \quad -\frac{1}{3}dy = dx$

$-\frac{1}{3}\int sin\, y\ \ dy = \ -\frac{1}{3} * \ -cos\, y + c \ \ \rightarrow \ +\frac{1}{3}cos(2-3x) + c$

---

26. $\int \frac{x\ dx}{\sqrt{1-x^4}}dx = \int \frac{x\ dx}{\sqrt{1-(x^2)^2}}dx$

$put\ x^2 = \ sin\, y$ $\qquad y = \ \sin^{-1} x^2$

$2x\, dx = cos\, y\ \ dy \quad \rightarrow x\, dx = \frac{1}{2}\ cos\, y\ \ dy$

$$\frac{1}{2}\int \frac{\cos y}{\sqrt{1-\cos^2 y}}\,dy = \frac{1}{2}\int \frac{\cos y}{\sqrt{\sin^2 y}}\,dx = \frac{1}{2}\int \frac{\cos y}{\cos y}\,dx$$

$$\frac{1}{2}\int dy = \frac{1}{2}\, y + c \rightarrow \frac{1}{2}\, \sin^{-1} x^2 + c$$

$$27.\int \frac{x\,dx}{\sqrt{x^4-1}}\,dx = \int \frac{x\,dx}{\sqrt{(x^2)^2-1}}\,dx$$

$$y = (\sec^{-1} x^2)$$

$$put\ x^2 = \sec y$$

$$2x\,dx = \sec y\ \tan y \rightarrow x\,dx = \frac{1}{2}\sec y\ \tan y$$

$$\frac{1}{2}\int \frac{\sec y\ \tan y}{\sqrt{\sec^2 y-1}}\,dx = \frac{1}{2}\int \frac{\sec y\ \tan y}{\sqrt{\tan^2 y}}\,dx = \frac{1}{2}\int \frac{\sec y\ \tan y}{\tan y}$$

$$= \frac{1}{2}\int \sec y\ dy == \frac{1}{2}\ \ln \mid \sec y + \tan y \mid + c$$

$$= \frac{1}{2}\ \ln \mid \sec(\sec^{-1} x^2) + \tan(\sec^{-1} x^2) \mid + c$$

ملاحظة / هناك حل اخر بتطبيق صيغة التكامل مباشرة اذا كان

$$\int \frac{dy}{\sqrt{y^2-a^2}} = \ln \mid y + \sqrt{y^2 - a^2} \mid + c$$

$$27.\int \frac{x\,dx}{\sqrt{x^4-1}} == \int \frac{x\,dx}{\sqrt{(x^2)^2-1}}\,dx$$

$$\boldsymbol{y} = x^2$$

$$dy = 2x\,dx \quad \rightarrow \frac{1}{2}dy = xdx$$

$$\frac{1}{2}\int \frac{dy}{\sqrt{y^2-1}} = \frac{1}{2}\ln \mid y + \sqrt{y^2 - 1} \mid + c$$

$$= \ln \mid x^2 + \sqrt{x^4 - 1} \mid + c$$

$$28. \int \frac{e^{\frac{x}{2}}}{\sqrt{16-e^x}} dx = \int \frac{e^{\frac{x}{2}}}{\sqrt{16-\left(e^{\frac{x}{2}}\right)^2}} dx$$

$$put\ y = e^{\frac{x}{2}}$$

$$dy = \frac{e^{\frac{x}{2}}}{2} dx \rightarrow 2dy = e^{\frac{x}{2}} dx$$

$$2 \int \frac{dy}{\sqrt{16-y^2}}$$

$$put\ y = 4 \sin t$$

$$dy = 4 \cos t\ dt$$

$$2 \int \frac{4 \cos t}{\sqrt{16-16 \sin^2 t}} dt = 2 \int \frac{4 \cos t}{4\sqrt{1-\sin^2 t}} dt$$

$$2 \int \frac{4 \cos t}{4\sqrt{1-\sin^2 t}} dt = 2 \int \frac{\cos t}{\sqrt{1-\sin^2 t}} dt$$

$$2 \int \frac{\cos t}{\sqrt{1-\sin^2 t}} dt = 2 \int \frac{\cos t}{\cos t} dt = 2 \int dt = 2t + c$$

$$2 \sin^{-1} \frac{y}{4} + c = 2 \sin^{-1} \left(\frac{e^{\frac{x}{2}}}{4} \right) + c$$

$$29. \int \frac{\sin 4x}{\cos^4 x + 4} dx = \int \frac{\sin 4x}{(\cos^2 2x)^2 + 4} dx = \int \frac{2 \sin 2x \cos 2x}{(\cos^2 2x)^2 + 4} dx$$

$$put\ \ y = \cos^2 2x \quad \rightarrow dy = 2 \times 2 \cos 2x \times -\sin 2x\ dx$$

$$-\frac{1}{2} dy = 2 \cos 2x\ \sin 2x\ dx$$

$$-\frac{1}{2}\int \frac{dy}{y^2+4} = \frac{1}{4}tan^{-1}\frac{y}{2} + c = \frac{1}{4}tan^{-1}\left(\frac{cos^2 2\,x}{2}\right) + c$$

30. $\int \frac{dx}{(x-7)\sqrt{x}} = \int \frac{dx}{(\sqrt{x}-\sqrt{7})(\sqrt{x}+\sqrt{7})\sqrt{x}}$

$put\ y = \sqrt{x}$

$dy = \frac{dx}{2\sqrt{x}} \rightarrow 2dy = \frac{dx}{\sqrt{x}}$

$2\int \frac{dy}{(y-\sqrt{7})(y+\sqrt{7})} = \frac{A}{(y-\sqrt{7})} + \frac{B}{(y+\sqrt{7})}$

$\frac{A(y+\sqrt{7})+B(y-\sqrt{7})}{(y-\sqrt{7})(y+\sqrt{7})}$

$y = \sqrt{7} \quad \rightarrow \quad 1 = A2\sqrt{7} \rightarrow A = \frac{1}{2\sqrt{7}}$

$y = -\sqrt{7} \rightarrow \quad 1 = -B2\sqrt{7} \rightarrow B = -\frac{1}{2\sqrt{7}}$

$2\int \frac{\frac{1}{2\sqrt{7}}}{(y-\sqrt{7})} + \frac{-\frac{1}{2\sqrt{7}}}{(y+\sqrt{7})}$

$= \frac{2}{2\sqrt{7}}\int \frac{dy}{(y-\sqrt{7})} - \frac{2}{2\sqrt{7}}\int \frac{dy}{(y+\sqrt{7})}$

$\frac{1}{\sqrt{7}} ln\,|(y-\sqrt{7})| - ln\,|(y+\sqrt{7})| + c$

$\frac{1}{\sqrt{7}} ln\left|\frac{(y-\sqrt{7})}{(y+\sqrt{7})}\right| + c = \frac{1}{\sqrt{7}} ln\left|\frac{(\sqrt{x}-\sqrt{7})}{(\sqrt{x}+\sqrt{7})}\right| + c$

$$31. \int \frac{\sqrt{2-x^2}+\sqrt{2+x^2}}{\sqrt{4-x^4}} dx = \int \frac{\sqrt{2-x^2}+\sqrt{2+x^2}}{\sqrt{(2-x^2)(2+x^2)}} dx$$

$$\int \frac{\sqrt{2-x^2}}{\sqrt{2-x^2} \times \sqrt{2+x^2}} dx + \int \frac{\sqrt{2+x^2}}{\sqrt{2-x^2} \times \sqrt{2+x^2}} dx$$

$$\int \frac{dx}{\sqrt{2+x^2}} + \int \frac{dx}{\sqrt{2-x^2}}$$

$\int \frac{dx}{\sqrt{2+x^2}} = ln \mid x + \sqrt{2+x^2} \mid$ هنا تطبيق صيغة

$\int \frac{dx}{\sqrt{2-x^2}} = sin^{-1} \frac{x}{\sqrt{2}}$ هنا تطبيق الصيغة العكسية

$$= ln \mid x + \sqrt{2+x^2} \mid + sin^{-1} \frac{x}{\sqrt{2}} + c$$

$$32. \int \frac{5x+3}{\sqrt{3-x^2}} dx = \int \frac{5x}{\sqrt{3-x^2}} dx + \int \frac{3}{\sqrt{3-x^2}} dx$$

$-\frac{5}{2} \int \frac{-2x}{\sqrt{3-x^2}} dx = -\frac{5}{2} \int -2x \ (3-x^2)^{\frac{-1}{2}} dx$ دالة × مشتقتها

$$= -\frac{5}{2} \times 2(3-x^2)^{\frac{1}{2}} += -5\sqrt{3-x^2} +$$

$\int \frac{3}{\sqrt{3-x^2}} dx = 3\, sin^{-1} \frac{x}{\sqrt{3}} + c$ هنا تطبيق الصيغة العكسية

$$\int \frac{5x+3}{\sqrt{3-x^2}} dx = -5\sqrt{3-x^2} + 3\, sin^{-1} \frac{x}{\sqrt{3}} + c$$

رأي / أحيانا تكون طريقتنا في التعامل مع الأخطاء أكبر من الخطأ نفسه ..

$33.\int \frac{x^2}{\sqrt{2-3x^3}}dx \qquad put\ y = 2-3x^3 \ \rightarrow dy = -9\,x^2 dx$

$-\frac{1}{9}dy = x^2 dx$

$-\frac{1}{9}\int \frac{dy}{y^{\frac{1}{2}}} = -\frac{1}{9}\int y^{\frac{-1}{2}}dy = -\frac{2}{9}y^{\frac{1}{2}} = -\frac{2}{9}\sqrt{2-3x^3}+c$

$34.\int x\sqrt{x-5}\,dx$

$put\ y^2 = x-5 \quad \rightarrow 2ydy = dx \qquad\qquad x = y^2+5$

$2\int (y^2+5)\ y^2 dy = \int y^4+5y^2\ \ dy = \frac{2}{5}y^5+\frac{10}{3}y^3+c$

$\frac{2}{5}\sqrt{(x-5)^5}+\frac{10}{3}\sqrt{(x-5)^3}+c$

$35.\int \frac{dx}{a^2\sin^2 x+b^2\cos^2 x} = \int \frac{\frac{1}{\cos^2 x}}{a^2\tan^2 x+b^2}dx$

$\int \frac{\sec^2 x}{a^2\tan^2 x+b^2}dx$

$put\ y = \tan x \quad \rightarrow dy = \sec^2 x\,dx$

$\int \frac{dy}{a^2y^2+b^2}dy \rightarrow put\ y = \frac{b}{a}\tan t \quad \rightarrow dy = \frac{b}{a}\sec^2 t\,dt$

$\int \frac{\frac{b}{a}\sec^2 t}{a^2\frac{b^2}{a^2}\tan^2 t+b^2}dt = \int \frac{\frac{b}{a}\sec^2 t}{b^2\tan^2 t+b^2}dt$

$\frac{1}{ab}\int \frac{\sec^2 t}{\tan^2 t+1}dt = \frac{1}{ab}\int \frac{\sec^2 t}{\sec^2 t}dt = \frac{1}{ab}\int dt = \frac{1}{ab}t+c$

$$= \frac{1}{ab} tan^{-1}\left(\frac{a}{b} tan\, x\right) + c$$

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36 $\int \frac{dx}{(cos^{-1} x)^5 \sqrt{1-x^2}}$ put $y = cos^{-1} x \rightarrow dy = \frac{-dx}{\sqrt{1-x^2}}$

$$-\int \frac{dy}{y^5} = y^{-5}dy = -\frac{1}{4} y^{-4} + c = \frac{1}{4y^4} + c$$

$$= \frac{1}{4(cos^{-1} x)^4} + c$$

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37. $\int \frac{sin\ 2x}{1+sin^2 x} dx = \int \frac{2\, sin\ x\, cos\ x}{1+sin^2 x} dx$ البسط مشتقة للمقام

$$= ln\ |\ 1 + sin^2 x\ \ | + c$$

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38. $\int \frac{\sqrt[3]{1+ln\ x}}{x}\ dx$ put $y = 1 + ln\, x \rightarrow dy = \frac{dx}{x}$

$$\int y^{\frac{1}{3}} dy = \frac{3}{4} y^{\frac{4}{3}} + c = \frac{3}{4} \sqrt[3]{(1 + ln\, x)^4} + c$$

---

39. $\int cos^5 x \sqrt{sin\, x}\, dx = \int cos\, x\, (cos^2 x)^2 \sqrt{sin\, x}\, dx$

$$\int cos\, x\, (1 - sin^2 x)^2 \sqrt{sin\, x}\, dx$$

put $y = sin\, x \rightarrow dy = cos\, x\, dx$

$$\int (1 - y^2)^2\, (y)^{\frac{1}{2}} dy = (\int 1 - 2y^2 + y^4)(y)^{\frac{1}{2}} dy$$

$$\int (y)^{\frac{1}{2}} - 2(y)^{\frac{5}{2}} + (y)^{\frac{9}{2}}\, dy = \frac{2}{3}(y)^{\frac{3}{2}} - 2 \times \frac{2}{7}(y)^{\frac{7}{2}} + \frac{2}{11}(y)^{\frac{11}{2}}$$

$\frac{2}{3}(\sin x)^{\frac{3}{2}} - 2 \times \frac{2}{7}(\sin x)^{\frac{7}{2}} + \frac{2}{11}(\sin x)^{\frac{11}{2}}$

$\left(\frac{2}{3} - \frac{4}{7}\sin^2 x + \frac{2}{11}\sin^4 x\right)\sqrt{\sin^3 x} + c$ هنا استخرجنا عامل مشترك

$40. \int \frac{x^3}{2+x^8}dx = \int \frac{x^3}{2+(x^4)^2}dx$

$put\ y = x^4 \quad \to \quad dy = 4x^3 dx \quad \to \frac{dy}{4} = x^3 dx$

$\frac{1}{4}\int \frac{dy}{2+y^2} = \frac{1}{\sqrt{2}}\tan^{-1}\frac{y}{\sqrt{2}}$ هنا تطبيق الصيغة العكسية

$\frac{1}{4\sqrt{2}}\tan^{-1}\frac{x^4}{\sqrt{2}} + c$

$41. \int \frac{x}{\sqrt{1+x^2+\sqrt{(1+x^2)^3}}}dx$

$= \int \frac{x}{\sqrt{1+x^2+(1+x^2)\sqrt{1+x^2}}}dx$

$= \int \frac{x}{\sqrt{1+x^2}\sqrt{1+\sqrt{1+x^2}}}dx$

$put\ y = 1 + x^2 \quad \to dy = 2x\,dx \to \frac{dy}{2} = x\,dx$

$\frac{1}{2}\int \frac{dy}{\sqrt{y}\sqrt{1+\sqrt{y}}} \quad put\ t = \sqrt{y} \ \to dt = \frac{dy}{2\sqrt{y}}$

$2dt = \frac{dy}{\sqrt{y}} \ \to \frac{1}{2}\int \frac{2dt}{\sqrt{1+t}} = \int \frac{dt}{\sqrt{1+t}}$

$$\int (1+t)^{\frac{-1}{2}}\,dt = 2(1+t)^{\frac{1}{2}} = 2\sqrt{1+t}+c$$

$$2\sqrt{1+t}+c = 2\sqrt{1+\sqrt{y}}+c = 2\sqrt{1+\sqrt{1+x^2}}+c$$

$$42.\ \int \frac{\cos x}{\sqrt{2+\cos 2x}}\,dx = \int \frac{\cos x}{\sqrt{2+(1-\sin^2 x)}}\,dx$$

$$\int \frac{\cos x}{\sqrt{3-2\sin^2 x}}\,dx \quad \rightarrow y = \sin x \quad \rightarrow dy = \cos x\,dx$$

$$\int \frac{dy}{\sqrt{3-2y^2}}\,dx \quad \rightarrow put\ y = \frac{\sqrt{3}}{\sqrt{2}}\sin t \rightarrow dy = \frac{\sqrt{3}}{\sqrt{2}}\cos t\ dt$$

$$\int \frac{\frac{\sqrt{3}}{\sqrt{2}}\cos t}{\sqrt{3-2\,\frac{3}{2}\sin^2 t}}\,dt = \int \frac{\frac{\sqrt{3}}{\sqrt{2}}\cos t}{\sqrt{3-3\sin^2 t}}\ dt$$

$$\frac{\sqrt{3}}{\sqrt{2}\times\sqrt{3}}\int \frac{\cos t}{\sqrt{1-\sin^2 t}}\,dt = \frac{1}{\sqrt{3}}\int \frac{\cos t}{\cos t}\,dt = \frac{1}{\sqrt{3}}\int dt$$

$$\frac{1}{\sqrt{3}}t + c = \frac{1}{\sqrt{3}}\sin^{-1}\frac{\sqrt{2}\,y}{\sqrt{3}} = \frac{1}{\sqrt{3}}\sin^{-1}\frac{\sqrt{2}\,(\sin x)}{\sqrt{3}} + c$$

$$43.\ \int \frac{\sin x\ \cos x}{\sin^4 x+\cos^4 x}\ dx\ \ (\cos^4 x)$$ نقسم ونضرب

$$\int \frac{\frac{\sin x\ \cos x}{\cos^4 x}}{\frac{\sin^4 x}{\cos^4 x}+\frac{\cos^4 x}{\cos^4 x}}\ dx = \int \frac{\tan x\ \sec^2 x}{\tan^4 x+1}\ dx$$

$$put\ y = \tan^2 x$$

$$dy = 2\tan x \sec^2 x \, dx \quad \rightarrow \frac{dy}{2} = \tan x \sec^2 x \, dx$$

$$\frac{1}{2}\int \frac{dy}{y^2+1} = \frac{1}{2}\tan^{-1} y + c$$

$$\frac{1}{2}\tan^{-1}(\tan^2 x) + c$$

$$44. \int \frac{\sin x+\cos x}{\sqrt[3]{\sin x-\cos x}}\, dx$$

$$put\ y = \sin x - \cos x$$

$$dy = (\sin x + \cos x)dx$$

$$\int \frac{dy}{\sqrt[3]{y}} = \int (y)^{\frac{-1}{3}} dy = \frac{3}{2}(y)^{\frac{2}{3}} + c$$

$$\frac{3}{2}(\sin x - \cos x)^{\frac{2}{3}} + c = \frac{3}{2}\sqrt[3]{(\sin x - \cos x)^2} + c$$

45. $\int x \ln x \, dx$ $u = \ln x$ $dv = x$

$du = \frac{dx}{x}$ $v = \frac{x^2}{2}$

$$\frac{x^2}{2}\ln x - \int \frac{x^2}{2} \times \frac{dx}{x} = \frac{x^2}{2}\ln x - \int \frac{x}{2} dx$$

$$= \frac{x^2}{2}\ln x - \frac{x^2}{4} + c$$

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46. $\int x^3 \ln x \, dx$

$u = \ln x$ $dv = x^3$

$du = \frac{dx}{x}$ $v = \frac{x^4}{4}$

$$= \frac{x^4}{4}\ln x - \frac{1}{4}\int \frac{x^4}{x} dx == \frac{x^4}{4}\ln x - \frac{1}{4}\int x^3 \, dx$$

$$= \frac{x^4}{4}\ln x - \frac{1}{16}x^4 + c$$

---

47. $\int x e^x \, dx$

$u = x$ $dv = e^x$

$du = dx$ $v = e^x$

$$xe^x - \int e^x dx = xe^x - e^x + c = e^x(x-1) + c$$

---

48. $\int (x+1) e^x dx$

$u = x + 1$ $dv = e^x$

$du = dx$ $v = e^x$

$$(x+1)e^x - \int e^x \, dx$$

$$(x+1)e^x - e^x + c = e^x(x+1-1) = xe^x + c$$

$49. \int (x^2 - 2x + 5)\, e^{-x} dx$

$u = x^2 - 2x + 5 \qquad dv = e^{-x}$

$du = 2x - \qquad v = -e^{-x}$

2

$u = 2x - 2 \qquad dv = e^{-x}$

$du = 2dx \qquad v = -e^{-x}$

$(x^2 - 2x + 5) - e^{x}$

$+ \int (2x - 2)\, e^{x} dx$

$-(2x - 2)e^{x} + 2 \int e^{x} dx$

$-(2x - 2)e^{x} + 2e^{x}$

$(x^2 - 2x + 5) - e^{x} \; - (2x - 2)e^{x} + 2e^{x}$

$-e^{x}(x^2 - 2x + 5 - 2x + 2 + 2) =$

$50. \int tan^{-1} x\, dx$

$u = \tan^{-1} x \qquad dv = dx$

$du = \frac{dx}{1+x^2} \qquad v = x$

$x\, tan^{-1}\, x \; - \int \frac{x\, dx}{1+x^2}$

$- \int \frac{x\, dx}{1+x^2} = -\frac{1}{2} \int \frac{2x}{1+x^2} dx = -\frac{1}{2} ln \mid 1 + x^2 \mid + c$

$x\, tan^{-1}\, x - \frac{1}{2} ln \mid 1 + x^2 \mid + c$

$51. \int x\, tan^{-1}\, x\, dx$

$u = \tan^{-1} x \qquad dv = x$

$du = \frac{dx}{1+x^2} \qquad v = \frac{x^2}{2}$

$\frac{x^2\, tan^{-1}\, x}{2} - \frac{1}{2} \int \frac{x^2}{1+x^2} dx$

$-\frac{1}{2} \int \frac{x^2}{1+x^2} dx = -\frac{1}{2} \int \frac{x^2+1-1}{1+x^2} dx = -\frac{1}{2} \int dx - \frac{1}{1+x^2}$

$-\frac{1}{2} \int dx - \frac{1}{1+x^2} == -\frac{1}{2} [x - \tan^{-1} x] = -\frac{1}{2} x + \frac{1}{2} \tan^{-1} x$

$$= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2}x + \frac{1}{2}\tan^{-1} x = \left(\frac{x^2+1}{2}\right)\tan^{-1} x - \frac{x}{2} + c$$

52. $\int x^2 \tan^{-1} x \, dx$

$u = \tan^{-1} x \qquad dv = x^2$

$du = \frac{dx}{1+x^2} \qquad v = \frac{x^3}{3}$

$$\frac{x^3 \tan^{-1} x}{3} - \frac{1}{3}\int \frac{x^3}{1+x^2}dx$$

$-\frac{1}{3}\int \frac{x^3}{1+x^2}dx$ (نقسم قسمة مطولة) البسط اكبر من درجة المقام

$$-\frac{1}{3}\int x dx - \frac{1}{3}\int \frac{-x}{1+x^2}dx = -\frac{x^2}{6} + \frac{1}{3}\times\frac{1}{2}\int \frac{2x}{1+x^2}dx$$

$$= \frac{x^3}{3}\tan^{-1} x - \frac{x^2}{6} + \frac{1}{6}\ln|1+x^2|$$

53. $\int x^3 \tan^{-1} x \, dx$

$u = \tan^{-1} x \qquad dv = x^3$

$du = \frac{dx}{1+x^2} \qquad v = \frac{x^4}{4}$

$$\frac{x^4 \tan^{-1} x}{4} - \frac{1}{4}\int \frac{x^4}{1+x^2}dx$$

$-\frac{1}{4}\int \frac{x^4}{1+x^2}dx$ (نقسم قسمة مطولة) البسط اكبر من درجة المقام

$$-\frac{1}{4}\int (x^2-1) + \frac{1}{1+x^2} = -\frac{1}{4}\left[\frac{x^3}{3} - x + \tan^{-1} x\right] + c$$

$$= \frac{x^4 \tan^{-1} x}{4} - \frac{1}{12}x^3 + \frac{1}{4}x - \frac{1}{4}\tan^{-1} x$$

$$= \frac{(x^4-1)}{4}\tan^{-1} x - \frac{1}{12}x^3 + \frac{1}{4}x + c$$

ملاحظة / استخرجنا هنا عامل مشترك وهو $\tan^{-1} x$

54. $\int (3x^2 + 6x + 5)\, tan^{-1} x\, dx$

$u = \tan^{-1} x$ $dv = 3x^2 + 6x + 5$

$du = \frac{dx}{1+x^2}$ $du = x^3 + 3x^2 + 5x$

$= (x^3 + 3x^2 + 5x) tan^{-1} x - \int \frac{x^3+3x^2+5x}{1+x^2} dx$

البسط اكبر من المقام نقسم قسمة مطولة

$-\int (x+3) + \frac{4x-3}{1+x^2} dx = -\frac{x^2}{2} - 3x - \int \frac{4x-3}{1+x^2} dx$

$-\int \frac{4x-3}{1+x^2} dx = -\int \frac{4x}{1+x^2} dx + \int \frac{3}{1+x^2} dx$

$-2\, ln\, |1 + x^2| + 3\, tan^{-1} x$

$= (x^3 + 3x^2 + 5x) tan^{-1} x - 2\, ln\, |1 + x^2| + 3\, tan^{-1} x$

$= (x^3 + 3x^2 + 5x + 3) tan^{-1} x - 2\, ln\, |1 + x^2| + c$

55. $\int sin^{-1} x\, dx$

$u = \sin^{-1} x$ $dv = dx$

$du = \frac{dx}{\sqrt{1-x^2}}$ $v = x$

$= x sin^{-1} - \int \frac{x}{\sqrt{1-x^2}} dx$

$-\int \frac{x}{\sqrt{1-x^2}} dx = \frac{1}{2} \int -2x(1-x^2)^{\frac{-1}{2}}$

نضرب ونقسم في 2- لنجعلها صورة دالة في مشتقتها

$= (1-x^2)^{\frac{1}{2}} = \sqrt{1-x^2}$

$= x sin^{-1} + \sqrt{1-x^2} + c$

56. $\int x^5 e^{x^2} dx = \int e^{x^2} . (x^2)^2 . x dx$

$put\ y = x^2 \quad \to dy = 2xdx \quad \to \frac{dy}{2} = xdx$

$\frac{1}{2} \int e^y . y^2 dy$

$u = y^2 \qquad dv = e^y$

$du = 2ydy \qquad v = e^y$

$(y^2 e^y) - 2 \int y e^y\, dy$

$-2(ye^y) + \int e^y\, dy = -2(ye^y) + 2e^y$

$u = y \qquad dv = e^y$

$du = dy \qquad v = e^y$

الحل النهائي

$= \frac{1}{2}[(y^2 e^y) - 2(ye^y) + 2e^y]$

نعوض بقيمة ال(y)

$= \frac{1}{2}\left[\left(x^4 e^{x^2}\right) - 2\left(x^2 e^{x^2}\right) + 2e^{x^2}\right]$

$= \frac{1}{2} e^{x^2}[x^4 - 2x^2 + 2] + c$

---

57. $\int (x^2 + 2x + 3) \cos x\, dx$

$u = (x^2 + 2x + 3) \qquad dv = \cos x$

$du = 2x + 2dx \qquad v = \sin x$

$(x^2 + 2x + 3) \sin x$

$- \int (2x + 2) \sin x\, dx$

$u = (2x + 2) \qquad dv = \sin x$

$du = 2dx \qquad v = -\cos x$

$= (2x + 2) \cos x - 2 \int \cos x\, dx$

$= (2x+2)\sin x - 2\sin x$

$= (x^2+2x+3)\sin x + (2x+2)\cos x - 2\sin x + c$

$= (x^2+2x+3-2)\sin x + (2x+2)\cos x + c$

$(x^2+2x+1)\sin x + (2x+2)\cos x + c$

$= (x+1)^2 \sin x + (2x+2)\cos x + c$

1- عامل مشترك

2- مربع كامل (x^2+2x+1)

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58. $\int e^{2x}\cos x\,dx = i$

$i = e^{2x}(\sin x) - 2\int e^{2x}\sin x\,dx$

$u = e^{2x}$ $\quad dv = \cos x$

$du = 2e^{2x}dx$ $\quad v = \sin x$

$i = 2e^{2x}\cos x - 4\int e^{2x}\cos x\,dx$

$u = e^{2x}$ $\quad dv = \sin x$

$du = 2e^{2x}dx$ $\quad v = -\cos x$

$5i = e^{2x}(\sin x) + 2e^{2x}\cos x$

$i = \frac{1}{5}e^{2x}(\sin x + 2\cos x) + c$

---

59. $\int \sin(\ln x)\,dx$

نأخذ (e) للطرفين

$put\ y = \ln x \ \rightarrow e^y = x \ \rightarrow e^y dy = dx$

$\int e^y.\sin y\,dy = i$

$u = e^y$ $\quad dv = \sin y$

$du = e^y dy$ $\quad v = -\cos y$

$i = -e^y(\cos y) + \int e^y\cos y$

$u = e^y$ $\quad dv = \cos y$

$du = e^y dy$ $\quad v = \sin y$

$i = e^y.\sin y - \int e^y.\sin y\,dy$

$2i = -e^y(\cos y) + e^y.\sin y$

$$i = \frac{1}{2} - e^{y}(cos\, y) + e^{y}.sin\, y + c$$

$$i = \frac{1}{2}e^{y}(sin\, y - cos\, y) + c$$

نأخذ عامل مشترك

اللوغاريتم مع e نختصره

$$i = \frac{1}{2}e^{ln\, x}(sin(ln\, x) - cos(ln\, x)) + c$$

$$i = \frac{1}{2}x(sin(ln\, x) - cos(ln\, x)) + c$$

$60. \int cos(ln\, x)\, dx$

$put\ y = ln\, x \ \rightarrow e^{y} = x \ \rightarrow e^{y}dy = dx$

$u = e^{y}$	$dv = \cos y$
$du = e^{y}dy$	$v = \sin y$

$\int e^{y}.cos\, y\, dy = i$

$u = e^{y}$	$dv = \sin y$
$du = e^{y}dy$	$v = -\cos y$

$i = e^{y}.sin\, y - \int e^{y}.sin\, y\, dy$

$i = e^{y}(cos\, y) - \int e^{y}\ cos\, y$

$2i = e^{y}.sin\, y + e^{y}(cos\, y)$

$i = \frac{1}{2}e^{ln\, x}(sin\, y + cos\, y) + c$

$i = \frac{x}{2}e^{y}(sin(ln\, x) + cos(ln\, x)) + c$

$61. \int sin\sqrt{x}\, dx\ put\ \ y = \sqrt{x}\ \rightarrow y^{2} = x\ \rightarrow 2ydy = dx$

$2\int y.sin\, y\, dy \qquad put\ \ y = \sqrt{x}\ \rightarrow y^{2} = x\ \rightarrow 2ydy = dx$

$u = y$	$dv = sin\, y$
$du = dy$	$v = -cos\, y$

$= 2[(-y\cos y) + \int \cos y dy]$

$= -2y\cos y + 2\sin y + c$

$= -2\sqrt{x}\cos\sqrt{x} + 2\sin\sqrt{x} + c$

---

62. $\int (x^2 - 2x + 3)\, \ln x\, dx$

$u = \ln x \qquad dv = x^2 - 2x + 3$

$du = \frac{dx}{x} \qquad v = \frac{x^3}{3} - x^2 + 3x$

$= \left(\frac{x^3}{3} - x^2 + 3x\right)\ln x - \int \frac{1}{x} \times \left(\frac{x^3}{3} - x^2 + 3x\right) dx$

$= \left(\frac{x^3}{3} - x^2 + 3x\right)\ln x - \int \left(\frac{x^2}{3} - x + 3\right) dx$

$= \left(\frac{x^3}{3} - x^2 + 3x\right)\ln x - \frac{x^3}{9} + \frac{x^2}{2} - 3x + c$

---

63. $\int (x^2 + 2x - 1)\, \sin 3x\, dx$

$u = (x^2 + 2x - 1) \qquad dv = \sin 3x$

$du = (2x + 2)dx \qquad v = -\frac{1}{3}\cos 3x$

$= -\frac{1}{3}\cos(3x)\,(x^2 + 2x - 1) + \frac{1}{3}\int (2x + 2)\cos 3x dx$

$u = (2x + 2) \qquad dv = \cos 3x$

$du = 2dx \qquad v = \frac{1}{3}\sin 3x$

ملاحظة / الحل إلى المنطقة المحددة يكفي وإذا أنت تريد تبسيط استخرج عامل مشترك

$$=\frac{1}{3}\left[\frac{1}{3}\sin 3x(2x+2)-\frac{2}{3}\int \sin 3x\ dx\right]$$

$$=\frac{1}{9}\sin 3x(2x+2)-\frac{2}{9}\times-\frac{1}{3}\cos 3x$$

$$=-\frac{1}{3}\cos 3x\,(x^2+2x-1)+\frac{1}{9}\sin 3x(2x+2)+\frac{2}{27}\cos 3x$$

$$=-\frac{1}{3}\cos(3x)\,(x^2+2x-1)+\frac{2}{9}\sin(3x)(x+1)+\frac{2}{27}\cos(3x)$$

استخرجنا عامل

$$-\frac{1}{27}(9x^2+18x-11)\cos 3x+\frac{2}{9}(x+1)\sin(3x)+c$$

$$64.\int(1+x^2)^2\cos x=(1+2x^2+x^4)\cos x\,dx=i$$

$u=(1+2x^2+x^4)$	$dv=\cos x$
$du=4x^3+4x$	$v=\sin x$

$$=(1+2x^2+x^4)\sin x-\int(4x^3+4x)\sin x\,dx$$

$u=(4x^3+4x)$	$dv=\sin x$
$du=12x^2+4$	$v=-\cos x$

$$-[(4x^3+4x)-\cos x+\int(12x^2+4)\cos x\ dx]$$

$$=(4x^3+4x)\cos x-\int(12x^2+4)\cos x\ dx$$

$u=(12x^2+4)$	$dv=\cos x$
$du=24x$	$v=\sin x$

$= -[(12x^2 + 4)\sin x - 24\int x \sin x \ \ dx]$

$= -(12x^2 + 4)\sin x + 24\int x \sin x \ \ dx$

$u = (x)$	$dv = \sin x$
$du = dx$	$v = -\cos x$

استخرجنا عامل مشترك

$24[-x\cos x + \int \cos x \ dx] = -24x\cos x + 24\sin x$

$= (1 + 2x^2 + x^4)\sin x + (4x^3 + 4x)\cos x - (12x^2 + 4)\sin x - 24x\cos x + 24\sin x$

$= (x^4 - 10x^2 + 21)\sin x + x(4x^2 - 20)\cos x + c$

---

65. $\int (x^3 - 2x^2 + 5)e^{3x} \ dx$

$u = (x^3 - 2x^2 + 5)$	$dv = e^{3x}$
$du = (3x^2 - 4x)dx$	$v = \frac{1}{3}e^{3x}$

$= \frac{1}{3}(x^3 - 2x^2 + 5)e^{3x} - \frac{1}{3}\int (3x^2 - 4x)\, e^{3x} dx$

$u = (3x^2 - 4x)$	$dv = e^{3x}$
$du = (6x - 4)dx$	$v = \frac{1}{3}e^{3x}$

$= -\frac{1}{3}\left[(3x^2 - 4x) \times \frac{1}{3}e^{3x} - \frac{1}{3}\int (6x - 4)e^{3x} \ dx\right]$

$= -\frac{1}{9}(3x^2 - 4x)e^{3x} + \frac{1}{9}\int (6x - 4)e^{3x} \ dx$

$u = (6x-4)$	$dv = e^{3x}$
$du = 6dx$	$v = \frac{1}{3}e^{3x}$

$$= \frac{1}{9}\left[\frac{1}{3}(6x-4)e^{3x} - \frac{6}{3}\int e^{3x}dx\right] = \frac{1}{27}(6x-4)e^{3x} - \frac{2}{9}\times\frac{1}{3}e^{3x}$$

$$= \frac{1}{27}(6x-4)e^{3x} - \frac{2}{27}e^{3x}$$

$$= \frac{1}{3}(x^3-2x^2+5)e^{3x} - \frac{1}{9}(3x^2-4x)e^{3x} + \frac{1}{27}(6x-4)e^{3x} - \frac{2}{27}e^{3x}$$

$$= \left(\frac{1}{3}x^3 - \frac{2}{3}x^2 + \frac{5}{3} - \frac{1}{3}x^2 + \frac{4}{9}x + \frac{2}{9}x - \frac{4}{27} - \frac{2}{27}\right)e^{3x}$$

$$= \left(\frac{1}{3}x^3 - x^2 + \frac{2}{3} + \frac{13}{9}\right)e^{3x} + c$$

$$66.\ \int \frac{x\cos x}{\sin^3 x}dx$$

$u = x$	$dv = (\sin x)^{-3}\cos x$ دالة × مشتقتها
$du = dx$	$v = \frac{-1}{2\sin^2 x}$

$$= \frac{-x}{2\sin^2 x} + \int \frac{1}{2\sin^2 x}dx = \frac{-x}{2\sin^2 x} + \frac{1}{2}\int \csc^2 x\,dx$$

$$= \frac{-x}{2\sin^2 x} - \frac{1}{2}\cot x +$$

$$= -\frac{1}{2}\left(\frac{x}{\sin^2 x} + \cot x\right) + c$$

$$67.\ \int \frac{\sin^{-1} x}{\sqrt{1+x}}dx$$

$u = \sin^{-1} x$	$dv = (1+x)^{\frac{-1}{2}}$ دالة × مشتقتها
$du = \frac{1}{\sqrt{1-x^2}}dx$	$v = 2\sqrt{1+x}$

$$= \left(2\sqrt{1-x^2}\right)\ sin^{-1} x - 2\int \frac{\sqrt{1-x}}{\sqrt{1-x^2}}\,dx$$

$$-2\int \frac{\sqrt{1-x}}{\sqrt{1-x^2}}\,dx = -2\int \frac{\sqrt{1+x}}{\sqrt{1-x}\times\sqrt{1+x}}\,dx$$

$2\int -(1-x)^{\frac{-1}{2}}\,dx$ مشتقتها × دالة $= 4\sqrt{1+x}$

$$= \left(2\sqrt{1+x}\right)\ sin^{-1} x + 4\sqrt{1+x} + c$$

68. $\int \sqrt[3]{x}\,(ln\,x)^2 dx$

$u = (\ln x)^2$	$dv = (x)^{\frac{1}{3}}$ مشتقتها × دالة
$du = \frac{2\ln x}{x}\,dx$	$v = \frac{3}{4}(x)^{\frac{4}{3}}$

$$= \frac{3}{4}(x)^{\frac{4}{3}}(ln\,x)^2 - \frac{3}{4}\times 2\int (ln\,x)\frac{(x)^{\frac{4}{3}}}{x}\,dx$$

$-\frac{3}{2}\int (ln\,x)(x)^{\frac{1}{3}}\,dx$

$u = \ln x$	$dv = (x)^{\frac{1}{3}}$ مشتقتها × دالة
$du = \frac{1}{x}\,dx$	$v = \frac{3}{4}(x)^{\frac{4}{3}}$

$$= -\frac{3}{2}\left[\frac{3}{4}(x)^{\frac{4}{3}}(ln\,x) - \frac{3}{4}\int \frac{(x)^{\frac{4}{3}}}{x}\,dx\right]$$

$$= -\frac{9}{8}(x)^{\frac{4}{3}}(ln\,x) + \frac{9}{8}\int (x)^{\frac{1}{3}}\,dx = \frac{9}{8}\times\frac{3}{4}(x)^{\frac{4}{3}}$$

$$= \frac{3}{4}(x)^{\frac{4}{3}}(ln\,x)^2 - \frac{9}{8}(x)^{\frac{4}{3}}(ln\,x) + \frac{27}{32}(x)^{\frac{4}{3}} + c$$

استخرجناعامل مشترك

$$= \frac{3}{4}\sqrt[3]{x^4}\left((\ln x)^2 - \frac{3}{2}(\ln x) + \frac{9}{8}\right) + c$$

69. $\int ln\left(x+\sqrt{1+x^2}\right)dx$

$u=\ln\left(x+\sqrt{1+x^2}\right)dx \qquad dv=dx$

$$du=\frac{1+\frac{2x}{2\sqrt{1+x^2}}}{x+\sqrt{1+x^2}}=\frac{\frac{x+\sqrt{1+x^2}}{\sqrt{1+x^2}}}{x+\sqrt{1+x^2}}$$

$du=\frac{dx}{\sqrt{1+x^2}} \qquad v=x$

$$=x\,ln\left(x+\sqrt{1+x^2}\right)-\int\frac{x}{\sqrt{1+x^2}}dx$$

$$-\frac{1}{2}\int 2x(1+x^2)^{\frac{-1}{2}}dx=-\frac{1}{2}\times 2\sqrt{1+x^2}$$

نضرب ونقسم في 2-

نجعلها دالة في مشتقتها

$$=x\,ln\left(x+\sqrt{1+x^2}\right)-\sqrt{1+x^2}+c$$

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70. $\int(sin^{-1}x)^2dx$

$u=(\sin^{-1}x)^2 \qquad dv=dx$

$du=\frac{2\sin^{-1}x}{\sqrt{1-x^2}}dx \qquad v=x$

$$=x(sin^{-1}x)^2-2\int\frac{x\,sin^{-1}x}{\sqrt{1-x^2}}$$

$u=\sin^{-1}x \qquad dv=x(1-x^2)^{\frac{-1}{2}}$

$du=\frac{dx}{\sqrt{1-x^2}} \qquad v=-\sqrt{1-x^2}$

$$=-2\left[-\sqrt{1-x^2}\ sin^{-1}x+\int\frac{\sqrt{1-x^2}}{\sqrt{1-x^2}}dx\right]$$

$$=x(sin^{-1}x)^2+2\sqrt{1-x^2}\ sin^{-1}x-2x+c$$

71. $\int x(\tan^{-1} x)^2\, dx$

$u = (\tan^{-1} x)^2$	$dv = x$
$du = \frac{2\tan^{-1} x}{1+x^2}dx$	$v = \frac{x^2}{2}$

$$= \frac{x^2}{2}(\tan^{-1} x)^2$$

$$-\frac{2}{2}\int \frac{x^2 \tan^{-1} x}{1+x^2}dx = -\int \frac{x^2 \tan^{-1} x}{1+x^2}dx$$

$u = \tan^{-1} x$	$dv = \frac{x^2}{1+x^2} = \frac{x^2+1-1}{1+x^2}$
$du = \frac{dx}{1+x^2}$	$v = x - \tan^{-1} x$

نضيف واحد ونطرح واحد ونوزع المقام

$$= -(x - \tan^{-1} x)\tan^{-1} x + \int \frac{x - \tan^{-1} x}{1+x^2}dx$$

نوزع المقام

$$\int \frac{x}{1+x^2}dx - \int \frac{(\tan^{-1} x)}{1+x^2}dx$$

$$\frac{1}{2}\int \frac{2x}{1+x^2}dx - \int \frac{(\tan^{-1} x)}{1+x^2}dx$$

نضرب ونقسم في 2 نجعلها دالة في مشتقتها

$\tan^{-1} x$ صورة دالة في مشتقتها

$$= \frac{1}{2}\ln|1+x^2| - \frac{(\tan^{-1} x)^2}{2}$$

$$= \frac{x^2}{2}(\tan^{-1} x)^2 - (x - \tan^{-1} x)\tan^{-1} x + \frac{1}{2}\ln|1+x^2| - \frac{(\tan^{-1} x)^2}{2}$$

$$= \frac{x^2}{2}(\tan^{-1} x)^2 - x\tan^{-1} x + (\tan^{-1} x)^2 - \frac{(\tan^{-1} x)^2}{2} + \frac{1}{2}\ln|1+x^2| + c$$

$$= \frac{x^2}{2}(\tan^{-1} x)^2 - x\tan^{-1} x + \frac{1}{2}\ln|1+x^2| + \frac{(\tan^{-1} x)^2}{2} + c$$

$$(\tan^{-1} x)^2 - \frac{(\tan^{-1} x)^2}{2} = \frac{(\tan^{-1} x)^2}{2}$$

ركز على عملية الطرح (يعني واحد ناقص نص يساوي نص)

72. $\int e^x \tan^{-1} e^x \, dx \quad put\ y = e^x \rightarrow dy = e^x dx$

$\int \tan^{-1} y \, dy$

$u = \tan^{-1} y$	$dv = dy$
$du = \frac{dy}{1+y^2}$	$v = y$

$= y \tan^{-1} y - \int \frac{y}{1+y^2} dy$

$-\int \frac{y}{1+y^2} dy = -\frac{1}{2} \int \frac{2y}{1+y^2}$ نضرب نقسم في 2 البسط مشتقة المقام

$= y \tan^{-1} y - \frac{1}{2} \ln |1 + y^2| + c$

$= e^x \tan^{-1} e^x - \frac{1}{2} \ln(|1 + e^{2x}|) + c$

---

73. $\int \frac{\tan^{-1} e^x}{e^x} dx = \int \frac{\tan^{-1} e^x}{e^x} \times \frac{e^x}{e^x} dx$

نضرب ونقسم في e^x

$put\ y = e^x \rightarrow dy = e^x dx$

$u = \tan^{-1} y$	$dv = y^{-2}$
$du = \frac{dy}{1+y^2}$	$v = -\frac{1}{y}$

$\int \frac{\tan^{-1} y}{y^2} dy$

$= -\frac{1}{y} \tan^{-1} y + \int \frac{dy}{y(1+y^2)}$

$\int \frac{dy}{y(1+y^2)} = \frac{A}{y} + \frac{By+c}{(1+y^2)} = \frac{A(1+y^2)+(By+c)y}{y(1+y^2)}$

$= A + Ay^2 + By^2 + cy$

$y^2 \rightarrow 0 = A + B \quad , y^1 \rightarrow 0 = c \quad , \quad y^0 \rightarrow 1 = A$

$0 = 1 + B \rightarrow B = -1$

$= \int \frac{1}{y} - \frac{1}{2} \int \frac{2y}{(1+y^2)} = \ln |y| - \frac{1}{2} \ln |(1 + y^2)|$

$$= -\frac{1}{y}\tan^{-1} y + \ln|y| - \frac{1}{2}\ln|(1+y^2)| + c$$

$$= -\frac{1}{e^x}\tan^{-1} e^x + \ln|e^x| - \frac{1}{2}\ln|(1+e^{2x})| + c$$

$$= -e^{-x}\tan^{-1} e^x + x - \frac{1}{2}\ln|(1+e^{2x})| + c$$

---

74. $\int \frac{xe^x}{(x+1)^2}dx$

$u = xe^x$	$dv = (x+1)^{-2}$
$du = e^x + xe^x$	$v = -\frac{1}{(x+1)}$

$$= -\frac{xe^x}{(x+1)} + \int \frac{e^x + xe^x}{(x+1)}dx = -\frac{xe^x}{(x+1)} + \int \frac{e^x(x+1)}{(x+1)}dx$$

$$= -\frac{xe^x}{(x+1)} + \int e^x\,dx == -\frac{xe^x}{(x+1)} + e^x + c$$

نوحد المقامات

$$= -\frac{xe^x + e^x(x+1)}{(x+1)} = \frac{-xe^x + xe^x + e^x}{(x+1)} + c$$

$$= \frac{e^x}{(x+1)} + c$$

---

75. $\int \frac{xe^{\tan^{-1}x}}{(1+x^2)^{\frac{3}{2}}}dx = \int \frac{xe^{\tan^{-1}x}}{(1+x^2)^{\frac{1}{2}}(1+x^2)} = i$

$(1+x^2)^{\frac{1}{2}}(1+x^2) = (1+x^2)^{\frac{3}{2}}$

$u = \frac{x}{\sqrt{1+x^2}}$　　　$dv = \frac{e^{\tan^{-1}x}}{(1+x^2)}$

$$du = \frac{\sqrt{1+x^2} + x\frac{2x}{\sqrt{1+x^2}}}{(1+x^2)} = \frac{1+x^2-x^2}{\sqrt{1+x^2}(1+x^2)}$$

$du = \frac{1}{\sqrt{1+x^2}(1+x^2)}dx$　　　$v = e^{\tan^{-1}x}$

$$i = \frac{x e^{\tan^{-1} x}}{\sqrt{1+x^2}} - \int \frac{e^{\tan^{-1} x}}{\sqrt{1+x^2}(1+x^2)} dx$$

$$u = \frac{1}{\sqrt{1+x^2}} \qquad dv = \frac{e^{\tan^{-1} x}}{(1+x^2)}$$

$$du = \frac{\frac{-2x}{2\sqrt{1+x^2}}}{(1+x^2)} = \frac{-x}{\sqrt{1+x^2}(1+x^2)} \qquad v = e^{\tan^{-1} x}$$

$$i = -\left[\frac{e^{\tan^{-1} x}}{\sqrt{1+x^2}} + \int \frac{x e^{\tan^{-1} x}}{\sqrt{1+x^2}(1+x^2)} dx\right]$$

$$i = -\frac{e^{\tan^{-1} x}}{\sqrt{1+x^2}} - \int \frac{x e^{\tan^{-1} x}}{\sqrt{1+x^2}(1+x^2)} dx$$

$$i = -\frac{e^{\tan^{-1} x}}{\sqrt{1+x^2}} - i$$

$$2i = -\frac{e^{\tan^{-1} x}}{\sqrt{1+x^2}} \rightarrow i = -\left(\frac{e^{\tan^{-1} x}}{\sqrt{1+x^2}}\right)$$

$$2i = \frac{x e^{\tan^{-1} x}}{\sqrt{1+x^2}} - \left(\frac{e^{\tan^{-1} x}}{\sqrt{1+x^2}}\right) + c$$

المقامات موحدة نجمع

نستخرج عامل مشترك

$$2i = \frac{x e^{\tan^{-1} x}}{\sqrt{1+x^2}} - \frac{e^{\tan^{-1} x}}{\sqrt{1+x^2}} = \frac{x e^{\tan^{-1} x} - e^{\tan^{-1} x}}{\sqrt{1+x^2}}$$

$$2i = \frac{(x-1) e^{\tan^{-1} x}}{\sqrt{1+x^2}}$$

$$i = \frac{(x-1) e^{\tan^{-1} x}}{2\sqrt{1+x^2}} + c$$

$$76.\int \frac{e^{\tan^{-1}x}}{(1+x^2)^{\frac{3}{2}}}dx = \int \frac{e^{\tan^{-1}x}}{(1+x^2)^{\frac{1}{2}}(1+x^2)}dx = i$$

$$u = \frac{1}{\sqrt{1+x^2}} \qquad dv = \frac{e^{\tan^{-1}x}}{(1+x^2)}$$

$$du = \frac{\frac{-2x}{2\sqrt{1+x^2}}}{(1+x^2)} = \frac{-x}{\sqrt{1+x^2}(1+x^2)} \qquad v = e^{\tan^{-1}x}$$

$$i = \frac{e^{\tan^{-1}x}}{\sqrt{1+x^2}} + \int \frac{x e^{\tan^{-1}x}}{\sqrt{1+x^2}(1+x^2)}dx$$

$$u = \frac{x}{\sqrt{1+x^2}} \qquad dv = \frac{e^{\tan^{-1}x}}{(1+x^2)}$$

$$du = \frac{\sqrt{1+x^2}+x\frac{2x}{\sqrt{1+x^2}}}{(1+x^2)} = \frac{1+x^2-x^2}{\sqrt{1+x^2}(1+x^2)}$$

$$du = \frac{1}{\sqrt{1+x^2}(1+x^2)}dx \qquad v = e^{\tan^{-1}x}$$

$$i = \frac{e^{\tan^{-1}x}}{\sqrt{1+x^2}} + \frac{x e^{\tan^{-1}x}}{\sqrt{1+x^2}} - \int \frac{e^{\tan^{-1}x}}{\sqrt{1+x^2}(1+x^2)}dx$$

$$i = \frac{e^{\tan^{-1}x}}{\sqrt{1+x^2}} + \frac{x e^{\tan^{-1}x}}{\sqrt{1+x^2}} - i$$

$$2i = \frac{e^{\tan^{-1}x}}{\sqrt{1+x^2}} + \frac{x e^{\tan^{-1}x}}{\sqrt{1+x^2}} \rightarrow \quad i = \frac{(x+1)e^{\tan^{-1}x}}{2\sqrt{1+x^2}} + c$$

قاعدة هامة جدا / إذا كان $\frac{Ax+B}{\sqrt{ax^2+bx+c}}$ أو $\frac{Ax+B}{(ax^2+bx+c)}$ فإن

$$Ax+B=\alpha(2ax+b)+\beta \rightarrow \quad \alpha=\frac{A}{2a} \quad و \quad \beta=b\alpha+B$$

يصبح التكامل كالتالي

$$\alpha\int\frac{(2ax+b)}{\sqrt{ax^2+bx+c}}+\beta\int\frac{1}{\sqrt{ax^2+bx+c}}$$

وبعدين نكامل بالراحة

$$77.\int\frac{dx}{x^2+6x+25}$$

بإكمال المربع نأخذ نصف مربع معامل x

$$\int\frac{dx}{x^2+6x+9-9+25}=\int\frac{dx}{(x+3)^2+16}$$

نطبق صيغة $\tan^{-1}x$ اذا كان

$$\int\frac{dx}{b^2x^2+a^2}=\frac{b}{a}\tan^{-1}\frac{bx}{a}$$

$$=\frac{1}{4}tan^{-1}\frac{x+3}{4}+c$$

$$78.\int\frac{3x-1}{x^2-4x+8}dx \rightarrow 3x-1=\alpha(2x-4)+\beta$$

$$3=2\alpha\rightarrow\alpha=\frac{3}{2} \qquad ,-1=-4\alpha+\beta\rightarrow\beta=5$$

$$\alpha\int\frac{(2ax+b)}{\sqrt{ax^2+bx+c}}+\beta\int\frac{1}{\sqrt{ax^2+bx+c}}$$

$$\frac{3}{2}\int\frac{(2x-4)}{x^2-4x+8}dx \text{ (البسط مشتقة للمقام) } +5\int\frac{dx}{x^2-4x+8}$$

$$=\frac{3}{2}ln\mid x^2-4x+8\mid+5\int\frac{dx}{x^2-4x+4-4+8}$$

$$=\frac{3}{2}ln\mid x^2-4x+8\mid+5\int\frac{dx}{(x-2)^2+4}$$

$$=\frac{3}{2}\ln\mid x^2-4x+8\mid+\frac{5}{2}\tan^{-1}\frac{x-2}{2}+c$$

$$79. \int \frac{x}{2x^2+2x+5} dx \quad \rightarrow x = \alpha(2 \times 2x + 2) + \beta$$

$$1 = 4\alpha \quad \rightarrow \alpha = \frac{1}{4} \quad , \quad 0 = 2\alpha + \beta \rightarrow \beta = -\frac{1}{2}$$

$$\frac{1}{4} \int \frac{(4x+2)}{2x^2+2x+5} dx - \frac{1}{4} \int \frac{dx}{x^2+x+\frac{5}{2}}$$

$$= \frac{1}{4} \mid 2x^2 + 2x + 5 \mid - \frac{1}{4} \int \frac{dx}{x^2+x+\frac{1}{4}-\frac{1}{4}+\frac{5}{2}}$$

$$= \frac{1}{4} \mid 2x^2 + 2x + 5 \mid - \frac{1}{4} \int \frac{dx}{\left(x+\frac{1}{2}\right)^2+\frac{9}{4}}$$

$$= \frac{1}{4} \mid 2x^2 + 2x + 5 \mid - \frac{1}{4} \times \frac{2}{3} tan^{-1} \frac{2\left(x+\frac{1}{2}\right)}{3}$$

$$= \frac{1}{4} \mid 2x^2 + 2x + 5 \mid - \frac{1}{6} tan^{-1} \frac{(2x+1)}{3} + c$$

$$80. \int \frac{2x^3+3x}{x^4+x^2+1} dx = \int \frac{2x^3+x+2x}{x^4+x^2+1} dx$$

$$= \int \frac{2x^3+x}{x^4+x^2+1} + \int \frac{2x}{x^4+x^2+1} dx$$

$$= \int \frac{\frac{1}{2}(4x^3+2x)}{x^4+x^2+1} + \int \frac{2x}{(x^2)^2+x^2+1} dx$$

$$= \frac{1}{2} ln \mid x^4 + x^2 + 1 \mid + \int \frac{2x}{(x^2)^2+x^2+1} dx$$

$$put\ y = x^2 \quad \rightarrow dy = 2xdx$$

$$\int \frac{dy}{y^2+y+1} = \int \frac{dy}{y^2+y+\frac{1}{4}+-\frac{1}{4}+1}$$

$$\int \frac{dy}{\left(y+\frac{1}{2}\right)^2+\frac{3}{4}} = \frac{2}{\sqrt{3}} tan^{-1} \frac{2\left(y+\frac{1}{2}\right)}{\sqrt{3}}$$

$$= \frac{1}{2} ln\ |x^4+x^2+1| + \frac{2}{\sqrt{3}} tan^{-1} \frac{2\left(x^2+\frac{1}{2}\right)}{\sqrt{3}}$$

$$= \frac{1}{2} ln\ |x^4+x^2+1| + \frac{2}{\sqrt{3}} tan^{-1} \frac{(2x^2+1)}{\sqrt{3}} + c$$

81. $\int \frac{dx}{(x-1)^4} dx = \int (x-1)^{-4}\, dx$

صورة دالة في مشتقتها

$$= -\frac{1}{3(x-1)^3} + c$$

82. $\int \frac{dx}{(2x+3)^3} = \frac{1}{2}\int 2\,(2x+3)^{-3}$

نضرب ونقسم في 2 لكي نجعلها دالة في مشتقتها

$$= -\frac{1}{4(2x+3)^2} + c$$

83. $\int \frac{dx}{x^2-6x+18} = \int \frac{dx}{x^2-6x+9-9+18}$

$$= \int \frac{dx}{(x-3)^2+9} = \frac{1}{3} tan^{-1} \frac{(x-3)}{3} + c$$

$$84.\int \frac{x^2}{x^6+2x^3+3}dx = \int \frac{x^2}{(x^3)^2+2x^3+3}dx$$

$$put\ y = x^3 \ \rightarrow dy = 3x^2dx \ \rightarrow \frac{1}{3}\ dy = x^2dx$$

$$\frac{1}{3}\int \frac{dy}{y^2+2y+3} = \frac{1}{3}\int \frac{dy}{y^2+2y+1-1+3}$$

$$= \frac{1}{3}\int \frac{dy}{(y+1)^2+2} = \frac{1}{3}\times\frac{1}{\sqrt{2}}tan^{-1}\frac{(y+1)}{\sqrt{2}} + c$$

$$= \frac{1}{3\sqrt{2}}tan^{-1}\frac{(x^3+1)}{\sqrt{2}} + c$$

$$85.\int \frac{x-2}{x^2-4x+7}dx \ = \int \frac{\frac{1}{2}(2x-4)}{x^2-4x+7}dx$$

استخرجنا $\frac{1}{2}$ عامل مشترك حتى يصبح البسط مشتقه للمقام

$$\frac{1}{2}\int \frac{(2x-4)}{x^2-4x+7}dx = \frac{1}{2}ln\ |\ x^2 - 4x + 7\ |\ + c$$

$$86.\int \frac{5x+3}{x^2+10x+29}dx \ \ \rightarrow 5x + 3 = \alpha(2x + 10) + \beta$$

$$5 = 2\alpha \rightarrow \alpha = \frac{5}{2} \quad , \quad 3 = 10\alpha + \beta \ \rightarrow \beta = -22$$

$$\frac{5}{2}\ \int \frac{(2x+10)}{x^2+10x+29}dx - 22\int \frac{dx}{x^2+10x+29}$$

$$= \frac{5}{2}ln\ |\ x^2 + 10x + 29\ | - 22\int \frac{dx}{x^2+10x+25-25+29}$$

$$= \frac{5}{2}ln\ |\ x^2 + 10x + 29\ | - 22\int \frac{dx}{(x+5)^2+4}$$

$$= \frac{5}{2} ln \mid x^2 + 10x + 29 \mid - \frac{22}{2} tan^{-1} \frac{(x+5)}{2}$$

$$= \frac{5}{2} ln \mid x^2 + 10x + 29 \mid - 11\, tan^{-1} \frac{(x+5)}{2} + c$$

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87. $\int \frac{x+1}{5x^2+2x+1} dx \ \rightarrow\ x + 1 = \alpha(10x + 2) + \beta$

$$1 = 10\alpha \ \rightarrow\ \alpha = \frac{1}{10} \quad , \qquad 1 = 2\alpha + \beta \rightarrow \beta = \frac{4}{5}$$

$$\frac{1}{10} \int \frac{(10x+2)}{5x^2+2x+1} dx + \frac{4}{5} \int \frac{dx}{5x^2+2x+1}$$

$$= \frac{1}{10} ln \mid 5x^2 + 2x + 1 \mid + \frac{4}{25} \int \frac{dx}{x^2+\frac{2}{5}x+\frac{1}{5}}$$

$$= \frac{1}{10} ln \mid 5x^2 + 2x + 1 \mid + \frac{4}{25} \int \frac{dx}{x^2+\frac{2}{5}x+\frac{1}{25}-\frac{1}{25}+\frac{1}{5}}$$

$$= \frac{1}{10} ln \mid 5x^2 + 2x + 1 \mid + \frac{4}{25} \int \frac{dx}{\left(x+\frac{1}{5}\right)^2+\frac{4}{25}}$$

$$= \frac{1}{10} ln \mid 5x^2 + 2x + 1 \mid + \frac{4}{25} \times \frac{5}{2} tan^{-1} \frac{5\left(x+\frac{1}{5}\right)}{2} + c$$

$$= \frac{1}{10} ln \mid 5x^2 + 2x + 1 \mid + \frac{2}{5} tan^{-1} \frac{(5x+1)}{2} + c$$

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88. $\int \frac{3x+2}{(x^2+2x+10)^2} dx \quad \rightarrow 3x + 2 = \alpha(2x + 2) + \beta$

$$3 = 2\alpha \ \rightarrow \alpha = \frac{3}{2} \quad , \quad 2 = 2\alpha + \beta \ \rightarrow\ \beta = -1$$

$$\frac{3}{2}\int\frac{(2x+2)}{(x^2+2x+10)^2}dx-\int\frac{dx}{(x^2+2x+10)^2}$$

$$\frac{3}{2}\int(x^2+2x+10)^{-2}\,(2x+2)dx-\int\frac{dx}{(x^2+2x+10)^2}$$

$$-\frac{3}{2(x^2+2x+10)^{-2}}-\int\frac{dx}{(x^2+2x+10)^2}$$

تحل بالصيغة التراجعية

بس يوجد حل اخر

$$-\int\frac{dx}{(x^2+2x+1-1+10)^2}=\int\frac{dx}{((x+1)^2+9)^2}$$

$put\ y=x+1\quad\rightarrow dy=dx$

نضرب ونقسم في 9

نضيف ونطرح y^2

$$\int\frac{dy}{(y^2+9)^2}=\frac{1}{9}\int\frac{9}{(y^2+9)^2}=\frac{1}{9}\int\frac{(9+y^2)-y^2}{(y^2+9)^2}$$

$$=\frac{1}{9}\int\frac{(9+y^2)}{(y^2+9)^2}dy-\frac{1}{9}\int\frac{y^2}{(y^2+9)^2}dy$$

نوزع المقام

بحيث نختصر

$$=\frac{1}{9}\int\frac{dy}{(y^2+9)}-\frac{1}{9}\int\frac{y^2}{(y^2+9)^2}$$

$$=\frac{1}{27}tan^{-1}\frac{y}{3}-\frac{1}{9}\int\frac{y^2}{(y^2+9)^2}$$

$u=y$	$dv=y(y^2+9)^{-2}$
$du=dy$	$v=-\frac{1}{2(y^2+9)}$

$$=-\frac{1}{9}\left[-\frac{y}{2(y^2+9)}+\frac{1}{2}\int\frac{dy}{(y^2+9)}\right]$$

$$=-\frac{y}{18(y^2+9)}+\frac{1}{18}\int\frac{dy}{(y^2+9)}$$

$$= -\frac{y}{18(y^2+9)} + \frac{1}{18} \times \frac{1}{3} tan^{-1}\frac{y}{3}$$

$$= -\frac{3}{2(x^2+2x+10)^{-2}} - \frac{x+1}{18((x+1)^2+9)} + \frac{1}{54} tan^{-1}\frac{(x+1)}{3} + c$$

$$= -\frac{3}{2(x^2+2x+10)^{-2}} - \frac{x+1}{18(x^2+2x+10)} + \frac{1}{54} tan^{-1}\frac{(x+1)}{3} + c$$

$$89. \int \frac{2x+3}{(x^2+2x+5)^2} dx \rightarrow 2x + 3 = \alpha(2x + 2) + \beta$$

$$2 = 2\alpha \rightarrow \alpha = 1 \quad , \quad 3 = 2\alpha + \beta \rightarrow \beta = 1$$

$$\int \frac{(2x+2)}{(x^2+2x+5)^2} dx + \int \frac{dx}{(x^2+2x+5)^2} dx$$

$$\int (x^2 + 2x + 5)^{-2} (2x + 2)dx + \int \frac{dx}{(x^2+2x+5)^2}$$

$$= \frac{-1}{(x^2+2x+5)} + \int \frac{dx}{(x^2+2x+1-1+5)^2}$$

$$\int \frac{dx}{((x+1)^2+4)^2} \quad put\ y = (x + 1) \rightarrow dy = dx$$

$$\int \frac{dy}{(y^2+4)^2} = \frac{1}{4}\int \frac{(4+y^2)-y^2}{(y^2+4)^2} = \frac{1}{8} tan^{-1}\frac{y}{2} - \frac{1}{4}\int \frac{y^2}{(y^2+4)^2}$$

$u = y$	$dv = y(y^2 + 4)^{-2}$
$du = dy$	$v = -\frac{1}{2(y^2+4)}$

$$= -\frac{1}{4}\left[-\frac{y}{2(y^2+4)} + \frac{1}{2}\int \frac{dy}{(y^2+4)}\right]$$

$$= \frac{y}{8(y^2+4)} - \frac{1}{4} \times \frac{1}{2} \times \frac{1}{2} tan^{-1} \frac{y}{2}$$

$$= \frac{-1}{(x^2+2x+15)} + \frac{1}{8} tan^{-1} \frac{y}{2} + \frac{y}{8(y^2+4)} - \frac{1}{16} \times tan^{-1} \frac{y}{2}$$

$$= \frac{-1}{(x^2+2x+15)} + \frac{(x+1)}{8((x+1)^2+4)} + \frac{1}{16} \times tan^{-1} \frac{(x+1)}{2}$$

$$= \frac{-1}{(x^2+2x+15)} + \frac{(x+1)}{8(x^2+2x+15)} + \frac{1}{16} \times tan^{-1} \frac{(x+1)}{2}$$

$$= \frac{-8+x+1}{(x^2+2x+15)} + \frac{1}{16} \times tan^{-1} \frac{(x+1)}{2} + c$$

$$= \frac{x-7}{(x^2+2x+15)} + \frac{1}{16} \times tan^{-1} \frac{(x+1)}{2} + c$$

$$= -\frac{(x+3)}{(x^2+2x+15)} + \frac{1}{16} \times tan^{-1} \frac{(x+1)}{2} + c$$

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90. $\int \frac{x^2+2x+6}{x^3-7x+14x-8} dx = \int \frac{x^2+2x+6}{x^3-8-7x^2+14x} dx$

$$= \int \frac{x^2+2x+6}{(x-2)(x^2+2x+4)-7x(x-2)} dx$$

$$= \int \frac{x^2+2x+6}{(x-2)[x^2+2x+4-7x]} dx$$

$$= \int \frac{x^2+2x+6}{(x-2)[x^2-5x+4]} dx == \int \frac{x^2+2x+6}{(x-2)(x-1)(x-4)} dx$$

$$= \frac{A}{(x-2)} + \frac{B}{(x-1)} + \frac{C}{(x-4)}$$

$$= \frac{A(x-1)(x-4)+B(x-2)(x-4)+C(x-2)(x-1)}{(x-2)(x-1)(x-4)}$$

$$x = 1 \rightarrow 9 = 3B \rightarrow B = \frac{9}{3} = 3 \quad \rightarrow B = 3$$

$$x = 4 \rightarrow 30 = 6C \rightarrow B = \frac{30}{6} = 5 \quad \rightarrow C = 5$$

$$x = 2 \rightarrow 14 = -2A \rightarrow A = -\frac{14}{2} = -7 \rightarrow A = -7$$

$$= \int \frac{-7}{(x-2)} dx + \int \frac{3}{(x-1)} dx + \int \frac{5}{(x-4)} dx$$

$$= -7\, ln\, |(x-2)| + 3\, ln\, |(x-1)| + 5\, ln\, |(x-4)| + c$$

$$= 3\, ln\, |(x-1)| + 5\, ln\, |(x-4)| - 7\, ln\, |(x-2)| + c$$

$$= ln\, |(x-1)^3| + ln\, |(x-4)^5| - ln\, |(x-2)^7| + c$$

$$= ln\, |(x-1)^3(x-4)^5| - ln\, |(x-2)^7| + c$$

$$= ln\, \left| \frac{(x-1)^3(x-4)^5}{(x-2)^7} \right| + c$$

من خواص اللوغاريتم الجمع يرجع ضرب والطرح قسمة

$$91. \int \frac{15x^2-4x-81}{x^3-13x+12} dx$$

المقام لايتحلل نشوق ماهو الرقم اللي يصفر المقام ونقسم قسمة خوارزمية

$x = 1$ يصفر المقام $1 \rightarrow x - 1 = 0$

$$= \int \frac{15x^2-4x-81}{(x-1)(x^2-x-12)} dx$$

$$= \int \frac{15x^2-4x-81}{(x-1)(x+4)(x-3)} dx$$

$$= \frac{A}{(x-1)} + \frac{B}{(x+4)} + \frac{C}{(x-3)}$$

$$= \frac{A(x+4)(x-3)+B(x-1)(x-3)+C(x-1)(x+4)}{(x-1)(x+4)(x-3)}$$

$$x = 1 \to -70 = -10A \to A = \frac{70}{10} = 7 \to A = 7$$

$$x = 3 \to 42 = 14C \to C = \frac{42}{14} = 3 \to C = 3$$

$$x = -4 \to 175 = 35\text{B} \to \text{B} = \frac{175}{35} = 5 \to \text{B} = 5$$

$$= \int \frac{7}{(x-1)} + \int \frac{5}{(x+4)} + \int \frac{3}{(x-3)}$$

$$= 7\,ln\,|(x-1)| + 5\,ln\,|(x+4)| + 3\,ln\,|(x-3)| + c$$

$$= ln\,|(x-1)^7| + ln\,|(x+4)^5| + ln\,|(x-3)^3| + c$$

$$= ln\,|(x-1)^7(x+4)^5(x-3)^3| + c$$

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92. $\int \frac{x^4}{(x+2)(x^2-1)} dx = \int \frac{x^4}{x^3+2x^2-x-2}$

$$= \int \frac{x^4}{x^3+2x^2-x-2}$$

$$= \int (x-2)\,dx + \int \frac{5x^2-4}{x^3+2x^2-x-2} dx$$

$$= \frac{x^2}{2} - 2x + \int \frac{5x^2-4}{(x+2)(x^2-1)} dx$$

$$= \int \frac{5x^2-4}{(x+2)(x-1)(x+1)} dx$$

$$= \frac{A}{(x+2)} + \frac{B}{(x-1)} + \frac{C}{(x+1)}$$

$$= \frac{A(x-1)(x+1)+B(x+2)(x+1)+C(x+2)(x-1)}{(x+2)(x-1)(x+1)}$$

$$x = 1 \rightarrow -1 = 6B \rightarrow B = \frac{1}{6}$$

$$x = -1 \rightarrow 1 = -2C \rightarrow C = -\frac{1}{2} \rightarrow C = -\frac{1}{2}$$

$$x = -2 \rightarrow 16 = 3A \rightarrow A = \frac{16}{3}$$

$$= \int \frac{\frac{16}{3}}{(x+2)} dx + \int \frac{\frac{1}{6}}{(x-1)} dx + \int \frac{-\frac{1}{2}}{(x+1)} dx$$

$$= \frac{16}{3} ln\ |(x+2)| + \frac{1}{6} ln\ |(x-1)| - \frac{1}{2} ln\ |(x+1)| + c$$

$$= \frac{1}{6}[ln\ |(x-1)| - 3\ ln\ |(x+1)|] + \frac{16}{3} ln\ |(x+2)|$$

$$= \frac{1}{6}[ln\ |(x-1)| - ln\ |(x+1)^3|] + \frac{16}{3} ln\ |(x+2)|$$

$$= \frac{x^2}{2} - 2x + \frac{1}{6} ln\ \left|\frac{(x-1)}{(x-1)^3}\right| + \frac{16}{3} ln\ |(x+2)| + c$$

$$93. \int \frac{x^4-3x^2-3x-2}{x^3-x^2-2x} dx$$

$$= \int (x+1)\, dx - \int \frac{2x+2}{x(x^2-x-2)} dx$$

$$= \frac{x^2}{2} + x - \int \frac{x+2}{x(x-2)(x+1)} dx$$

$$= \frac{A}{x} + \frac{B}{(x-2)} + \frac{C}{(x+1)}$$

$$\frac{A(x-2)(x+1)+Bx(x+1)+Cx(x-2)}{x(x-2)(x+1)}$$

$$= \frac{A(x^2-x-2)+Bx^2+Bx+Cx^2-2Cx}{x(x-2)(x+1)}$$

$$= \frac{Ax^2-Ax-2A+Bx^2+Bx+Cx^2-2Cx}{x(x-2)(x+1)}$$

$x^2 \to 0 = A + B + C$, $x \to 1 = -A + B - 2C$

$x^0 \to 2 = -2A \to A = -1$

$1 = -1 + B - 2C$, $C = -A - B \to C = 1 - B$

$1 = -1 + B - 2(1 - B) \to 1 = -1 + B - 2 + 2B$

$B = \frac{2}{3}$, $C = 1 - B \to C = \frac{1}{3}$

$$= -\left[\int \frac{-1}{x} dx + \int \frac{\frac{2}{3}}{(x-2)} dx + \int \frac{\frac{1}{3}}{(x+1)} dx\right]$$

$$= ln\ |x| - \frac{2}{3}\ |(x-2)|\ - \frac{1}{3}\ |(x+1)|\ + c$$

$$= \frac{x^2}{2} + x + ln\ |x| - \frac{2}{3}\ |(x-2)|\ - \frac{1}{3}\ |(x+1)|\ + c$$

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$$94. \int \frac{x^2+1}{(x-1)^3(x+3)} dx$$

$$= \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3} + \frac{D}{(x+3)}$$

$$= \frac{A(x-1)^2(x+3)+B(x-1)(x+3)+C(x+3)+D(x-1)^3}{(x-1)^3(x+3)}$$

$x = 1 \to 2 = 4C \to C = \frac{1}{2}$ $, x = -3 \to 10 = -64D \to D = -\frac{5}{32}$

$$= \frac{A(x^2-2x+1)(x+3)+B(x^2+2x-3)+cx+3c+D(x^3-3x^2+3x-1)}{(x-1)^3(x+3)}$$

$$= \frac{A(x^3+x^2-5x+3)+B(x^2+2x-3)+cx+3c+D(x^3-3x^2+3x-1)}{(x-1)^3(x+3)}$$

$= Ax^3 + Ax^2 - 5Ax + 3A + Bx^2 + 2Bx - 3B + cx + 3c + Dx^3 - 3Dx^2 + 3Dx - D$

$x^3 \to 0 = A + D$ $\quad , x^2 \to 1 = A + B + -3D$

$x \to 0 = -5A + 2B + C + 3D$ $\quad , \quad x^0 \to 1 = 3A - 3B + 3C - D$

$$0 = A + D \to 0 = A - \frac{5}{32} \to A = \frac{5}{32}$$

$$1 = 3A - 3B + 3C - D \to 1 = 3 \times \frac{5}{32} - 3B + 3 \times \frac{1}{2} - \left(-\frac{5}{32}\right)$$

$$1 = \frac{15}{32} - 3B + \frac{3}{2} + \frac{5}{32} \to 1 = \frac{20}{32} + \frac{3}{2} - 3B \to 1 = \frac{5}{8} + \frac{3}{2} - 3B$$

$$1 = \frac{17}{8} - 3B \to 1 - \frac{17}{8} = -3B \to -\frac{9}{8} = -3B \to B = \frac{3}{8}$$

$$= \int \frac{\frac{5}{32}}{(x-1)} dx + \int \frac{\frac{3}{8}}{(x-1)^2} dx + \int \frac{\frac{1}{2}}{(x-1)^3} dx + \int \frac{-\frac{5}{32}}{(x+3)} dx$$

$$= \frac{5}{32} ln \mid (x-1) \mid - \frac{3}{8(x-1)} - \frac{1}{4(x-1)^2} - \frac{5}{32} ln \mid (x+3) \mid + c$$

$$= -\frac{1}{4(x-1)^2} - \frac{3}{8(x-1)} + \frac{5}{32}[ln \mid (x-1) \mid - ln \mid (x+3) \mid] + c$$

$$= -\frac{1}{4(x-1)^2} - \frac{3}{8(x-1)} + \frac{5}{32} ln \mid \frac{(x-1)}{(x+3)} \mid + c$$

95. $\int \frac{2x^2-3x+3}{x^3-2x^2+x}dx = \int \frac{2x^2-3x+3}{x(x^2-2x+1)}dx = \int \frac{2x^2-3x+3}{x(x-1)^2}dx$

$= \frac{A}{x} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2} = \frac{A(x-1)^2+Bx(x-1)+cx}{x(x-1)^2}$

$= \frac{A(x^2-2x+1)+Bx^2-Bx+Cx}{x(x-1)^2}$

$\frac{Ax^2-2Ax+A+Bx^2-Bx+Cx}{x(x-1)^2}$

$x^2 \to 2 = A + B$, $x \to -3 = -2A - B + C$, $x^0 \to 3 = A$

$A = 3$, $2 = 3 + B \to B = -1$, $-3 = -6 + 1 + C \to C = 2$

$A = 3$, $B = -1$, $C = 2$

$= \int \frac{3}{x}dx + \int \frac{-1}{(x-1)}dx + \int \frac{2}{(x-1)^2}dx$

$= 3\ln|x| - \ln|(x-1)| - \frac{2}{(x-1)} + c$

---

96. $\int \frac{x^3+1}{x(x-1)^3}dx = \frac{A}{x} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2} + \frac{D}{(x-1)^3}$

$= \frac{A(x-1)^3+Bx(x-1)^2+Cx(x-1)+Dx}{x(x-1)^3}$

$= \frac{A(x^3-3x^2+2x-1)+Bx(x^2-2x+1)+Cx(x-1)+Dx}{x(x-1)^3}$

$= \frac{Ax^3-3Ax^2+2Ax-A+Bx^3-2Bx^2+Bx+Cx^2-Cx+Dx}{x(x-1)^3}$

$x^3 \to 1 = A + B$, $x^2 \to 0 = -3A - 2B + C$, $x \to 0 = 2A + B - C + D$

$x^0 \to 1 = -A \to A = -1$, $1 = A + B \to 1 = -1 + B \to B = 2$

$0 = -3A - 2B + C \;\rightarrow 0 = 3 - 4 + C \rightarrow C = 1$

$0 = 2A + B - C + D \rightarrow 0 = -2 + 2 + D \rightarrow D = 0$

$A = 1 \quad , \; B = 0 \quad , \; C = 3 \quad , D = 1$

$= \int \frac{-1}{x} dx + \int \frac{2}{(x-1)} dx + \int \frac{1}{(x-1)^2} dx + \int \frac{0}{(x-1)^3} dx$

$= ln \mid x \mid - \frac{3}{(x-1)} - \frac{1}{(x-1)^2}$

$97. \int \frac{x}{x^3+1} dx = \int \frac{x}{(x+1)(x^2-x+1)} dx$

$= \frac{A}{(x+1)} dx + \frac{Bx+C}{(x^2-x+1)} dx = \frac{A(x^2-x+1)+Bx+C(x+1)}{(x+1)(x^2-x+1)}$

$= \frac{Ax^2-Ax+A+Bx^2+Cx+Bx+C}{(x+1)(x^2-x+1)}$

$x^2 \rightarrow 0 = A + B \quad , \; x \rightarrow 1 = -A + C + B \quad , \; x^0 \rightarrow 0 = A + C$

$B = -A \quad , \; C = -A \; , 1 = -A + C + B \rightarrow 1 = -A - A - A$

$1 = -3A \rightarrow A = -\frac{1}{3} \quad , B = -A \;\rightarrow B = \frac{1}{3} \quad , \; C = \frac{1}{3}$

$= \int \frac{-\frac{1}{3}}{(x+1)} dx + \int \frac{\frac{1}{3}x+\frac{1}{3}}{(x^2-x+1)} dx$

$= -\frac{1}{3} \int \frac{dx}{(x+1)} + \frac{1}{3} \int \frac{x+1}{(x^2-x+1)} dx$

$-\frac{1}{3} ln \mid (x + 1) \mid + \frac{1}{3} \int \frac{x+1}{(x^2-x+1)} dx \;\rightarrow x + 1 = \alpha(2x - 1) + \beta$

$$1 = 2\alpha \to \alpha = \frac{1}{2} \quad , \quad 1 = -\alpha + \beta \to 1 = -\frac{1}{2} + \beta \to \beta = \frac{3}{2}$$

$$= \frac{1}{3}\left[\frac{1}{2}\int \frac{(2x-1)}{(x^2-x+1)}dx + \frac{3}{2}\int \frac{dx}{(x^2-x+1)}\right]$$

$$= \frac{1}{6} \mid (x^2 - x + 1) \mid + \frac{1}{2}\int \frac{dx}{\left(x^2-x+\frac{1}{4}-\frac{1}{4}+1\right)}$$

$$= \frac{1}{6} \mid (x^2 - x + 1) \mid + \frac{1}{2}\int \frac{dx}{\left(x-\frac{1}{2}\right)^2+\frac{3}{4}}$$

$$= \frac{1}{6} \mid (x^2 - x + 1) \mid + \frac{1}{2} \times \frac{2}{\sqrt{3}} tan^{-1} \frac{2\left(x-\frac{1}{2}\right)}{\sqrt{3}}$$

$$= -\frac{1}{3} ln \mid (x + 1) \mid + \frac{1}{6} \mid (x^2 - x + 1) + \frac{1}{\sqrt{3}} tan^{-1} \frac{(2x-1)}{\sqrt{3}} + c$$

---

98. $\int \frac{dx}{x^5-x^2} = \int \frac{dx}{x^2(x^3-1)} = \int \frac{dx}{x^2(x-1)(x^2+x+1)}$

$$= \frac{Ax+B}{x^2} + + \frac{C}{(x-1)} + \frac{Dx+E}{(x^2+x+1)}$$

$$= \frac{(Ax+B)(x-1)(x^2+x+1)+cx^2(x^2+x+1)+(Dx+E)x^2(x-1)}{x^2(x-1)(x^2+x+1)}$$

$$= \frac{(Ax+B)(x^3-1)+Cx^4+cx^3+cx^2+(Dx+E)(x^3-x^2)}{x^2(x-1)(x^2+x+1)}$$

$$= \frac{Ax^4-Ax+Bx^3-B+Cx^4+Cx^3+Cx^2+Dx^4-Dx^3+Ex^3-Ex^2}{x^2(x-1)(x^2+x+1)}$$

$x^4 \to 0 = A + C + D \quad , \quad x^3 \to 0 = B + C - D + E$

$x^2 \to 0 = C - E \quad , x \to 0 = -A \quad , x^0 \to 1 = -B$

$B=-1 \quad , \quad 0=C+D \rightarrow D=-C \quad , \; 0=C-E \rightarrow E=C$

$0=B+C-D+E \rightarrow 0=-1+C+C+C \rightarrow 3C=1 \rightarrow C=\frac{1}{3}$

$E=\frac{1}{3} \quad ,D=-\frac{1}{3} \quad ,B=-1 \qquad ,A=0 \quad ,C=\frac{1}{3}$

$$=\int \frac{Ax+B}{x^2}dx+\int \frac{C}{(x-1)}dx+\int \frac{Dx+E}{(x^2+x+1)}dx$$

$$=\int \frac{-1}{x^2}dx+\int \frac{\frac{1}{3}}{(x-1)}dx+\int \frac{-\frac{1}{3}x+\frac{1}{3}}{(x^2+x+1)}dx$$

$$=-\int \frac{1}{x^2}dx+\frac{1}{3}\int \frac{1}{(x-1)}dx-\frac{1}{3}\int \frac{x-1}{(x^2+x+1)}dx$$

$$=\frac{1}{x}+\frac{1}{3}ln\ |(x-1)|\ -\frac{1}{3}\int \frac{x-1}{(x^2+x+1)}dx$$

$$-\frac{1}{3}\int \frac{x-1}{(x^2+x+1)}dx \qquad x-1=\alpha(2x+1)+\beta$$

$1=2\alpha \rightarrow \alpha=\frac{1}{2} \quad , \quad -1=\alpha+\beta \rightarrow -1=\frac{1}{2}+\beta \rightarrow \beta=-\frac{3}{2}$

$$=-\frac{1}{3}\left(\frac{1}{2}\int \frac{(2x+1)}{(x^2+x+1)}dx-\frac{3}{2}\int \frac{dx}{(x^2+x+1)}\right)$$

$$=-\frac{1}{6}\int \frac{(2x+1)}{(x^2+x+1)}dx+\frac{1}{2}\int \frac{dx}{(x^2+x+1)}$$

$$=-\frac{1}{6}ln\ |(x^2+x+1)|\ +\frac{1}{2}\int \frac{dx}{\left(x^2+x+\frac{1}{4}-\frac{1}{4}+1\right)}$$

$$=-\frac{1}{6}ln\ |(x^2+x+1)|\ +\frac{1}{2}\int \frac{dx}{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}}$$

$$=-\frac{1}{6}ln\ |(x^2+x+1)|\ +\frac{1}{2}\times\frac{2}{\sqrt{3}}tan^{-1}\frac{2\left(x+\frac{1}{2}\right)}{\sqrt{3}}$$

$$= -\frac{1}{6} ln\ |(x^2 + x + 1)|\ + \frac{1}{\sqrt{3}} tan^{-1} \frac{(2x+1)}{\sqrt{3}}$$

$$= \frac{1}{x} + \frac{1}{3} ln\ |(x-1)| - \frac{1}{6} ln\ |(x^2 + x + 1)|\ + \frac{1}{\sqrt{3}} tan^{-1} \frac{(2x+1)}{\sqrt{3}}$$

$$\frac{1}{x} + \frac{1}{6}(2\,ln\ |(x-1) - ln\ |(x^2 + x + 1)|\ |) + \frac{1}{\sqrt{3}} tan^{-1} \frac{(2x+1)}{\sqrt{3}}$$

$$= \frac{1}{x} + \frac{1}{6} ln\ \left|\frac{(x-1)^2}{(x^2+x+1)}\right| + \frac{1}{\sqrt{3}} tan^{-1} \frac{(2x+1)}{\sqrt{3}} + c$$

---

99. $\int \frac{dx}{(x^2+1)(x^2+4)} = \frac{Ax+B}{(x^2+1)} + \frac{Cx+D}{(x^2+4)}$

$$= \frac{(Ax+B)(x^2+4)+(Cx+D)(x^2+1)}{(x^2+1)(x^2+4)}$$

$$= \frac{Ax^3+4Ax+Bx^2+4B+Cx^3+Cx+Dx^2+D}{(x^2+1)(x^2+4)}$$

$x^3 \to 0 = A + C \quad , \quad x^2 \to 0 = B + D \quad \to D = -B$

$x \to 0 = 4A + C \quad , \quad x^0 \to 1 = 4B + D \to 1 = 4B - B \to B = \frac{1}{3}$

$D = -\frac{1}{3} \quad , C = -A \quad ,0 = 4A + C \quad \to 0 = 4A - A \to A = 0$

$A = 0 \quad , \quad B = \frac{1}{3} \quad , \quad D = -\frac{1}{3} \quad , \quad C = 0$

$$= \int \frac{\frac{1}{3}}{(x^2+1)} dx + \int \frac{-\frac{1}{3}}{(x^2+4)} dx$$

$$= \frac{1}{3} tan^{-1} x - \frac{1}{3} \times \frac{1}{2} tan^{-1} \frac{x}{2}$$

$$= \frac{1}{3} tan^{-1} x - \frac{1}{6} tan^{-1} \frac{x}{2}$$

$$100. \int \frac{x^3-2x}{(x^2+1)^2} dx = \frac{Ax+B}{(x^2+1)} + \frac{Cx+D}{(x^2+1)^2}$$

$$= \frac{(Ax+B)(x^2+1)+Cx+D}{(x^2+1)^2} = \frac{Ax^3+Ax+Bx^2+B+Cx+D}{(x^2+1)^2}$$

$x^3 \to 1 = A$, $x^2 \to 0 = B$, $x \to -2 = A + C$, $x^0 \to 0 = B+D$

$A = 1$, $-2 = A + C \to -2 = 1 + C \to C = -3$, $B = 0$, $D = 0$

$$= \int \frac{Ax+B}{(x^2+1)} dx + \int \frac{Cx+D}{(x^2+1)^2} dx = \int \frac{x}{(x^2+1)} dx + \int \frac{-3x}{(x^2+1)^2} dx$$

$$= \frac{1}{2}\int \frac{2x}{(x^2+1)} dx - \frac{3}{2}\int 2x\,(x^2+1)^2 dx$$

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$$= \frac{1}{2} ln\ |(x^2+1)| + \frac{3}{2(x^2+1)} + c$$

$$101. \int \frac{x^3+x^2+5x+7}{x^2+2} dx$$

$$\int (x+3)\,dx + \int \frac{3x+1}{x^2+2} dx = \int (x+3)\,dx + \int \frac{3x}{x^2+2} dx + \int \frac{dx}{x^2+2}$$

$$= \frac{x^2}{2} + 3x + \frac{3}{2}\int \frac{3x}{x^2+2} dx + \int \frac{dx}{x^2+2}$$

$$== \frac{x^2}{2} + 3x + \frac{3}{2} ln\ |(x^2+2)| + \frac{1}{\sqrt{2}} tan^{-1} \frac{x}{\sqrt{2}} + c$$

$102. \int \frac{x+4}{x^3+6x^2+11x+6} dx$ يصفر المقام 2 $\to x = 2 \to x - 2 = 0$

$$= \int \frac{x+4}{(x+1)(x^2+5x+6)} dx$$

$$= \frac{x+4}{(x+1)(x+3)(x+2)} dx$$

$$= \frac{A}{(x+1)} + \frac{B}{(x+3)} + \frac{C}{(x+2)}$$

$$= \frac{A(x+3)(x+2)+B(x+1)(x+2)+C(x+1)(x+3)}{(x+1)(x+3)(x+2)}$$

$x = -2 \quad \to 2 = -C \to C = -2 \;\; , x = -3 \;\; \to 1 = 2B \to B = \frac{1}{2}$

$x = -1 \quad \to 3 = 2A \to A = \frac{3}{2}$

$A = \frac{3}{2} \qquad , \quad B = \frac{1}{2} \quad , \;\; C = -2$

$$= \int \frac{\frac{3}{2}}{(x+1)} dx + \int \frac{\frac{1}{2}}{(x+3)} dx + \int \frac{-2}{(x+2)} dx$$

$$= \frac{3}{2} ln \;|(x+1)|\; + \frac{1}{2} ln \;|(x+3)|\; - 2 ln \;|(x+2)|\; + c$$

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103. $\int \frac{x^2}{(x-1)^5} dx$

$put\ y = x - 1 \quad \to dy = dx \qquad , x = y - 1$

$$\int \frac{(y-1)^2}{y^5} dy = \int \frac{y^2-2y+1}{y^5} dy = \int \frac{y^2}{y^5} dy - 2\int \frac{y}{y^5} dy + \int \frac{1}{y^5} dy$$

$$= \int \frac{dy}{y^3} - 2\int \frac{dy}{y^4} dy + \int \frac{1}{y^5} dy = -\frac{1}{2y^2} - \frac{2}{3y^3} - \frac{1}{4y^4} + c$$

$$= \frac{-6y^2-8y-3}{12y^4} = -\frac{6(x-1)^2+8(x-1)+3}{12(x-1)^4}$$

$$= -\frac{6\left(x^2-2x+1\right)+8x-8+3}{12(x-1)^4} = -\frac{6x^2-12x+6+8x-8+3}{12(x-1)^4}$$

$$= -\frac{6x^2-4x+1}{12(x-1)^4} + c$$

$$104. \int \frac{x}{x^4+6x^2+5}\,dx = \int \frac{x}{(x^2)^2+6x^2+5}\,dx$$

$$put\ y = x^2 \quad \to dy = 2xdx \ \to \frac{1}{2}dy = xdx$$

$$\frac{1}{2}\int \frac{dy}{y^2+6y+5}dy = \frac{1}{2}\int \frac{dy}{y^2+6y+9-9+5}$$

$$= \frac{1}{2}\int \frac{dy}{(y+3)^2-4} \qquad put\ u = y+3 \to du = dy$$

$$= \frac{1}{2}\int \frac{du}{u^2-4} = \frac{1}{2}\int \frac{du}{(u+2)(u-2)} = \frac{A}{(u+2)} + \frac{B}{(u-2)}$$

$$= \frac{A(u-2)+B(u+2)}{(u+2)(u-2)}$$

$$u = 2 \to 1 = 4B \to\to B = \frac{1}{4} \quad , u = -2 \to 1 = -4A \to A = -\frac{1}{4}$$

$$= \frac{1}{2}\left(\int \frac{-\frac{1}{4}}{(u+2)}du - \int \frac{\frac{1}{4}}{(u-2)}du\right) = -\frac{1}{8}\int \frac{du}{(u+2)} + \frac{1}{8}\int \frac{du}{(u-2)}$$

$$= \frac{1}{8}ln\ |(u-2)| - \frac{1}{8}ln\ |(u+2)| = \frac{1}{8}ln\ \left|\frac{(u-2)}{(u+2)}\right|$$

$$= \frac{1}{8}ln\ \left|\frac{(y+3-2)}{(y+3+2)}\right| = \frac{1}{8}ln\ \left|\frac{(y+1)}{(y+5)}\right| = \frac{1}{8}ln\ \left|\frac{(x^2+1)}{(x^2+5)}\right| + c$$

$$105. \int \frac{x+2}{x(x-3)}dx = \frac{A}{x} + \frac{B}{(x-3)}$$

$$= \frac{A(x-3)+Bx}{x(x-3)} = \frac{Ax-3A+Bx}{x(x-3)}$$

$$x \to 1 = A+B \quad , \quad x^0 \to 2 = -3A \to A = -\frac{2}{3}$$

$$A = -\frac{2}{3} \quad ,1 = -\frac{2}{3} + B \to B = \frac{5}{3}$$

$$= -\frac{2}{3}\int \frac{dx}{x} + \frac{5}{3}\int \frac{B}{(x-3)}dx = \frac{2}{3} ln\ |x|\ \frac{5}{3} ln\ |(x-3)| + c$$

106. $\int \frac{2x^2+x+3}{(x+2)(x^2+x+1)}dx$

$$= \frac{A}{(x+2)} + \frac{Bx+C}{(x^2+x+1)} = \frac{A(x^2+x+1)+(Bx+C)(x+2)}{(x+2)(x^2+x+1)}$$

$$= \frac{Ax^2+Ax+A+Bx^2+2Bx+Cx+2C}{(x+2)(x^2+x+1)}$$

$$x^2 \to 2 = A + B \quad \to B = 2 - A$$

$$x \to 1 = A + 2B + C$$

$$x^0 \to 3 = A + 2C \quad \to \frac{1}{2}(3 - A)$$

$$1 = A + 2B + C \to 1 = A + 4 - 2A + \frac{1}{2}(3 - A)$$

$$1 = 4 - A + \frac{3}{2} - \frac{A}{2} \to 1 = -\frac{3A}{2} - 4A + \frac{11}{2}$$

$$1 - \frac{11}{2} = -\frac{3A}{2} = -\frac{9}{2} = -\frac{3A}{2} = 9 = 3A \to A = 3$$

$$B = 2 - A \to B = 2 - 3 \to B = -1 \quad , \quad C = 0$$

$$= 3\int \frac{dx}{(x+2)} + \int \frac{-x}{(x^2+x+1)}$$

$$3\ ln\ |(x+2)| + \int \frac{-x}{(x^2+x+1)}dx$$

$$\int \frac{-x}{(x^2+x+1)}dx \quad -x = \alpha(2x+1) + \beta$$

$-1 = 2\alpha \to \alpha = -\frac{1}{2} \quad , \quad 0 = \alpha + \beta \to \beta = \frac{1}{2}$

$-\frac{1}{2}\int \frac{(2x+1)}{(x^2+x+1)}dx + \frac{1}{2}\int \frac{dx}{(x^2+x+1)}$

$= -\frac{1}{2}ln\ |(x^2+x+1)|\ + \frac{1}{2}\int \frac{dx}{\left(x^2+x+\frac{1}{4}-\frac{1}{4}+1\right)}$

$= -\frac{1}{2}ln\ |(x^2+x+1)|\ + \frac{1}{2}\int \frac{dx}{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}}$

$= -\frac{1}{2}ln\ |(x^2+x+1)|\ + \frac{1}{2}\times\frac{2}{\sqrt{3}}tan^{-1}\frac{2\left(x+\frac{1}{2}\right)}{\sqrt{3}}$

$= 3\,ln\ |(x+2)|\ - \frac{1}{2}ln\ |(x^2+x+1)|\ + \frac{1}{\sqrt{3}}tan^{-1}\frac{(2x+1)}{\sqrt{3}}$

---

107. $\int \frac{x^3+x+1}{x^4-81}dx = .\int \frac{x^3+x+1}{(x^2+9)(x-3)(x+3)}dx$

$= \frac{Ax+B}{(x^2+9)} + \frac{C}{(x-3)} + \frac{D}{(x+3)}$

$= \frac{(Ax+B)(x-3)(x+3)+C\left(x^2+9\right)(x+3)+D\left(x^2+9\right)(x-3)}{(x^2+9)(x-3)(x+3)}$

$x = 3 \to 31 = 108C \to C = \frac{31}{108}$

$x = -3 \to -29 \to -108D \to D = \frac{29}{108}$

$= \frac{Ax^3-9Ax+Bx^2-9B+C\left(x^3+3x^2+9x+27\right)+D\left(x^3-3x^2+9x-27\right)}{(x^2+9)(x-3)(x+3)}$

$= \frac{Ax^3-9Ax+Bx^2-9B+Cx^3+3Cx^2+9Cx+27C+Dx^3-3Dx^2+9Dx-27D}{(x^2+9)(x-3)(x+3)}$

$x^3 \rightarrow 1 = A + C + D \qquad \rightarrow A = \frac{31}{108} + \frac{29}{108} = 1 - \frac{60}{108} \rightarrow A = \frac{48}{108}$

$$x^2 \rightarrow 0 = B + 3C - 3D \rightarrow 0 = B + 3 \times \frac{31}{108} - 3 \times \frac{29}{108}$$

$$0 = B + \frac{93}{108} - \frac{87}{108} \rightarrow B = \frac{6}{108}$$

$$= \int \frac{\frac{48}{108}x + \frac{6}{108}}{(x^2+9)} dx + \int \frac{\frac{31}{108}}{(x-3)} dx + \int \frac{\frac{29}{108}}{(x+3)} dx$$

$$= \frac{6}{108} \int \frac{8x+1}{(x^2+9)} dx + \frac{31}{108} \int \frac{dx}{(x-3)} + \frac{29}{108} \int \frac{dx}{(x+3)} dx$$

$$= \frac{6}{108} \int \frac{8x}{(x^2+9)} + \frac{1}{108} \int \frac{dx}{(x^2+9)} + \frac{31}{108} \int \frac{dx}{(x-3)} + \frac{29}{108} \int \frac{dx}{(x+3)}$$

$$= \frac{31}{108} ln \mid (x-3) \mid + \frac{29}{108} ln \mid (x+3) \mid$$

$$+ \frac{48}{2 \times 108} \int \frac{2x}{(x^2+9)} dx + \frac{6}{108} \int \frac{dx}{(x^2+9)} dx$$

$$= \frac{24}{108} \int \frac{2x}{(x^2+9)} + \frac{6}{108} \times \frac{1}{3} tan^{-1} \frac{x}{3}$$

$$= \frac{2}{9} ln \mid (x^2+9) \mid + \frac{1}{54} tan^{-1} \frac{x}{3}$$

$$= \frac{31}{108} ln \mid (x-3) \mid + \frac{29}{108} ln \mid (x+3) \mid \frac{2}{9} ln \mid (x^2+9) \mid + \frac{1}{54} tan^{-1} \frac{x}{3}$$

---

108. $\int \frac{dx}{x^3-8} = \int \frac{dx}{(x-2)(x^2+2x+4)}$

$$= \frac{A}{(x-2)} + \frac{Bx+C}{(x^2+2x+4)} = \frac{A(x^2+2x+4)+(Bx+C)(x-2)}{(x-2)(x^2+2x+4)}$$

$$= \frac{Ax^2+2Ax+4A+Bx^2-2Bx+Cx-2C}{(x-2)(x^2+2x+4)}$$

$x^2 \to 0 = A + B \quad , \; B = -A$

$x \to 0 = 2A - 2B + C$

$x^0 \to 1 = 4A - 2C \quad , \quad C = \frac{1}{2}(4A - 1)$

$0 = 2A - 2B + C \to 0 = 2A + 2A + 2A - \frac{1}{2}$

$\frac{1}{2} = 6A \to A = \frac{1}{12} \quad ,B = -\frac{1}{12} \; ,C = \frac{1}{2}(4A - 1) \to C = 1$

$A = \frac{1}{12} \quad ,B = -\frac{1}{12} \quad , \; C = 1$

$= \int \frac{\frac{1}{12}}{(x-2)} dx + \int \frac{-\frac{1}{12}x+1}{(x^2+2x+4)} dx$

$= \frac{1}{12} ln \; |(x - 2)| \; + \frac{1}{12} \int \frac{-x+12}{(x^2+2x+4)} dx$

$-x + 12 = \; \alpha(2x + 2) + \beta$

$-1 = 2\alpha \to \alpha = -\frac{1}{2} \quad , \quad 12 = \; 2\alpha + \beta \to \beta = 13$

$= \frac{1}{12}\left(-\frac{1}{2} \int \frac{(2x+2)}{(x^2+2x+4)} dx + 13 \int \frac{dx}{(x^2+2x+4)}\right)$

$= -\frac{1}{24} ln \; |(x^2 + 2x + 4)| \; + \frac{13}{12} \int \frac{dx}{(x^2+2x+1-1+4)}$

$= -\frac{1}{24} ln \; |(x^2 + 2x + 4)| \; + \frac{13}{12} \int \frac{dx}{(x+1)^2+3}$

$= -\frac{1}{24} ln \; |(x^2 + 2x + 4)| \; + \frac{13}{12} \times \frac{1}{\sqrt{3}} tan^{-1} \frac{(x+1)}{\sqrt{3}}$

$= \frac{1}{12} \ln |(x - 2)| - \frac{1}{24} \ln |(x^2 + 2x + 4)| + \frac{13}{12\sqrt{3}} \tan^{-1} \frac{(x+1)}{\sqrt{3}} + c$

$$109.\int \frac{x+1}{(x^2+1)(x^2+9)}dx = \frac{Ax+B}{(x^2+1)} + \frac{Cx+D}{(x^2+9)}$$

$$= \frac{(Ax+B)\left(x^2+9\right)+(Cx+D)\left(x^2+1\right)}{(x^2+1)(x^2+9)}$$

$$= \frac{Ax^3+9Ax+Bx^2+9B+Cx^3+Cx+Dx^2+D}{(x^2+1)(x^2+9)}$$

$$x^3 \to 0 = A + B \quad , \quad C = -A$$

$$x^2 \to 0 = B + D \quad ,D = -B$$

$$x \to 1 = 9A + C \to 1 = 9A - A \to A = \frac{1}{8} \quad , C = -\frac{1}{8}$$

$$x^0 \to 1 = 9B + D \to 1 = 9B - B \to B = \frac{1}{8} ,D = -\frac{1}{8}$$

$$= \int \frac{\frac{1}{8}x+\frac{1}{8}}{(x^2+1)}dx + \int \frac{-\frac{1}{8}x-\frac{1}{8}}{(x^2+9)}dx$$

$$= \frac{1}{8}\int \frac{x+1}{(x^2+1)}dx - \frac{1}{8}\int \frac{x+1}{(x^2+9)}dx$$

$$= \frac{1}{8\times2}\int \frac{2x}{(x^2+1)} + \frac{1}{8}\int \frac{1}{(x^2+1)} - \frac{1}{8\times2}\int \frac{2x}{(x^2+9)} - \frac{1}{8}\int \frac{1}{(x^2+9)}$$

$$= \frac{1}{16} ln \mid (x^2 + 1) \mid + \frac{1}{8} tan^{-1} x - \frac{1}{16} ln \mid (x^2 + 9) \mid - \frac{1}{24} tan^{-1}\frac{x}{3} + c$$

$$= \frac{1}{16} ln \mid \frac{(x^2+1)}{(x^2+9)} \mid + \frac{1}{8} tan^{-1} x - \frac{1}{24} tan^{-1}\frac{x}{3} + c$$

$$110.\int \frac{x^2+2}{x^4+4}dx$$

ملاظة / هناك عدة حلول لهذه المسألة على حسب الفرض الذي تفرضه 1- نقوم باستخراج x^2 من البسط والمقام

2- نأخذ البسط ونكامله ونفرضه ب y بعدها نربع y

$$\frac{x^2}{x^2}\int \frac{1+\frac{2}{x^2}}{x^2+\frac{4}{x^2}}dx \qquad put\ y = x-\frac{2}{x} \quad \to dy = 1+\frac{2}{x^2}$$

$$y^2 = x^2-4+\frac{4}{x^2} \to y^2+4 = x^2+\frac{4}{x^2}$$

$$= \int \frac{dy}{y^2+4}dy = \frac{1}{2}tan^{-1}\frac{y}{2}+c$$

$$= \frac{1}{2}tan^{-1}\frac{\left(x-\frac{2}{x}\right)}{2}+c$$

$$111. \int \frac{5x^3+9x^2-22x-8}{x^3-4x}dx$$

$$= 5x + \int \frac{9x^2-2x-8}{x^3-4x}dx$$

$$= \int \frac{9x^2-2x-8}{x(x-2)(x+2)}dx$$

$$= \frac{A}{x}+\frac{B}{(x-2)}+\frac{C}{(x+2)}$$

$$= \frac{A(x-2)(x+2)+Bx(x+2)+Cx(x-2)}{x(x-2)(x+2)}$$

$$x = 2 \to 24 = 8B \to B = 3 \quad , x = -2 \to 32 = 8C \to C = 4$$

$$\frac{Ax^2-4A+Bx^2+2Bx+Cx^2-2Cx}{x(x-2)(x+2)}$$

$$x^2 \to 9 = A+B+C \quad \to 9 = A+3+4 \to A = 2$$

$$A = 2 \quad , B = 3 \quad , \quad C = 4$$

$$= \int \frac{2}{x}dx + \int \frac{3}{(x-2)}dx + \int \frac{4}{(x+2)}dx$$

$$= 2\,ln\,|x| + 3\,ln\,|(x-2)| + 4\,ln\,|(x+2)| + c$$

$$= 5x + ln\,|x|^2 + ln\,|(x-2)^3| + ln\,|(x+2)^4| + c$$

$$= 5x + ln\,|x^2(x-2)^3(x+2)^4| + c$$

$$113. \int \frac{dx}{(x^2-4x+4)(x^2-4x+5)}$$

$$= \int \frac{dx}{(x-2)^2(x^2-4x+5)}$$

$$= \frac{A}{(x-2)} + \frac{B}{(x-2)^2} + \frac{Cx+D}{(x^2-4x+5)}$$

$$= \frac{A(x-2)(x^2-4x+5)+B(x^2-4x+5)+(Cx+D)(x-2)^2}{(x-2)^2(x^2-4x+5)}$$

$x = 2 \rightarrow 1 = B \rightarrow B = 1$ قبل ما نفك الاقواس نستطيع نجيب مجهول

$$= \frac{Ax^3-6Ax^2+13Ax-10A+Bx^2-4Bx+5B+Cx^3-2Cx^2+4Cx+Dx^2-2Dx+4D}{(x-2)^2(x^2-4x+5)}$$

$$x^3 \rightarrow 0 = A + C \quad \rightarrow C = -A$$

$$x^2 \rightarrow 0 = -6A + B - 2C$$

$$x \rightarrow 0 = 13A - 4B + 4C - 2D$$

$$x^0 \rightarrow 1 = -10A + 5B + 4D \rightarrow 1 = -10A + 5 + 4D$$

$$-4 = -10A - 4D \quad \rightarrow 4D = 10A - 4 \rightarrow D = \frac{5}{2}A - 1$$

$$0 = -6A + B - 2C \rightarrow 0 = -6A + 1 + 2A + \frac{5}{2}A - 1 \rightarrow A = 0$$

$$A = 0 \quad , \quad B = 1 \quad , \quad C = 0 \ , \quad D = -1$$

$$= \int \frac{0}{(x-2)} dx + \int \frac{1}{(x-2)^2} dx + \int \frac{-1}{(x^2-4x+5)} dx$$

$$= \int (x-2)^{-2}\, dx - \int \frac{dx}{(x^2-4x+4-4+5)}$$

$$= -\frac{1}{(x-2)} - \int \frac{dx}{(x-2)^2+1} = -\frac{1}{(x-2)} - tan^{-1}(x-2)$$

$$= \frac{1}{(2-x)} - tan^{-1}(x-2) + c$$

$$115. \int \frac{x^3+3}{(1+x)(1+x^2)} dx$$

*نفك الاقواس في المقام نلاحظ درجة البسط والمقام متساوية نقسم قسمة خوارزمية

$$= \int \frac{x^3+3}{x^3+x^2+x+1} dx$$

$$= \int dx + \int \frac{-x^2-x+2}{x^3+x^2+x+1} dx$$

$$= x + \int \frac{-x^2-x+2}{(1+x)(1+x^2)} dx$$

$$= \frac{A}{(1+x)} + \frac{Bx+C}{(1+x^2)} = \frac{A(1+x^2)+(Bx+C)(1+x)}{(1+x)(1+x^2)}$$

$$= \frac{A+Ax^2+Bx+Bx^2+C+Cx}{(1+x)(1+x^2)}$$

$$x^2 \rightarrow -1 = A + B \quad , \quad B = -1 - A$$

$$x \rightarrow -1 = B + C \quad , \quad x^0 \rightarrow 2 = A + C \ \rightarrow C = 2 - A$$

$$-1 = B + C \rightarrow -1 = -1 - A + 2 - A \rightarrow -2 = -2A \rightarrow A = 1$$

$$B = -1 - A \to -1 - 1 = -2 \quad , C = 2 - A \to 2 - 1 = 1$$

$$A = 1 \quad , \quad B = -2 \quad , \quad C = 1$$

$$= \int \frac{1}{(1+x)} dx + \int \frac{-2x+1}{(1+x^2)} dx$$

$$= \int \frac{1}{(1+x)} dx - \int \frac{2x}{(1+x^2)} dx + \int \frac{dx}{(1+x^2)} dx$$

$$= \ln |(1+x)| - \ln |(1+x^2)| + \tan^{-1} x + c$$

ملاحظة/ اذا كانت خطوات التكامل صح فالتكامل صح

$$= 116. \int \frac{x^2}{(1-x)^{100}} dx$$

$$put \quad y = 1 - x \quad \to dy = dx \quad , \quad x = 1 - y$$

$$\int \frac{(1-y)^2}{y^{100}} dy = \int \frac{1-2y+y^2}{y^{100}} dy$$

$$= \int \frac{1}{y^{100}} - \int \frac{2y}{y^{100}} + \int \frac{y^2}{y^{100}} == \int \frac{1}{y^{100}} - \int \frac{2}{y^{99}} + \int \frac{1}{y^{98}}$$

$$= -\frac{1}{99y^{99}} + \frac{2}{98y^{98}} - \frac{1}{97y^{97}} + c$$

$$= -\frac{1}{99(1-x)^{99}} + \frac{1}{49(1-x)^{98}} - \frac{1}{97(1-x)^{97}} + c$$

$$117. \int \frac{\sqrt{x}+\sqrt[3]{x}}{\sqrt[4]{x^5}-\sqrt[6]{x^7}} dx$$

نلا حظ ان الذي يقبل القسمة على 2و3و4و هو العدد 12 لذلك نفرض

$$y^{12} = x$$

$$\int \frac{x^{\frac{1}{2}}+x^{\frac{1}{3}}}{x^{\frac{5}{4}}-x^{\frac{7}{6}}} dx$$

$y^{12} = x \quad \rightarrow 12y^{11} = dx$

$12\int \frac{y^{11}(y^6+y^4)}{y^{15}-y^{14}} dy = 12\int \frac{y^{11}\times y^3(y^3+y)}{y^{14}(y-1)} dy$

$= 12\int \frac{(y^3+y)}{(y-1)}$

نلاحظ ان درجة البسط أكبر من درجة المقام

نقسم قسمة خوارزمية

$12\int(y^2+y+2)dy \; + 12\int \frac{2}{(y-1)} dy$

$= 12\left(\frac{y^3}{3}+\frac{y^2}{2}+2y\right) + 24\, ln \mid (y-1) \mid + c$

$= 4y^3 + 6y^2 + 24y + 24\, ln \mid (y-1) \mid + c$

$= 4\sqrt[4]{x} + 6\sqrt[6]{x} + 24\sqrt[24]{x} + 24\, ln \mid \left(\sqrt[24]{x} - 1\right) \mid + c$

118. $\int \frac{\sqrt[6]{x}}{1+\sqrt[3]{x}} dx = \int \frac{x^{\frac{1}{6}}}{1+x^{\frac{1}{3}}} dx \quad put\ y^6 = x \rightarrow 6y^5 = dx$

$6\int \frac{y\times y^5}{1+y^2} dy = 6\int \frac{y^6}{1+y^2} dy$

$6\int (y^4 - y^2 + 1)\, dy - 6\int \frac{1}{1+y^2} dy$

$= \frac{6}{5}y^5 - \frac{6}{3}y^3 + 6y - 6\, tan^{-1} y + c$

$= \frac{6}{5}\sqrt[6]{x^5} - 2\sqrt{x} + 6\sqrt[6]{x} - 6\, tan^{-1} \sqrt[6]{x} + c$

$$119. \int \frac{dx}{(1-x)\sqrt{1-x^2}} = \int \frac{dx}{(1-x)\sqrt{(1-x)(1+x)}}$$

$$\int \frac{dx}{(1-x)(1+x)\sqrt{\frac{1-x}{1+x}}}$$

*استخرجنا(1+x) ووضعنا (1+x) في المقام تحت الجذر

يعني $\sqrt{1+x} = (1+x)^1(1+x)^{\frac{-1}{2}}$

$$put\ y = \sqrt{\frac{1-x}{1+x}} \to y^2 = \frac{1-x}{1+x}$$

$$y^2(1+x) = 1-x \quad \to y^2 + xy^2 = 1-x$$

$$x + xy^2 = 1-y^2 \to x(1+y^2) = 1-y^2$$

$$x = \frac{1-y^2}{1+y^2} \quad ,(1+x) = 1 + \frac{1-y^2}{1+y^2} \to, \quad (1+x) = \frac{2}{1+y^2}$$

$$(1-x) = 1 - \frac{1-y^2}{1+y^2} \to (1-x) = \frac{1+y^2-1+y^2}{1+y^2} = \frac{2y^2}{1+y^2}$$

$$-dx = \frac{4y(1+y^2)-\left(2y(2y^2)\right)}{(1+y^2)^2} = \frac{4y+4y^2-4y^2}{(1+y^2)^2}$$

$$(1+x) = \frac{2}{(1+y^2)}, \quad (1-x) = \frac{2y^2}{(1+y^2)} \ ,dx = \frac{-4y}{(1+y^2)^2}$$

$$= \int \frac{4y}{\frac{2y^2}{(1+y^2)} \times \frac{2}{(1+y^2)} \times y(1+y^2)^2} dy$$

$$= -\int \frac{4y(1+y^2)(1+y^2)}{4y^2 \times y(1+y^2)^2} dy = \int \frac{dy}{y^2} = \frac{1}{y} + c$$

$$= \frac{1}{y} + c \quad = -\frac{1}{\sqrt{\frac{1-x}{1+x}}} = \sqrt{\frac{1+x}{1-x}} + c$$

120. $\int \frac{dx}{\sqrt[3]{(x+1)^2(x-1)^4}}$

$\sqrt[3]{(x-1)^4} = (x-1)^1(x-1)^{\frac{1}{3}}$

$\sqrt[3]{(x+1)^2} \ = (x+1)^1(x+1)^{\frac{-1}{3}}$

$$\int \frac{dx}{(x+1)(x-1)\sqrt[3]{\frac{(x-1)}{(x+1)}}}$$

$$put\ y = \sqrt[3]{\frac{(x-1)}{(x+1)}} \rightarrow y^3 = \frac{(x-1)}{(x+1)} \rightarrow y^3(x+1) = (x-1)$$

$$xy^3 + y^3 = x - 1 \quad , \quad \rightarrow 1 + y^3 = x - xy^3$$

$$1 + y^3 = x(1-y^3) \quad , \quad \rightarrow x = \frac{1+y^3}{1-y^3}$$

$$(x+1) = 1 + \frac{1+y^3}{1-y^3} \ \rightarrow= \frac{1-y^3+1+y^3}{1-y^3} = \frac{2}{1-y^3}$$

$$(x-1) = -1 + \frac{1+y^3}{1-y^3} \ \rightarrow= \frac{-1+y^3+1+y^3}{1-y^3} = \frac{2y^3}{1-y^3}$$

$$dx = \frac{6y^2}{(1-y^3)^2} \quad ,(x+1) = \frac{2}{(1-y^3)} \quad ,(x-1) = \frac{2y^3}{(1-y^3)}$$

$$\int \frac{6y^2}{\frac{2}{(1-y^3)} \times \frac{2y^3}{(1-y^3)} \times y(1-y^3)^2} dy$$

$$\int \frac{6y^2(1-y^3)(1-y^3)}{4y^4(1-y^3)^2} dy = \frac{3}{2}\int \frac{dy}{y^2} = -\frac{3}{2y} + c$$

$$= -\frac{3}{2\sqrt[3]{\frac{(x-1)}{(x+1)}}} + c = -\ \frac{3}{2}\sqrt[3]{\frac{(x+1)}{(x-1)}} + c$$

$$121. \int \frac{dx}{\sqrt{1-2x}-\sqrt[4]{1-2x}} = \int \frac{dx}{(1-2x)^{\frac{1}{2}}-(1-2x)^{\frac{1}{4}}}$$

$put\ y^4 = 1-2x \quad \rightarrow 4y^3 = -2dx \ \rightarrow -2y^3 = dx$

$$= \int \frac{-2y^3}{y^2-y} dy = -2\int \frac{y^3}{y^2-y}$$

$$= -2\int (y+1)\, dy - 2\int \frac{y}{(y^2-y)} dy$$

$$= -2\int (y+1) - 2\int \frac{y}{y(y-1)} dy$$

$$= -2\left(\frac{y^2}{2}+y\right) - 2\ln|y-1| + c$$

$$-y^2 - 2y - 2\ln|y-1| + c$$

$$-\sqrt{1-2x} - 2\sqrt[4]{1-2x} - 2\ln|\sqrt[4]{1-2x}-1| + c$$

$$122. \int \frac{dx}{(x+2)\sqrt{x^2+2x}} = \int \frac{dx}{(x+2)\sqrt{x(x+2)}}$$

$$= \int \frac{dx}{x(x+2)\sqrt{\frac{(x+2)}{x}}} \quad put\ y = \sqrt{\frac{(x+2)}{x}} \rightarrow y^2 = \frac{(x+2)}{x}$$

$$xy^2 = x+2 \rightarrow xy^2 - x = 2 \quad \rightarrow x(y^2-1) = 2$$

$$x = \frac{2}{(y^2-1)} \quad , \quad x+2 = 2 + \frac{2}{(y^2-1)} = \frac{2(y^2-1)+2}{(y^2-1)}$$

$$(x+2) = \frac{2y^2-2+2}{(y^2-1)} \quad , (x+2) = \frac{2y^2}{(y^2-1)}$$

$$dx = \frac{4y(y^2-1)-(2y\times 2y^2)}{(y^2-1)^2} = \frac{4y^3-4y-4y^3}{(y^2-1)^2}$$

$$dx = \frac{-4y}{(y^2-1)^2} \quad , \quad (x+2) = \frac{2y^2}{(y^2-1)} \quad , \quad \frac{2}{(y^2-1)}$$

$$= \int \frac{-4y}{\frac{2}{(y^2-1)}\times\frac{2y^2}{(y^2-1)}\times y\,(y^2-1)^2}\,dy$$

$$= \int \frac{-4y(y^2-1)(y^2-1)}{4y^3(y^2-1)^2}\,dy = -\int \frac{1}{y^2} = \frac{1}{y} + c$$

$$= \frac{1}{\sqrt{\frac{(x+2)}{x}}} + c \quad \rightarrow= \sqrt{\frac{x}{(x+2)}} + c$$

$$123. \int \frac{dx}{\sqrt{x}\left(1+\sqrt[4]{x}\right)^{10}} = \int \frac{dx}{x^{\frac{1}{2}}\left(1+x^{\frac{1}{4}}\right)^{10}}$$

$$put\ y^4 = x \quad \rightarrow 4y^3 = dx$$

$$\int \frac{4y^3}{y^2(1+y)^{10}}\,dy = 4\int \frac{y}{(1+y)^{10}}\,dy$$

$$put\ u = 1+y \quad , \quad du = dy \quad , \quad y = u-1$$

$$4\int \frac{(u-1)}{u^{10}}\,du = 4\int \frac{u}{u^{10}}\,du - 4\int \frac{1}{u^{10}}\,du$$

$$4\int \frac{1}{u^9}\,du - 4\int \frac{1}{u^{10}}\,du = -\frac{4}{8u^8} + \frac{4}{9u^9} + c$$

$$= \frac{-1}{2(1+y)^8} + \frac{4}{9(1+y)^9} = -\frac{1}{2\left(1+\sqrt[4]{x}\right)^8} + \frac{4}{9\left(1+\sqrt[4]{x}\right)^9} + c$$

$$124. \int \frac{dx}{x\sqrt{1+x^3}} = \int \frac{dx}{x\sqrt{1+x^3}} \times \frac{x^2}{x^2}$$

$$= \int \frac{x^2}{x^3\sqrt{1+x^3}} dx$$

$put\ y = 1 + x^3 \ \rightarrow dy = 3x^2\ dx \quad \rightarrow \frac{1}{3} dy = x^2\ dx$,

$x^3 = y - 1$

$$= \frac{1}{3}\int \frac{dy}{(y-1)\sqrt{y}} = \frac{1}{3}\int \frac{dy}{(\sqrt{y}-1)(\sqrt{y}+1)\sqrt{y}}$$

$put\ u = \sqrt{y} \quad \rightarrow \quad du = \frac{1}{2\sqrt{y}} dy \rightarrow 2du = \frac{1}{\sqrt{y}} dy$

$$= \frac{1}{3}\int \frac{2du}{(u-1)(u+1)} = \frac{2}{3}\int \frac{du}{(u-1)(u+1)}$$

$$= \frac{A}{(u-1)} + \frac{B}{(u+1)} = \frac{A(u+1)+B(u-1)}{(u-1)(u+1)}$$

$u = 1 \rightarrow 1 = 2A \ \rightarrow A = 1/2$

$u = -1 \rightarrow 1 = -2B \rightarrow B = -\frac{1}{2}$

$$= \frac{2}{3}\left[\frac{1}{2}\int \frac{du}{(u-1)} - \frac{1}{2}\int \frac{du}{(u+1)}\right]$$

$$= \frac{1}{3} ln\ |(u-1)| - \frac{1}{3} ln\ |(u+1)| = \frac{1}{3} ln\ \left|\frac{(u-1)}{(u+1)}\right| + c$$

$$= \frac{1}{3} ln\ \left|\frac{(\sqrt{y}-1)}{(\sqrt{y}+1)}\right| + c = \frac{1}{3} ln\ \left|\frac{(\sqrt{1+x^3}-1)}{(\sqrt{1+x^3}+1)}\right| + c$$

$$125.\int \frac{\sqrt{x}}{\sqrt{\sqrt{x}+1}}dx \quad put\ \ y^2 = x \quad \to 2ydy = dx$$

$$= \int \frac{2y}{\sqrt{y+1}}dy \ \ put\ u = y+1 \ \ \to du = dy \quad , \quad y = u-1$$

$$= 2\int \frac{u-1}{u^{\frac{1}{2}}}du = 2\int u^{\frac{1}{2}}(u-1)\,du = 2\int u^{\frac{3}{2}} - u^{\frac{1}{2}}\,du$$

$$= 2\left(\frac{2}{5}u^{\frac{5}{2}} - \frac{2}{3}u^{\frac{3}{2}}\right) = \frac{4}{5}(y+1)^{\frac{5}{2}} - \frac{4}{3}(y+1)^{\frac{3}{2}} + c$$

$$= \frac{4}{5}\left(\sqrt{x}+1\right)^{\frac{5}{2}} - \frac{4}{3}\left(\sqrt{x}+1\right)^{\frac{3}{2}} + c$$

$$= \frac{4}{5}\sqrt{\left(\sqrt{x}+1\right)^5} - \frac{4}{3}\sqrt{\left(\sqrt{x}+1\right)^3} + c$$

نستخرج عامل مشترك

$$= 4\sqrt{\sqrt{x}+1}\left[\frac{1}{5}\left(\sqrt{x}+1\right)^2 - \frac{1}{3}\left(\sqrt{x}+1\right)\right] + c$$

$$126.\int \sqrt[3]{x}\sqrt{5x\sqrt[3]{x}+3}\,dx = \int x^{\frac{1}{3}}\sqrt{5x.x^{\frac{1}{3}}+3}\,dx$$

$$= \int .\,x^{\frac{1}{3}}\sqrt{5x^{\frac{4}{3}}+3}\,dx \quad put\ \ y \ = \ 5x^{\frac{4}{3}}+3$$

$$dy = 5\times\frac{4}{3}.x^{\frac{1}{3}}dx \quad \to \frac{3}{20}dy = x^{\frac{1}{3}}dx$$

$$= \frac{3}{20}\int\sqrt{y}\,dy \ = \ \frac{3}{20}\int y^{\frac{1}{2}}dy = \frac{3}{20}\times\frac{2}{3}y^{\frac{3}{2}} = \frac{1}{2}\sqrt{y^3} + c$$

$$= \frac{1}{10}\sqrt{\left(5x^{\frac{4}{3}}+3\ \right)^3} + c$$

127. $\int \frac{dx}{x^3 \sqrt[3]{2-x^3}}$ put $y = \frac{1}{x} \quad \rightarrow x = \frac{1}{y} \qquad , dx = -\frac{1}{y^2}$

$$= -\int \frac{1}{y^2 \times \frac{1}{y^3} \times \sqrt[3]{2-\frac{1}{y^3}}} dy$$

$$= -\int \frac{y}{\sqrt[3]{\frac{2y^3-1}{y^3}}} dy = \int \frac{y^2}{\sqrt[3]{2y^3-1}} dy$$

$$= -\frac{1}{6}\int 6y^2 (2y^3-1)^{\frac{-1}{3}} dy = -\frac{1}{6} \times \frac{3}{2} (2y^3-1)^{\frac{2}{3}} + c$$

$$= -\frac{1}{4}\sqrt[3]{(2y^3-1)^2}$$

$$= -\frac{1}{4}\sqrt[3]{\left(2 \times \frac{1}{x^3} - 1\right)^2} = -\frac{1}{4}\sqrt[3]{\left(\frac{2}{x^3} - 1\right)^2}$$

$$= \frac{1}{4}\sqrt[3]{\left(\frac{2-x^3}{x^3}\right)^2} = \frac{1}{4}\sqrt[3]{\frac{(2-x^3)^2}{(x^3)^2}} = \frac{1}{4}\sqrt[3]{\frac{(2-x^3)^2}{(x^2)^3}}$$

$$= \frac{\sqrt[3]{(2-x^3)^2}}{4x^2} + c = \frac{(2-x^3)^{\frac{2}{3}}}{4x^2} + c$$

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128. $\int \frac{\sqrt{1+\sqrt[3]{x}}}{\sqrt[3]{x^2}} dx = \int \frac{\sqrt{1+x^{\frac{1}{3}}}}{x^{\frac{2}{3}}} dx$

put $y = 1 + x^{\frac{1}{3}} \rightarrow dy = \frac{1}{3} x^{\frac{-2}{3}} dx \rightarrow 3dy = \frac{dx}{x^{\frac{2}{3}}}$

$$= 3\int \sqrt{y}\, dy = 3\int (y)^{\frac{1}{2}} dy = 3 \times \frac{2}{3} (y)^{\frac{3}{2}} + c$$

$$= 2\left(1 + x^{\frac{1}{3}}\right)^{\frac{3}{2}} + c = 2(1 + \sqrt[3]{x})^{\frac{3}{2}} + c$$

$$129. \int \sqrt[3]{x}\ \sqrt[4]{2 + \sqrt[3]{x^2}}\, dx = \int x^{\frac{1}{3}} \sqrt[4]{2 + x^{\frac{2}{3}}} \quad \times \frac{x^{\frac{1}{3}}}{x^{\frac{1}{3}}}\, dx$$

$$= \int \frac{x^{\frac{2}{3}} \sqrt[4]{2+x^{\frac{2}{3}}}}{x^{\frac{1}{3}}} dx \qquad put\ y = 2 + x^{\frac{2}{3}} \ \ \rightarrow dy = \frac{2}{3}\ x^{\frac{-1}{3}} dx$$

$$\frac{3}{2} dy = \frac{dx}{x^{\frac{1}{3}}} \qquad\qquad ,\ \ x^{\frac{2}{3}} = y - 2$$

$$= \frac{3}{2}\int (y-2)\sqrt[4]{y}\, dy = \frac{3}{2}\int (y-2)\, y^{\frac{1}{4}} = \frac{3}{2}\left(y^{\frac{5}{4}} - 2y^{\frac{1}{4}}\right) dy$$

$$= \frac{3}{2}\left[\frac{4}{9} y^{\frac{9}{4}} - 2 \times \frac{4}{5} y^{\frac{5}{4}}\right] = \frac{2}{3}\left(2 + x^{\frac{2}{3}}\right)^{\frac{9}{5}} - \frac{12}{5}\left(2 + x^{\frac{2}{3}}\right)^{\frac{5}{4}} + c$$

$$= \frac{2}{3}\ \sqrt[4]{\left(2 + \sqrt[3]{x^2}\right)^9} - \frac{12}{5}\sqrt[4]{\left(2 + \sqrt[3]{x^2}\right)^5} + c$$

$$130. \int x^5 (1 + x^2)^{\frac{2}{3}}\, dx = \int x . (x^2)^2\, (1 + x^2)^{\frac{2}{3}}\, dx$$

$$put\ y = 1 + x^2 \quad \rightarrow dy = 2xdx \quad \rightarrow \frac{1}{2}\ dy = xdx$$

$$x^2 = y - 1$$

$$\frac{1}{2}\int (y-1)^2\, y^{\frac{2}{3}} = \frac{1}{2}\int (y^2 - 2y + 1) y^{\frac{2}{3}} dy$$

$$= \frac{1}{2}\int \left(y^{\frac{8}{3}} - 2y^{\frac{5}{3}} + y^{\frac{2}{3}}\right) dy$$

$$= \frac{1}{2}\left[\frac{3}{11}y^{\frac{11}{3}} - 2\times\frac{3}{8}y^{\frac{8}{3}} + \frac{3}{5}y^{\frac{5}{3}}\right] = \frac{3}{11}y^{\frac{8}{3}} - \frac{3}{8}y^{\frac{8}{3}} + \frac{3}{10}y^{\frac{5}{3}}$$

$$= \frac{3}{11}(1+x^2)^{\frac{11}{3}} - \frac{3}{8}(1+x^2)^{\frac{8}{3}} + \frac{3}{10}(1+x^2)^{\frac{5}{3}} + c$$

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131. $\int \frac{dx}{x^{11}\sqrt{1+x^4}}$ put $y = \frac{1}{x} \rightarrow x = \frac{1}{y} \rightarrow dx = -\frac{1}{y^2}dy$

$$-\int \frac{dy}{y^2\times\frac{1}{y^{11}}\times\sqrt{1+\frac{1}{y^4}}} = -\int \frac{y^9}{\sqrt{\frac{y^4+1}{y^4}}}dy = -\int \frac{y^{11}dy}{\sqrt{1+y^4}}$$

$$= -\int \frac{y^3.y^8}{\sqrt{1+y^4}}dy = -\int \frac{y^3.(y^4)^2}{\sqrt{1+y^4}}dy$$

put $u = 1 + y^4 \rightarrow du = 4y^3 \rightarrow \frac{1}{4}du = y^3dy$, $y^4 = u - 1$

$$= -\frac{1}{4}\int \frac{(u-1)^2}{u^{\frac{1}{2}}}du = -\frac{1}{4}\int u^{\frac{-1}{2}}(u^2 - 2u + 1)du$$

$$= -\frac{1}{4}\left(\int u^{\frac{3}{2}} - 2u^{\frac{1}{2}} + u^{\frac{-1}{2}}\right)du = -\frac{1}{4}\left(\frac{2}{5}u^{\frac{5}{2}} - 2\times\frac{2}{3}u^{\frac{3}{2}} + 2u^{\frac{1}{2}}\right)$$

$$= -\frac{1}{10}u^{\frac{5}{2}} + \frac{1}{3}u^{\frac{3}{2}} - \frac{1}{2}u^{\frac{1}{2}} + c$$

$$= -\frac{1}{10}(1+y^4)^{\frac{5}{2}} + \frac{1}{3}(1+y^4)^{\frac{3}{2}} - \frac{1}{2}(1+y^4)^{\frac{1}{2}}$$

$$= -\frac{1}{10}\sqrt{(1+y^4)^5} + \frac{1}{3}\sqrt{(1+y^4)^3} - \frac{1}{2}\sqrt{1+y^4}$$

$$= -\frac{1}{10}\sqrt{\left(1+\frac{1}{x^4}\right)^5} + \frac{1}{3}\sqrt{\left(1+\frac{1}{x^4}\right)^3} - \frac{1}{2}\sqrt{1+\frac{1}{x^4}}$$

$$= -\frac{1}{10}\sqrt{\left(\frac{x^4+1}{x^4}\right)^5} + \frac{1}{3}\sqrt{\left(\frac{x^4+1}{x^4}\right)^3} - \frac{1}{2}\sqrt{\left(\frac{x^4+1}{x^4}\right)}$$

$$= -\frac{1}{10}\sqrt{\frac{(x^4+1)^5}{x^{20}}} + \frac{1}{3}\sqrt{\frac{(x^4+1)^3}{x^{12}}} - \frac{1}{2}\sqrt{\frac{(x^4+1)}{x^4}}$$

$$= -\frac{1}{10}\frac{\sqrt{(x^4+1)^5}}{\sqrt{(x^{10})^2}} + \frac{1}{3}\frac{\sqrt{(x^4+1)^3}}{\sqrt{(x^6)^2}} - \frac{1}{2}\frac{\sqrt{(x^4+1)}}{\sqrt{(x^2)^2}}$$

$$= \frac{-1}{10x^{10}}\sqrt{(x^4+1)^5} + \frac{1}{3x^6}\sqrt{(x^4+1)^3} - \frac{1}{2x^2}\sqrt{(x^4+1)} + c$$

$$132.\ \int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{\sqrt{x}}\,dx \;=\; \int \frac{\sqrt[3]{1+x^{\frac{1}{4}}}}{x^{\frac{1}{2}}}\,dx$$

$$put\ y^4 = x \quad \rightarrow 4y^3dy = dx$$

$$= 4\int \frac{y^3\ \sqrt[3]{1+y}}{y^2}\ dy = 4\int y\ \sqrt[3]{1+y}\,dy$$

$$put\ u = 1+y \ \rightarrow du = dy \qquad , \quad y = u-1$$

$$= 4\int (u-1)\,u^{\frac{1}{3}}du = 4\int u^{\frac{4}{3}} \ - u^{\frac{1}{3}}du = 4\left(\frac{3}{7}u^{\frac{7}{3}} - \frac{3}{4}u^{\frac{4}{3}}\right) + c$$

$$= 4\left(\frac{3}{7}(1+y)^{\frac{7}{3}} - \frac{3}{4}(1+y)^{\frac{4}{3}}\right) == \frac{12}{7}\left(1+\sqrt[4]{x}\right)^{\frac{7}{3}} - 3\left(1+\sqrt[4]{x}\right)^{\frac{4}{3}} + \mathrm{c}$$

$$133. \int \frac{dx}{x\left(1+\sqrt[3]{x}\right)^2} = \int \frac{dx}{x\left(1+x^{\frac{1}{3}}\right)^2} \qquad put\ y^3 = x$$

$$3y^2 dy = dx$$

$$3\int \frac{y^2}{y^3(1+y)^2}\, dy = 3\int \frac{dy}{y(1+y)^2}$$

$$= \frac{A}{y} + \frac{B}{(1+y)} + \frac{C}{(1+y)^2} = \frac{A(1+y)^2 + By(1+y) + Cy}{y(1+y)^2}$$

$$= \frac{A\left(y^2+2y+1\right)+By(1+y)+Cy}{y(1+y)^2}$$

$$= \frac{Ay^2+2Ay+A+By+By^2+Cy}{y(1+y)^2}$$

$$y^2 \rightarrow 0 = A + B \quad , \quad B = -A$$

$$y \rightarrow 0 = 2A + B + C$$

$$y^0 \rightarrow 1 = A \quad , B = -A \rightarrow B = -1$$

$$0 = 2A + B + C \rightarrow 0 = 2 - 1 + C \rightarrow C = -1$$

$$= 3\left[\int \frac{1}{y} dy + \int \frac{-1}{(1+y)} dy + \int \frac{-1}{(1+y)^2} dy\right]$$

$$= 3\, ln\ |\, y \,| - 3\, ln\ |\,(1+y)\,| + \frac{3}{(1+y)}$$

$$= 3\, ln\ \left|\, \frac{y}{(1+y)} \,\right| + \frac{3}{(1+y)} + c$$

$$= 3\, ln\ \left|\, \frac{\sqrt[3]{x}}{(1+\sqrt[3]{x})} \,\right| + \frac{3}{(1+\sqrt[3]{x})} + c$$

$$134. \int x^3 \sqrt{1+x^2}\,dx = \int x.x^2 \sqrt{1+x^2}\,dx$$

$$put\ y = 1 + x^2 \ \rightarrow dy = 2xdx \ \rightarrow \frac{1}{2}dy = xdx \ \ , x^2 = y - 1$$

$$= \frac{1}{2}\int (y-1)\, y^{\frac{1}{2}} dy = \frac{1}{2}\int \left(y^{\frac{3}{2}} - y^{\frac{1}{2}}\right) dy = \frac{1}{2}\left(\frac{2}{5}y^{\frac{5}{2}} - \frac{2}{3}y^{\frac{3}{2}}\right)$$

$$= \frac{1}{5}(1+x^2)^{\frac{5}{2}} - \frac{1}{3}(1+x^2)^{\frac{3}{2}}$$

$$= \frac{1}{15}\left[3(1+x^2)^{\frac{5}{2}} - 5(1+x^2)^{\frac{3}{2}}\right]$$

$$= \frac{1}{15}\left[3(1+x^2)(1+x^2)^{\frac{3}{2}} - 5(1+x^2)^{\frac{3}{2}}\right] +$$

$$= \frac{1}{15}(1+x^2)^{\frac{3}{2}}[3(1+x^2) - 5]$$

$$= \frac{1}{15}(1+x^2)^{\frac{3}{2}}[3 + 3x^2 - 5] = \frac{1}{15}(1+x^2)^{\frac{3}{2}}(3x^2 - 2) + c$$

$$135. \int \frac{dx}{x^4\sqrt{1+x^2}} \quad put\ \ y = \frac{1}{x} \quad \rightarrow x = \frac{1}{y} \rightarrow dx = -\frac{1}{y^2}$$

$$= \int -\frac{1}{y^2 \times \frac{1}{y^4}\sqrt{1+\frac{1}{y^2}}}dy = -\int \frac{y^2}{\sqrt{\frac{y^2+1}{y^2}}}dy$$

$$= -\int \frac{y.y^2}{\sqrt{y^2+1}}dy \quad put\ u = y^2 + 1 \ \rightarrow du = 2ydy$$

$$= \frac{1}{2}du = ydy \qquad , \qquad y^2 = u - 1$$

$$= -\frac{1}{2}\int \frac{(u-1)}{u^{\frac{1}{2}}}du = -\frac{1}{2}\int u^{\frac{-1}{2}}(u-1)\,du$$

$$= -\frac{1}{2}\int u^{\frac{1}{2}} - u^{\frac{-1}{2}} du = -\frac{1}{2}\left[\frac{2}{3}u^{\frac{3}{2}} - 2u^{\frac{1}{2}}\right]$$

$$= -\frac{1}{3}(y^2+1)^{\frac{3}{2}} + (y^2+1)^{\frac{1}{2}} == -\frac{1}{3}\left(\frac{1}{x^2}+1\right)^{\frac{3}{2}} + \left(\frac{1}{x^2}+1\right)^{\frac{1}{2}}$$

$$= -\frac{1}{3}\left(\frac{1+x^2}{x^2}\right)^{\frac{3}{2}} + \left(\frac{1+x^2}{x^2}\right)^{\frac{1}{2}}$$

$$= -\frac{1}{3}\frac{\sqrt{(1+x^2)^3}}{\sqrt{(x^2)^3}} + \frac{\sqrt{1+x^2}}{\sqrt{x^2}} = -\frac{1}{3}\frac{\sqrt{(1+x^2)^3}}{\sqrt{(x^3)^2}} + \frac{\sqrt{1+x^2}}{\sqrt{x^2}}$$

$$= -\frac{1}{3x^3}\sqrt{(1+x^2)^3} + \frac{1}{x}\sqrt{1+x^2} + c$$

$$= \frac{-\sqrt{(1+x^2)^3} + 3x^2\sqrt{1+x^2}}{3x^3}$$

$$= \frac{-(1+x^2)\sqrt{1+x^2} + 3x^2\sqrt{1+x^2}}{3x^3}$$

$$= \frac{\sqrt{1+x^2}(-1-x^2+3x^2)}{3x^3} = \frac{\sqrt{1+x^2}(2x^2-1)}{3x^3} + c$$

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136. $\int \sqrt[3]{x}\ \ \sqrt[7]{1+\sqrt[3]{x^4}}\, dx = \int x^{\frac{1}{3}}\ \ \sqrt[7]{1+x^{\frac{4}{3}}}\, dx$

$put\ y = 1 + x^{\frac{4}{3}} \rightarrow dy = \frac{4}{3}x^{\frac{1}{3}}\, dx \rightarrow \frac{3}{4}dy = x^{\frac{1}{3}}dx$

$$= \frac{3}{4}\int \sqrt[7]{y}\, dy = \frac{3}{4}\int y^{\frac{1}{7}}dy = \frac{3}{4}\times\frac{7}{8}y^{\frac{8}{7}} = \frac{21}{32}\sqrt[7]{\left(1+\sqrt[3]{x^4}\right)^2} + c$$

$$137. \int \frac{dx}{x^3 \sqrt[5]{1+\frac{1}{x}}} = \int \frac{dx}{x.x^2 \sqrt[5]{1+\frac{1}{x}}} \qquad put\ y = 1 + \frac{1}{x}$$

$$dy = -\frac{dx}{x^2} \quad \rightarrow -dy = \frac{dx}{x^2} \qquad , \quad \frac{1}{x} = y - 1 \ \rightarrow x = \frac{1}{(y-1)}$$

$$= -\int \frac{dy}{\frac{1}{(y-1)} \times y^{\frac{1}{5}}} = -\int \frac{(y-1)}{y^{\frac{1}{5}}} dy = -\int y^{\frac{-1}{5}} (y-1) dy$$

$$= -\int \left(y^{\frac{4}{5}} - y^{\frac{-1}{5}} \right) dy = -\left[\frac{5}{9} y^{\frac{9}{5}} - \frac{5}{4} y^{\frac{4}{5}} \right] + c$$

$$= -\frac{5}{9}\left(1 + \frac{1}{x}\right)^{\frac{9}{5}} + \frac{5}{4}\left(1 + \frac{1}{x}\right)^{\frac{4}{5}} + c$$

$$= \frac{5}{4}\left(1 + \frac{1}{x}\right)^{\frac{4}{5}} + -\frac{5}{9}\left(1 + \frac{1}{x}\right)^{\frac{9}{5}} + c$$

$$139. \int \frac{\left(x+\sqrt{1+x^2}\right)^{15}}{\sqrt{1+x^2}} dx$$

$$= \int \frac{\left(x+\sqrt{1+x^2}\right)^{14} \left(x+\sqrt{1+x^2}\right)}{\sqrt{1+x^2}} dx$$

Put $y = \left(x + \sqrt{1 + x^2}\right) \quad \rightarrow dy = 1 + \frac{2x}{2\sqrt{1+x^2}}$

$$dy = 1 + \frac{x}{\sqrt{1+x^2}} dx = \frac{\left(x+\sqrt{1+x^2}\right)}{\sqrt{1+x^2}} dx$$

$$= \int y^{14}\, dy = \frac{1}{15} y^{15} = \frac{1}{15}\left(x + \sqrt{1 + x^2}\right)^{15} + c$$

$$140. \int \frac{dx}{\sqrt{1-x^2}-1} = \int \frac{dx}{\sqrt{1-x^2}-1} \times \frac{\sqrt{1-x^2}+1}{\sqrt{1-x^2}+1}$$

$$= \int \frac{\sqrt{1-x^2}+1}{1-x^2-1} dx = \int \frac{\sqrt{1-x^2}+1}{-x^2} = -\int \frac{\sqrt{1-x^2}+1}{x^2} dx$$

$$= -\int \frac{1}{x^2} dx - \int \frac{\sqrt{1-x^2}}{x^2} dx = \frac{1}{x} - \int \frac{\sqrt{1-x^2}}{x^2} dx$$

$$-\int \frac{\sqrt{1-x^2}}{x^2} dx \qquad put\ \ x = \sin y \qquad dx = \cos y\, dy$$

$$-\int \frac{\sqrt{1-\sin^2 y}}{\sin^2 y} \times \cos y\, dy = -\int \frac{\cos^2 y}{\sin^2 y} dy$$

$$= -\int \frac{1-\sin^2 y}{\sin^2 y} dy = -\int \frac{1}{\sin^2 y} dy + \int \frac{\sin^2 y}{\sin^2 y} dy$$

$$= -\int \csc^2 y\, dy + \int dy = -(-\cot y) + y$$

$$= \cot(\sin^{-1} x) + (\sin^{-1} x)$$

$$= \frac{1}{x} + \cot(\sin^{-1} x) + (\sin^{-1} x) + c$$

$$141. \int \frac{dx}{x-\sqrt{x^2+2x+4}} \qquad a > 0$$

$$(t - x) = -\sqrt{x^2 + 2x + 4}$$

$$(t - x)^2 = x^2 + 2x + 4$$

$t^2 - 2tx + x^2 = x^2 + 2x + 4 \quad ,\rightarrow t^2 - 2tx = 2x + 4$

$t^2 - 4 = 2x + 2tx \qquad ,\rightarrow\ t^2 - 4 = x(2 + 2t)$

$$x = \frac{t^2-4}{2+2t} \quad , \quad dx = \frac{2t(2+2t)-2(t^2-4)}{(2+2t)^2}$$

$$dx = \frac{4t+4t^2-2t^2+8}{(2+2t)^2} = \frac{2t^2+4t+8}{(2+2t)^2}$$

$$= \int \frac{2t^2+4t+8}{t(2+2t)^2} = 2\int \frac{t^2+2t+4}{t(2+2t)^2}$$

$t - x = -\sqrt{x^2+2x+4}$

$t = x - \sqrt{x^2+2x+4}$

$$= \frac{A}{t} + \frac{B}{(2+2t)} + \frac{C}{(2+2t)^2} = \frac{A(2+2t)^2 + Bt(2+2t) + Ct}{t(2+2t)^2}$$

$$= \frac{A(4t^2+8t+4)+2Bt+2Bt^2+Ct}{t(2+2t)^2}$$

$$= \frac{4At^2+8At+4A+2Bt+2Bt^2+Ct}{t(2+2t)^2}$$

$t^2 \to 1 = 4A + 2B \quad , \quad t \to 2 = 8A + 2B + C$

$t^0 \to 4 = 4A \quad \to A = 1 \quad , \quad B = -\frac{3}{2}$

$2 = 8A + 2B + C \to 2 = 8 - 3 + C \to C = -3$

$A = 1 \quad , \quad B = -\frac{3}{2} \quad , C = -3$

$$= 2\left[\int \frac{1}{t}dt + \int \frac{-\frac{3}{2}}{(2+2t)}dy + \int \frac{-3}{(2+2t)^2}dy\right]$$

$$= 2\int \frac{1}{t}dt - \frac{3}{2}\int \frac{2}{(2+2t)}dy - \frac{3}{2}\int \frac{2}{(2+2t)^2}dy$$

$$= 2\ln|t| - \frac{3}{2}\ln|(2+2t)| + \frac{3}{(2+2t)}$$

$= 2\,ln\,|\,x - \sqrt{x^2 + 2x + 4}\,| - \frac{3}{2}ln\,|\left(2 + 2x - 2\sqrt{x^2 + 2x + 4}\right)| + \frac{3}{\left(2+2x-2\sqrt{x^2+2x+4}\right)}$

$== 2\,ln\,|\,x - \sqrt{x^2 + 2x + 4}\,| - \frac{3}{2}ln\,|\left(2 + 2x - 2\sqrt{x^2 + 2x + 4}\right)| + \frac{3}{2\left(1+x-\sqrt{x^2+2x+4}\right)} + c$

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142. $\int \frac{dx}{x+\sqrt{x^2-x+1}} \qquad , \quad a > 0$

$(t - x) = \sqrt{x^2 - x + 1}$

$(t - x)^2 = x^2 - x + 1 \rightarrow t^2 - 2tx + x^2 = x^2 - x + 1$

$t^2 - 1 = 2tx - x \qquad ,t^2 - 1 = x(2t - 1)$

$x = \frac{(t^2-1)}{(2t-1)} \qquad , \qquad dx = \frac{2t(2t-1)-2(t^2-1)}{(2t-1)^2}$

$dx = \frac{4t^2-2t-2t^2+2}{(2t-1)^2} = \frac{2t^2-2t+2}{(2t-1)^2}$

$= \int \frac{2t^2-2t+2}{t(2t-1)^2} = 2\int \frac{t^2-t+1}{t(2t-1)^2}dt$

$t - x = \sqrt{x^2 - x + 1}$

$t = x + \sqrt{x^2 - x + 1}$

$= \frac{A}{t} + \frac{B}{(2t-1)} + \frac{C}{(2t-1)^2} = \frac{A(2t-1)^2+Bt(2t-1)+Ct}{t(2t-1)^2}$

$= \frac{A\left(4t^2-4t+1\right)+Bt(2t-1)+Ct}{t(2t-1)^2}$

$t^2 \rightarrow 1 = 4A + 2B \qquad , \quad t \rightarrow -1 = -4A - B + C$

$$t^0 \to 4 = 4A \quad \to A = 4 \quad , \ B = -\frac{3}{2}$$

$$-1 = -4A - B + C = -1 = -4 + \frac{3}{2} + C \to C = -\frac{3}{2}$$

$$A = 1 \quad , \qquad B = -\frac{3}{2} \qquad , C = -\frac{3}{2}$$

$$= 2\left[\int \frac{1}{t} dt + \int \frac{-\frac{3}{2}}{(2t-1)} dy + \int \frac{-\frac{3}{2}}{(2t-1)^2} dy\right]$$

$$= 2\int \frac{1}{t} dt - \frac{3}{2}\int \frac{2}{(2t-1)} dy - \frac{3}{2}\int \frac{2}{(2t-1)^2} dy$$

$$= 2\, ln\ | t | - \frac{3}{2} ln\ | (2t-1) | + \frac{3}{(2t-1)}$$

$$= 2\, ln\ | x + \sqrt{x^2 - x + 1} | - \frac{3}{2} ln\ | \left(2x + 2\sqrt{x^2 - x + 1} - 1\right) | + \frac{3}{(2x+2\sqrt{x^2-x+1}-1)} + c$$

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146. $\int \frac{dx}{\sqrt{2+x-x^2}}$

نلاحظ ان ماتحت الجذر نستطيع ان نحلله

لذلك نفرض $t(x+1)$

$$2 + x - x^2 = (x+1)(2-x)$$

$$\sqrt{2 + x - x^2} = t(x+1)$$

$$2 + x - x^2 = t^2(x+1)^2 \to (x+1)(2-x) = t^2(x+1)^2$$

$$(2-x) = t^2(x+1) \to 2 - x = t^2 x + t^2$$

$$2 - t^2 = t^2 x + x \quad \to 2 - t^2 = x(t^2+1)$$

$$x = \frac{(2-t^2)}{(t^2+1)} \qquad \to dx = \frac{-2t(t^2+1) - 2t(2-t^2)}{(t^2+1)^2}$$

$$dx=\frac{-2t^3-2t-4t+2t^3}{(t^2+1)^2}=\frac{-6t}{(t^2+1)^2}$$

$$\int\frac{-6t}{(t^2+1)^2\times\frac{3t}{(t^2+1)}}dt$$

$$\sqrt{2+x-x^2}=t(x+1)$$

$$t(x+1)=t\left(\frac{(2-t^2)}{(t^2+1)}+1\right)=\frac{3t}{(t^2+1)}$$

$$-\int\frac{6t\times(t^2+1)}{(t^2+1)^2\times3t}=-\int\frac{2}{(t^2+1)}dt=-2\tan^{-1}t$$

$$\sqrt{2+x-x^2}=t(x+1)$$

$$t=\frac{\sqrt{2+x-x^2}}{(x+1)}=\frac{\sqrt{(x+1)(2-x)}}{(x+1)}=\frac{\sqrt{(x+1)}\sqrt{(2-x)}}{(x+1)}$$

$$\frac{\sqrt{(x+1)}\sqrt{(2-x)}}{(x+1)}\times\frac{\sqrt{(x+1)}}{\sqrt{(x+1)}}=\frac{(x+1)\sqrt{(2-x)}}{(x+1)\sqrt{(x+1)}}=\frac{\sqrt{(2-x)}}{\sqrt{(x+1)}}$$

$$t=\sqrt{\frac{(2-x)}{(x+1)}}$$

$$\int\frac{dx}{\sqrt{2+x-x^2}}=-2\tan^{-1}t=-2\tan^{-1}\sqrt{\frac{(2-x)}{(x+1)}}+c$$

---

143. $\int\frac{dx}{\sqrt{x^2-x-1}}$

$(t-x)=\sqrt{x^2-x-1}$ $\quad ,t=x+\sqrt{x^2-x-1}$

$$(t-x)^2=x^2-x-1$$

$$t^2-2tx+x^2=x^2-x-1\rightarrow t^2-2tx=-x-1$$

$$t^2+1=2tx-x \rightarrow t^2+1=x(2t-1)$$

$$x=\frac{(t^2+1)}{(2t-1)} \quad \rightarrow dx=\frac{2t(2t-1)-2(t^2+1)}{(2t-1)^2}$$

$$dx=\frac{4t^2-2t-2t^2-2}{(2t-1)^2}=\frac{2t^2-2t-2}{(2t-1)^2}=\frac{2(t^2-t-1)}{(2t-1)^2}$$

$$\sqrt{x^2-x-1}=(t-x)$$

$$=\left(t-\frac{(t^2+1)}{(2t-1)}\right)=\frac{2t^2-t-t^2-1}{(2t-1)}=\frac{t^2-t-1}{(2t-1)}$$

$$\int \frac{2(t^2-t-1)}{(2t-1)^2\times\frac{t^2-t-1}{(2t-1)}}dt=\int \frac{2(t^2-t-1)(2t-1)}{(2t-1)^2\times(t^2-t-1)}dt$$

$$=\int \frac{2}{(2t-1)}dt=ln\ |(2t-1)|$$

$$=ln\ |(2t-1)|\ =ln\ |\left(2x+2\sqrt{x^2-x-1}-1\right)|$$

$$ln\ |2\left(x-\frac{1}{2}+\sqrt{x^2-x-1}\right)|\ +c$$

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145. $\int \frac{dx}{\sqrt{-x^2-2x+8}}$

نلاحظ ان ماتحت الجذر نستطيع ان نحلله

لذلك نفرض $t(x+4)$

$-x^2-2x+8=(x+4)(2-x)$

$$\sqrt{-x^2-2x+8}=t(x+4)$$

$$-x^2-2x+8=t^2(x+4)^2$$

$$(x+4)(2-x)=t^2(x+4)^2 \rightarrow (2-x)=t^2(x+4)$$

$$(2-x)=t^2x+4t^2 \qquad \rightarrow 2-4t^2=t^2x+x$$

$$2-4t^2 = x(t^2+1) \quad \rightarrow x = \frac{(2-4t^2)}{(t^2+1)}$$

$$dx = \frac{-8t(t^2+1)-2t(2-4t^2)}{(t^2+1)^2} = \frac{-8t^3-8t-4t+8t^3}{(t^2+1)^2}$$

$$dx = \frac{-12t}{(t^2+1)^2}dt$$

$$\sqrt{-x^2-2x+8} = t(x+4)$$

$$= t\left(\frac{(2-4t^2)}{(t^2+1)}+4\right) = \frac{t(2-4t^2+4t^2+4)}{(t^2+1)} = \frac{6t}{(t^2+1)}$$

$$= \int \frac{-12t}{(t^2+1)^2 \times \frac{6t}{(t^2+1)}}dt = \int \frac{-12t \times (t^2+1)}{(t^2+1)^2 \times 6t}$$

$$= -\int \frac{2}{(t^2+1)}dt = -2\tan^{-1}t + c$$

$$\sqrt{-x^2-2x+8} = t(x+4) \rightarrow t = \frac{\sqrt{-x^2-2x+8}}{(x+4)}$$

$$t = \frac{\sqrt{(x+4)(2-x)}}{(x+4)} = \frac{\sqrt{(x+4)}\sqrt{(2-x)}}{(x+4)} \times \frac{\sqrt{(x+4)}}{\sqrt{(x+4)}}$$

$$t = \frac{(x+4)\sqrt{(2-x)}}{(x+4)\sqrt{(x+4)}} = \frac{\sqrt{(2-x)}}{\sqrt{(x+4)}} = \sqrt{\frac{(2-x)}{(x+4)}}$$

$$= -2\tan^{-1}\sqrt{\frac{(2-x)}{(x+4)}} + c$$

147. $\int \sin^3 x \cos^4 x\, dx = \int \sin x\,(\sin^2 x) \cos^4 x\, dx$

$= \int \sin x\,(1 - \cos^2 x) \cos^4 x$

put $y = \cos x \quad \rightarrow dy = -\sin x \quad \rightarrow -dy = \sin x\, dx$

$= -\int (1 - y^2)\, y^4 dy = -\int (y^4 - y^6)\, dy$

$= -\left(\frac{1}{5} y^5 - \frac{1}{7} y^7\right) = \frac{1}{7} y^7 - \frac{1}{5} y^5 = \frac{1}{7} \cos^7 x - \frac{1}{5} \cos^5 x + c$

---

148. $\int \sin^3 x \cos x\, x\, dx = \int \sin x\,(\sin^2 x) \cos x\, dx$

put $y = \sin x \quad \rightarrow dy = \cos x \;\; dx$

$\int y^3\, dy = \frac{1}{4} y^4 + c = \frac{1}{4} \sin^4 x + c$

---

149. $\int \cos^3 x\, dx = \int (\cos^2 x) \cos x\, dx$

$= \int (1 - \sin^2 x) \cos x\, dx \qquad$ put $y = \sin x \rightarrow dy = \cos x\, dx$

$= \int (1 - y^2)\, dy = y - \frac{1}{3} y^3 = \sin x - \frac{1}{3} \sin^3 x + c$

---

150. $\int \sin^5 x\, dx = \int (\sin^2 x)^2 \sin x\, dx$

$= \int (1 - \cos^2 x)^2 \sin x\, dx \qquad$ put $y = \cos x \quad \rightarrow dy = -\sin x$

$= -\int (1 - y^2)^2\, dy = -\int (1 - 2y^2 + y^4)\, dy$

$= -\left(y - \frac{2}{3} y^3 + \frac{1}{5} y^5\right) = -\cos x + \frac{2}{3} \cos^3 x - \frac{1}{5} \cos^5 x + c$

$$151. \int sin^4 x\,dx = \int (\sin^2 x)^2\,dx$$

$$sin^2 x = \frac{1}{2}(1 - cos2\,x)$$

$$cos^2 x = \frac{1}{2}(1 + cos2\,x)$$

$$= \int \left(\frac{1}{2}(1 - cos2\,x)\right)^2 dx = \frac{1}{4}\int (1 - cos\,2x)^2\,dx$$

$$= \frac{1}{4}\int (1 - 2\,cos\,2x + cos^2\,2x)$$

$$= \frac{1}{4}x - \frac{1}{4} \times 2 \times \frac{1}{2} sin\,2x + \frac{1}{4}\int cos^2\,2x\,dx$$

$$\frac{1}{4}\int cos^2\,2x\,dx = \frac{1}{4} \times \frac{1}{2}\int (1 + cos4\,x)\,dx = \frac{1}{8}x + \frac{1}{8} \times \frac{1}{4} sin\,4x$$

$$\int sin^4 x\,dx = \frac{1}{4}x - \frac{1}{4}sin\,2x + \frac{1}{8}x + \frac{1}{32}sin\,4x$$

$$= \frac{3}{8}x - \frac{1}{4}sin\,2x + \frac{1}{32}sin\,4x + c$$

$$152. \int cos^6 x\,dx = \int (cos^2 x)^3\,dx$$

$$\frac{1}{8}\int (1 + cos\,2x)^3\,dx$$

$$= \frac{1}{8}\int (1 + 3\,cos\,2x + 3\,cos^2\,2x + cos^3\,2x)\,dx$$

$$I_1 = \int (1 + 3\,cos\,2x)\,dx = x + \frac{3}{2}sin\,2x$$

$$I_2 = 3\int cos^2\,2x\,dx = 3\int \frac{1}{2}(1 + cos4\,x)\,dx$$

$$I_2 = \frac{3}{2}\int (1 + cos4\,x)\,dx = \frac{3}{2}\left(x + \frac{1}{4}sin\,4x\right) =$$

$$I_2 = \frac{3}{2}x + \frac{3}{8}\sin 4x$$

$$I_3 = \int \cos^3 2x\,dx = \int (\cos^2 2\,x)\cos 2x\,dx$$

$$I_3 = \int (1 - \sin^2 2\,x)\cos x\,dx = \int (\cos 2x - \sin^2 2x\cos 2\,x)$$

$$I_3 = \frac{1}{2}\sin 2x - \frac{1}{6}\sin^3 2\,x$$

$$\int \cos^6 x\,dx = \frac{1}{8}[I_1 + I_2 + I_3]$$

$$= \frac{1}{8}\left[x + \frac{3}{2}\sin 2x + \frac{3}{2}x + \frac{3}{8}\sin 4x + \frac{1}{2}\sin 2x - \frac{1}{6}\sin^3 2\,x\right]$$

$$= \frac{1}{8}\left[\frac{5}{2}x + 2\sin 2x + \frac{3}{8}\sin 4x - \frac{1}{6}\sin^3 2\,x\right]$$

$$= \frac{5}{16}x + \frac{1}{4}\sin 2x + \frac{3}{64}\sin 4x - \frac{1}{48}\sin^3 2\,x + c$$

---

153. $\int \sin^2 x \cos^4 x\,dx = \int \left(\frac{1}{2}(1 - \cos 2x)\left(\frac{1}{2}(1 + \cos 2x)\right)^2\right)$

$$\frac{1}{8}\int (1 - \cos 2x)(1 + \cos 2x)^2 = \frac{1}{8}(1 - \cos^2 2x)(1 + \cos 2x)$$

$$= \frac{1}{8}\int (\overbrace{1 + \cos 2x} - \overbrace{\cos^2 2x} - \overbrace{\cos^3 2x})dx$$

$$I_1 = \int (1 + \cos 2x)\,dx = x + \frac{1}{2}\sin 2x$$

$$I_2 = -\int \cos^2 2x\,dx = -\frac{1}{2}\int (1 + \cos 4x)\,dx$$

$$I_2 = -\frac{1}{2}\left[x + \frac{1}{4}\sin 4x\right] = -\frac{1}{2}x - \frac{1}{8}\sin 4x$$

$I_3 = -\int cos^3 2x\,dx = -\int (cos^2 2x)\, cos\, 2x\, dx$

$I_3 = -\int (1 - sin^2 2x)\, cos\, 2x\, dx$

$= -\int (cos\, 2x - sin^2 2x\;\; cos\, 2x)\, dx$

$I_3 = -\left(\frac{1}{2} sin\, 2x - \frac{1}{6} sin^3 2x\right) = -\frac{1}{2} sin\, 2x + \frac{1}{6} sin^3 2x$

$\int sin^2 x\, cos^4 x\, dx = \frac{1}{8}[I_1 + I_2 + I_3]$

$= \frac{1}{8}\left[x + \frac{1}{2} sin\, 2x - \frac{1}{2} x - \frac{1}{8} sin\, 4x - \frac{1}{2} sin\, 2x + \frac{1}{6} sin^3 2x\right]$

$= \frac{1}{8}\left(\frac{1}{2} x - \frac{1}{8} sin\, 4x + \frac{1}{6} sin^3 2x\right)$

$= \frac{1}{16} x - \frac{1}{64} sin\, 4x + \frac{1}{48} sin^3 2x + c$

---

155. $\int \frac{cos^3 x}{sin^6 x} dx = \int \frac{(cos^2 x)\, cos\, x}{sin^6 x} dx = \int \frac{(1 - sin^2 x)\, cos\, x}{sin^6 x}$

$put\ y = sin\, x \quad \rightarrow dy = cos\, x$

$\int \frac{(1-y^2)}{y^6} dy = \int \frac{1}{y^6} - \frac{1}{y^4} dy = -\frac{1}{5y^5} + \frac{1}{3y^3} + c$

$= \frac{1}{3\, sin^3 x} - \frac{1}{5\, sin^5 x} + c$

---

156. $\int \frac{cos^2 x}{sin^4 x} dx = \int \frac{cos^2 x}{sin^2 x\, sin^2 x} dx$

$= \int cot^2 x\ .csc^2 x\, dx\ \ put\ y = cot\, x \quad \rightarrow dy = -csc^2 x\, dx$

$$-\int y^2\,dy = -\frac{1}{3}y^3 = -\frac{1}{3}cot^3 x + c$$

157. $\int \frac{cos^4 x}{sin^2 x}dx = \int \frac{(cos^2 x)^2}{sin^2 x}dx = \int \frac{(1-sin^2 x)^2}{sin^2 x}dx$

$$= \int \frac{(1-2\,sin^2 x+sin^4 x)}{sin^2 x}dx = \int \frac{1}{sin^2 x} - 2 + sin^2 x\ \ dx$$

$$= \int csc^2 x - 2 + sin^2 x\ \ dx = -\cot x - 2x + \int sin^2 x\ dx$$

$$\int sin^2 x\ \ dx = \frac{1}{2}\int(1-\cos 2x)\,dx = \frac{1}{2}x - \frac{1}{4}\sin 2x$$

$$\int \frac{cos^4 x}{sin^2 x}dx = -\cot x - 2x + \frac{1}{2}x - \frac{1}{4}\sin 2x + c$$

$$= -\cot x - \frac{1}{4}\sin 2x - \frac{3}{2}x + c$$

158. $\int \frac{dx}{cos^2 x} = \int sec^6 x\,dx = \int sec^2 x\,sec^4 x\,dx$

$$= \int (1+tan^2 x)^2 sec^2 x\,dx$$

$put\ y = \tan x \qquad \rightarrow dy = sec^2 x\,dx$

$$\int(1+y^2)^2\,dy = \int(1+2y^2+y^4)\,dy = y + \frac{2}{3}y^3 + \frac{1}{5}y^5 + c$$

$$= \tan x + \frac{2}{3}tan^3 x + \frac{1}{5}tan^5 x + c$$

$$159. \int \frac{dx}{\sin^4 x} = \int csc^4 x\, dx = \int (1 + cot^2 x)\, csc^2 x\, dx$$

$$put\ y = \cot x \qquad \rightarrow dy = -csc^2 x\, dx \rightarrow -dy = csc^2 x\, dx$$

$$= -\int (1 + y^2)\, dy = -\left(y + \frac{1}{3} y^3\right) + c$$

$$= -\cot x + \frac{1}{3} cot^3 x + c$$

$$160. \int \frac{dx}{\cos^4 x} = \int (1 + tan^2 x)\, sec^2 x\, dx$$

$$put\ y = \tan x \qquad \rightarrow dy = sec^2 x\, dx$$

$$\int (1 + y^2)\, dy = y + \frac{1}{3} tan^3 x + c$$

$$161. \int \frac{\cos^5 x}{\sin x} dx = \int \frac{(\cos^2 x)^2 \cos x}{\sin x} dx = \int \frac{(1 - \sin^2 x)^2 \cos x}{\sin x} dx$$

$$put\ y = \sin x \quad \rightarrow dy = \cos x\, dx$$

$$\int \frac{(1 - y^2)^2}{y} dy = \int \frac{(1 - 2y^2 + y^4)}{y} dy = \int \left(\frac{1}{y} - 2y + y^3\right) dy$$

$$= \ln |y| - y^2 + \frac{1}{4} y^4 = \ln |\sin x| - \sin^2 x + \frac{1}{4} \sin^4 x + c$$

$$162. \int \frac{\sin^2 x}{\cos^6 x} dx = \int \frac{\sin^2 x}{\cos^2 x \cos^4 x} dx$$

$$= \int tan^2 x\, (1 + tan^2 x)\, sec^2 x\, dx = \int tan^2 x\, (1 + tan^2 x)\, sec^2 x\, dx$$

$$put\ y = \tan x \quad \rightarrow dy = sec^2 x\ dx$$

$$= \int y^2(1+y^2)\,dy = \int y^2 + y^4 dy = \frac{1}{3}y^3 + \frac{1}{5}y^5$$

$$= \frac{1}{3}\tan^3 x + \frac{1}{5}\tan^5 x + c$$

163. $$\int \frac{\cos^3 x+\cos^5 x}{\sin^2 x+\sin^5 x}dx = \int \frac{\cos x\left(\cos^2 x+(\cos^2 x)^2\right)}{\sin^2 x(1+\sin^3 x)}dx$$

$$\int \frac{(1-\sin^2 x)(1-\sin^2 x)^2 \cos x}{\sin^2 x(1+\sin^3 x)}dx$$

put $y = \sin x \quad \rightarrow dy = \cos x\, dx$

$$= \int \frac{(1-y^2)(1-y^2)^2}{(y^2+y^4)}dy = \int \frac{1-y^2+1-2y^2+y^4}{(y^2+y^4)}dy$$

$$= \int \frac{y^4-3y^2+2}{(y^2+y^4)}dy$$

درجة البسط تساوي درجة المقام

نقسم قسمة خوارزمية

$$= \int dy + \int \frac{-4y^2+2}{(y^2+y^4)}dy = y + \int \frac{-4y^2+2}{(y^2+y^4)}$$

$$\int \frac{-4y^2+2}{y^2(1+y^2)}dy = \frac{Ay+B}{y^2} + \frac{Cy+D}{(1+y^2)}$$

$$= \frac{(Ay+B)(1+y^2)+(Cy+D)y^2}{y^2(1+y^2)}$$

$$= \frac{Ay+Ay^3+B+By^2+Cy^3+Dy^2}{y^2(1+y^2)}$$

$y^3 \rightarrow 0 = A + C$

$y^2 \rightarrow -4 = B + D$

$$y \to 0 = A \quad \to A = 0 \quad , \quad C = 0$$

$$y^0 \to 2 = B \quad B = 2 \quad , -4 = 2 + D \quad \to D = -6$$

$$= \int \frac{2}{y^2} dy + \int \frac{-6}{(1+y^2)} dy = -\frac{2}{y} - 6 \tan^{-1} y$$

$$\int \frac{\cos^3 x + \cos^5 x}{\sin^2 x + \sin^5 x} dx = y - \frac{2}{y} - 6 \tan^{-1} y + c$$

$$= \sin x - \frac{2}{\sin x} - 6 \tan^{-1}(\sin x) + c$$

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164. $\int \frac{\sin x + \sin^3 x}{\cos 2x} dx = \int \frac{\sin x(1+\sin^2 x)}{2\cos^2 x - 1} dx$

$$= \int \frac{\sin x(1+1-\cos^2 x)}{2\cos^2 x - 1} dx = \int \frac{\sin x(2-\cos^2 x)}{2\cos^2 x - 1} dx$$

$put\ y = \cos x \quad \to dy = -\sin x\, dx \quad \to -dy = \sin x\, dx$

درجة البسط تساوي درجة المقام

نقسم قسمة خوارزمية

$$= -\int \frac{(2-y^2)}{(2y^2-1)} dy$$

$$= \frac{1}{2}\int dy - \frac{3}{2}\int \frac{dy}{2y^2-1} = \frac{1}{2}y - \frac{3}{2}\int \frac{dy}{(\sqrt{2}y-1)(\sqrt{2}y+1)}$$

$$= \frac{A}{(\sqrt{2}y-1)} + \frac{B}{(\sqrt{2}y+1)} = \frac{A(\sqrt{2}y+1)+B(\sqrt{2}y-1)}{(\sqrt{2}y-1)(\sqrt{2}y+1)}$$

$$= \frac{\sqrt{2}Ay + A + \sqrt{2}By - B}{(\sqrt{2}y-1)(\sqrt{2}y+1)}$$

$$y \to 0 = \sqrt{2}A + \sqrt{2}B \quad , \quad y^0 \to 1 = A - B \quad , B = A - 1$$

$$0=\sqrt{2}A+\sqrt{2}B \quad \rightarrow 0=\sqrt{2}A+\sqrt{2}(A-1)$$

$$0=\sqrt{2}A+\sqrt{2}A-\sqrt{2} \qquad \rightarrow 2\sqrt{2}A=\sqrt{2} \rightarrow A=\frac{1}{2}$$

$$B=\frac{1}{2}-1 \rightarrow B=-\frac{1}{2}$$

$$=-\frac{3}{2}\left[\int \frac{\frac{1}{2}}{(\sqrt{2}y-1)}dy+\int \frac{-\frac{1}{2}}{(\sqrt{2}y+1)}dy\right]$$

$$=-\frac{3}{2}\left[\frac{1}{2\sqrt{2}}ln\,|(\sqrt{2}y-1)|-\frac{1}{2\sqrt{2}}ln\,|(\sqrt{2}y+1)|\right]$$

$$=-\frac{3}{4\sqrt{2}}\left[ln\,|(\sqrt{2}y-1)|-ln\,|(\sqrt{2}y+1)|\right]$$

$$=-\frac{3}{4\sqrt{2}}\left[ln\frac{(\sqrt{2}y-1)}{(\sqrt{2}y+1)}\right]=-\frac{3}{4\sqrt{2}}\left[ln\frac{(\sqrt{2}\cos x-1)}{(\sqrt{2}\cos x+1)}\right]$$

$$\int \frac{\sin x+\sin^3 x}{\cos 2x}dx=\frac{1}{2}\cos x-\frac{3}{4}\left[ln\frac{(\sqrt{2}\cos x-1)}{(\sqrt{2}\cos x+1)}\right]+c$$

166. $\int \frac{\cos^3 x}{\sqrt{\sin^3 x}}dx=\int \frac{(\cos^2 x)\cos x}{\sqrt{\sin^3 x}}dx$

$$=\int \frac{(1-\sin^2 x)\cos x}{\sqrt{\sin^3 x}}dx \qquad put\ y=\sin x \quad \rightarrow dy=\cos x\,dx$$

$$=\int \frac{(1-y^2)}{\sqrt{y^3}}dy=\int y^{\frac{-3}{2}}(1-y^2)dy=\int \left(y^{\frac{-3}{2}}-y^{\frac{1}{2}}\right)dy$$

$$=-2y^{\frac{-1}{2}}-\frac{2}{3}y^{\frac{3}{2}}+c=\frac{-2}{\sqrt{\sin x}}-\frac{2}{3}\sqrt{\sin^3 x}+c$$

$167. \int \frac{\sin x}{\sqrt{\cos x}} dx \qquad put\ y = \cos x \quad \rightarrow -dy = \sin x\, dx$

$= -\int \frac{dy}{\sqrt{y}} = -\int y^{\frac{-1}{2}}\, dy = -2y^{\frac{1}{2}} = -2\sqrt{\cos x} + c$

$168. \int \frac{\sin^5 x}{\sqrt[3]{\cos x}} dx = \int \frac{(\sin^2 x)^2 \sin x}{\sqrt[3]{\cos x}} dx$

$= \int \frac{(1-\cos^2 x)^2 \sin x}{\sqrt[3]{\cos x}} dx$

$put\ y = \cos x \quad \rightarrow dy = -\sin x \quad \rightarrow -dy = \sin x\, dx$

$= -\int \frac{(1-y^2)^2}{\sqrt[3]{y}} = -\int y^{\frac{-1}{3}} (1 - 2y^2 + y^4) dy$

$= -\int \left(y^{\frac{-1}{3}} - 2y^{\frac{5}{3}} + y^{\frac{11}{3}}\right) dy$

$= -\left[\frac{3}{2} y^{\frac{2}{3}} - \frac{3}{8} \times 2y^{\frac{8}{3}} + \frac{3}{14} y^{\frac{14}{3}}\right] + c$

$= -\frac{3}{2}\sqrt[3]{\cos^2 x} + \frac{6}{8}\sqrt[3]{\cos^8 x} - \frac{3}{14}\sqrt[3]{\cos^{14} x} + c$

$169. \int \frac{\sin^3 x}{\cos x \sqrt[3]{\cos x}} dx = \int \frac{(\sin^2 x) \sin x}{\cos x \sqrt[3]{\cos x}} dx$

$= \int \frac{(1-\cos^2 x) \sin x}{\cos x \sqrt[3]{\cos x}} dx$

$put\ y = \cos x \quad \rightarrow dy = -\sin x dx \quad \rightarrow -dy = \sin x\, dx$

$$= -\int \frac{(1-y^2)}{y\sqrt[3]{y}} dy = -\int \frac{(1-y^2)}{y \times y^{\frac{1}{3}}} dy = -\int \frac{(1-y^2)}{y^{\frac{4}{3}}} dy$$

$$= -\int y^{\frac{-4}{3}}(1-y^2)\, dy = \left(y^{\frac{4}{3}} - y^{\frac{2}{3}}\right) dy$$

$$= -\left[-3y^{\frac{-1}{3}} - \frac{3}{5} y^{\frac{5}{3}}\right] + c$$

$$= \frac{3}{\sqrt[3]{\sin x}} + \frac{3}{5}\sqrt[3]{\sin^5 x} + c$$

$$170. \int \frac{dx}{\sin^2 x \cos^4 x} dx = \int \frac{(1+\tan^2 x)\sec^2 x}{\sin^2 x} dx$$

$$= \int \frac{(1+\tan^2 x)\sec^2 x}{\sin^2 x \times \frac{\cos^2 x}{\cos^2 x}} dx = \int \frac{(1+\tan^2 x)^2 \sec^2 x}{\tan^2 x} dx$$

$$put\ y = \tan x \to dy = \sec^2 x dx$$

$$= \int \frac{(1+y^2)^2}{y^2} dx = \int \frac{(1+2y^2+y^4)}{y^2} dy$$

$$= \int \left(\frac{1}{y^2} + 2 + y^2\right) dy = -\frac{1}{y} + 2y + \frac{1}{3}y^3 + c$$

$$= -\frac{1}{\tan x} + 2\tan x + \frac{1}{3}\tan^3 x + c$$

$$= 2\tan x + \frac{1}{3}\tan^3 x - \cot x + c$$

$$171. \int \frac{dx}{\sqrt[4]{\sin^5 x \cos^3 x}} = \int \frac{dx}{\sin^{\frac{5}{4}} x \cos^{\frac{3}{4}} x} dx$$

$$= \int \frac{dx}{\sin^{\frac{5}{4}} x \cos^{\frac{3}{4}} x \times \frac{\cos^{\frac{5}{4}} x}{\cos^{\frac{5}{4}} x}} = \int \frac{dx}{\tan^{\frac{5}{4}} x \times \cos^{\frac{3}{4}+\frac{5}{4}} x}$$

$$= \int \frac{dx}{\tan^{\frac{5}{4}} x \times \cos^2 x} = \int \frac{\sec^2 x}{\tan^{\frac{5}{4}} x} dx$$

$$put\ y = \tan x \rightarrow dy = \sec^2 x\ dx$$

$$= \int \frac{dy}{y^{\frac{5}{4}}} = \int y^{\frac{-5}{4}} dy = -4y^{\frac{-1}{4}} + c$$

$$= -\frac{4}{\sqrt[4]{\tan x}} = -4\sqrt[4]{\cot x} + c$$

$$173. \int \tan^4 x\, dx = \int \tan^4 x \times \frac{\cos^2 x}{\cos^2 x} dx$$

$$= \int \frac{\tan^4 x \sec^2 x}{\sec^2 x} dx = \int \frac{\tan^4 x \sec^2 x}{(1+\tan^2 x)} dx$$

$$put\ y = \tan x \rightarrow dy = \sec^2 x\ dx$$

$$= \int \frac{y^4}{(1+y^2)} dy$$

درجة البسط اكبر من درجة المقام

نقسم قسمة خوارزمية

$$= \int (y^2 - 1) dy + \int \frac{1}{1+y^2} dy$$

$$= \frac{y^3}{3} - y + \tan^{-1} y + c$$

$= \frac{1}{3}\tan^3 x - \tan x + \tan^{-1}(\tan x) + c$

$= \frac{1}{3}\tan^3 x - \tan x + x + c$

---

174. $\int \tan^5 x\, dx = \int \tan^5 x \times \frac{\cos^2 x}{\cos^2 x} dx$

$= \int \frac{\tan^5 x \sec^2 x}{\sec^2 x} dx = \int \frac{\tan^5 x \sec^2 x}{(1+\tan^2 x)} dx$

$put\ y = \tan x \to dy = \sec^2 x\ dx$

$= \int \frac{y^5}{(1+y^2)} dy = \int (y^3 - y) dy + \int \frac{y}{1+y^2} dy$

$= \frac{1}{4}y^4 - \frac{1}{2}y^2 + \frac{1}{2}\int \frac{2y}{1+y^2} dy$

$= \frac{1}{4}y^4 - \frac{1}{2}y^2 + \frac{1}{2}\ln|1+y^2| + c$

$= \frac{1}{4}\tan^4 x - \frac{1}{2}\tan^2 x + + \frac{1}{2}\ln|1+\tan^2 x| + c$

---

172. $\int \frac{\sin^3 x}{\sqrt[3]{\cos^2 x}} dx = \int \frac{(\sin^2 x)\sin x}{\sqrt[3]{\cos^2 x}} dx$

$= \int \frac{(1-\cos^2 x)\sin x}{\sqrt[3]{\cos^2 x}} dx$

$put\ y = \cos x \quad \to dy = -\sin x \to -dy = \sin x\, dx$

$= -\int \frac{(1-y^2)}{\sqrt[3]{y^2}} dy = -\int y^{\frac{-2}{3}}(1-y^2)\, dy = -\int y^{\frac{-2}{3}} - y^{\frac{4}{3}}\, dy$

$$= -3y^{\frac{1}{3}} + \frac{3}{7}y^{\frac{7}{3}} = -3\sqrt[3]{\cos x} + \frac{3}{7}\sqrt[3]{\cos^7 x} + c$$

$$= 3\sqrt[3]{\cos x}\left(\frac{1}{7}\cos^2 x - 1\right) + c$$

---

175. $\int \cot^6 x\, dx = \int \cot^6 x \times \frac{\sin^2 x}{\sin^2 x} dx$

$$= \int \frac{\cot^6 x \csc^2 x}{\csc^2 x} dx = \int \frac{\cot^6 x \csc^2 x}{(1+\cot^2 x)} dx$$

put $y = \cot x \rightarrow dy = -\csc^2 x\, dx \rightarrow -dy = \csc^2 x\, dx$

$$= -\int \frac{y^6}{(1+y^2)} dy = -\int (y^4 - y^2 - 1) dy - \int \frac{dy}{1+y^2}$$

$$= -\frac{1}{5}y^5 + \frac{1}{3}y^3 - y - \tan^{-1} y + c$$

$$= -\frac{1}{5}\cot^5 x + \frac{1}{3}\cot^3 x - \cot x - \tan^{-1}(\cot x) + c$$

$$= -\cot x + \frac{1}{3}\cot^3 x - \frac{1}{5}\cot^5 x - x + c$$

---

176. $\int \sin 3x \sin x\, dx$

$\cos(x+y) - \cos(x-y) = -2\sin x \sin y$

$$= -\frac{1}{2}\int -2\sin 3x \sin x\, dx$$

$$= -\frac{1}{2}\cos(3x + x) - \cos(3x - x)$$

$$= -\frac{1}{2}\int \cos(4x) - \cos(2x)\, dx$$

$= -\frac{1}{2}\left(\frac{1}{4}\sin x - \frac{1}{2}\sin 2x\right) + c$

$-\frac{1}{8}\sin x + \frac{1}{4}\sin x + c$

---

177. $\int \sin^2 \frac{x}{4} \cos^2 \frac{x}{4}\, dx$

$= \frac{1}{4}\int \left(1 - \cos\frac{x}{2}\right)\left(1 + \cos\frac{x}{2}\right) dx$

$\frac{1}{4}\int \left(1 - \cos^2\frac{x}{2}\right) dx = \frac{1}{4}\int \sin^2\frac{x}{2}\, dx$

$= \frac{1}{4}\int \frac{1}{2}(1 - \cos x)dx = \frac{1}{8}\int (1 - \cos x)\, dx$

$= \frac{1}{8}(x - \sin x) + c = \frac{x}{8} - \frac{\sin x}{8} + c$

---

178. $\int \cos\frac{x}{2}\cos\frac{x}{3}\, dx$

$= \frac{1}{2}\int 2\cos\frac{x}{2}\cos\frac{x}{3}\, dx$

$= \frac{1}{2}\int \left[\cos\left(\frac{x}{2} + \frac{x}{3}\right) + \cos\left(\frac{x}{2} - \frac{x}{3}\right)\right] dx$

$= \frac{1}{2}\int \left[\cos\left(\frac{5x}{6}\right) + \cos\left(\frac{x}{6}\right)\right] dx$

$= \frac{1}{2}\left(\frac{6}{5}\sin\left(\frac{5x}{6}\right) + 6\sin\left(\frac{x}{6}\right)\right) + c$

$= \frac{3}{5}\sin\left(\frac{5x}{6}\right) + 3\sin\left(\frac{x}{6}\right) + c$

$$179. \int \frac{dx}{1-\sin x}\ dx$$

$$y = \tan\frac{x}{2} \quad , \sin x = \frac{2y}{1+y^2} \quad , \quad dx = \frac{2}{1+y^2}dy$$

$$= \int \frac{2}{(1+y^2)\times\left(1-\frac{2y}{(1+y^2)}\right)}dy$$

$$= \int \frac{2}{(1+y^2)-2y}dy = \int \frac{2}{y^2-2y+1}dy$$

$$= \int \frac{2}{(y-1)^2}dy = 2\int (y-1)^{-2}\,dy = -\frac{2}{(y-1)} + c$$

$$= -\frac{2}{\left(\tan\frac{x}{2}-1\right)} + c$$

$$179. \int \frac{dx}{1-\sin x}\ dx$$

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$$\int \frac{dx}{1-\sin x} \times \frac{1+\sin x}{1+\sin x}dx$$

$$= \int \frac{1+\sin x}{1-\sin^2 x}dx = \int \frac{1+\sin x}{\cos^2 x}dx = \int \frac{dx}{\cos^2 x} + \int \frac{\sin x}{\cos^2 x}$$

$$= \int \sec^2 x\,dx + \int \tan x \sec x\,dx$$

$$= \tan x + \sec x + c$$

---

180. $\int \frac{dx}{3+5\sin x+3\cos x}$ $\qquad y = \tan\frac{x}{2}$

$$\sin x = \frac{2y}{1+y^2} \quad ,\cos x = \frac{1-y^2}{1+y^2} \quad ,dx = \frac{2}{1+y^2}dy$$

$$= \int \frac{2}{(1+y^2)\left[3+5\times\frac{2y}{(1+y^2)}+3\times\frac{(1-y^2)}{(1+y^2)}\right]}dy$$

$$= \int \frac{2}{3+3y^2+10y+3-3y^2}dy = 2\int \frac{1}{(6+10y)}dy$$

$$= 2\int \frac{1}{2(3+5y)}dy = \int \frac{1}{(3+5y)}dy = \frac{1}{5}\int \frac{5}{(3+5y)}dy$$

$$= \frac{1}{5}\ln|(3+5y)| + c = \frac{1}{5}\ln\left|\left(3+\tan\frac{x}{2}\right)\right| + c$$

---

181. $\int \frac{dx}{5+\sin x+3\cos x}$ $\qquad y = \tan\frac{x}{2}$

$$\sin x = \frac{2y}{1+y^2} \quad ,\cos x = \frac{1-y^2}{1+y^2} \quad ,dx = \frac{2}{1+y^2}dy$$

$$= \int \frac{2}{(1+y^2)\left[5+\frac{2y}{(1+y^2)}+3\times\frac{(1-y^2)}{(1+y^2)}\right]}dy$$

$$= \int \frac{2}{5+5y^2+2y+3-3y^2}dy = 2\int \frac{1}{2y^2+2y+8}dy$$

$$= 2\int \frac{1}{2(y^2+y+4)}dy = \int \frac{1}{\left(y^2+y+\frac{1}{4}-\frac{1}{4}+4\right)}dy$$

$$= \int \frac{dy}{\left(y+\frac{1}{2}\right)^2+\frac{15}{4}} = \frac{2}{\sqrt{15}} tan^{-1} \frac{2\left(y+\frac{1}{2}\right)}{\sqrt{15}}$$

$$= \frac{2}{\sqrt{15}} tan^{-1} \frac{(2y+1)}{\sqrt{15}} + c = \frac{2}{\sqrt{15}} tan^{-1} \frac{\left(2\, tan \frac{x}{2}+1\right)}{\sqrt{15}} + c$$

182. $\int \frac{cos^3 x}{sin^2 x+sin\ x} dx = \int \frac{(cos^2 x)\, cos\ x}{sin^2 x+sin\ x} dx$

$$= \int \frac{(1-sin^2 x)\, cos\ x}{sin^2 x+sin\ x} dx$$

$put\ y = \ sin\, x \qquad \rightarrow dy = cos\, x\, dx$

$$= \int \frac{(1-y^2)}{y^2+y} dy = \int \frac{(1-y)(1+y)}{y(1+y)} dy = \int \frac{1}{y} - \frac{y}{y} dy$$

$$= ln\ |y| - y + c == ln\ |\ sin\, x\ | - sin\, x + c$$

183. $\int \frac{sin\ x}{1+sin\ x} dx = \int \frac{sin\ x}{1+sin\ x} \times \frac{1-sin\ x}{1-sin\ x} dx$

$$= \int \frac{sin\ x - sin^2 x}{1-sin^2 x} dx = \int \frac{sin\ x-sin^2 x}{cos^2 x} dx$$

$$= \int \frac{sin\ x}{cos^2 x} dx - \int \frac{sin^2 x}{cos^2 x} dx$$

$$= \int tan\, x\, sec\, x\, dx - \int \frac{1-cos^2 x}{cos^2 x} dx$$

$$= \int tan\, x\, sec\, x\, dx - \int \frac{1}{cos^2 x} + \int dx$$

$$= sec\, x - tan\, x + x + c$$

$$184. \int \frac{x^2-2}{x^2+1} tan^{-1} x\, dx$$

$u = x^2 - 2$ $dv = \frac{tan^{-1} x}{x^2+1}$

$du = 2x\, dx$ $v = \frac{(tan^{-1} x)^2}{2}$

$$= \frac{(x^2-2)(tan^{-1} x)^2}{2} - \int x\, (tan^{-1} x)^2 dx$$

$u = (tan^{-1} x)^2$ $dv = x$

$du = \frac{2 \tan^{-1} x}{1+x^2}\, dx$ $v = \frac{x^2}{2}$

$$= -\left[\frac{x^2}{2}(tan^{-1} x)^2 - \int \frac{x^2 (tan^{-1} x)}{1+x^2} dx\right]$$

$$= -\frac{x^2}{2}(tan^{-1} x)^2 + \int \frac{x^2 (tan^{-1} x)}{1+x^2} dx$$

$u = \tan^{-1} x$ $dv = \frac{x^2}{1+x^2}$

$du = \frac{dx}{1+x^2}$ $v = x - \tan^{-1} x$

$$= (x - tan^{-1} x)\, tan^{-1} x - \int \frac{x - tan^{-1} x}{1+x^2} dx$$

$$-\int \frac{x - tan^{-1} x}{1+x^2} dx = -\int \frac{x}{1+x^2} dx + \int \frac{tan^{-1} x}{1+x^2} dx$$

$$= -\frac{1}{2}\int \frac{2x}{1+x^2}dx + \int \frac{\tan^{-1}x}{1+x^2}dx$$

$$-\frac{1}{2}\ln|1+x^2| + \frac{(\tan^{-1}x)^2}{2}$$

$$= \frac{(x^2-2)(\tan^{-1}x)^2}{2} - \frac{x^2}{2}(\tan^{-1}x)^2 + (x-\tan^{-1}x)\tan^{-1}x$$

$$-\frac{1}{2}\ln|1+x^2| + \frac{(\tan^{-1}x)^2}{2}$$

$$= \frac{x^2}{2}(\tan^{-1}x)^2 - (\tan^{-1}x)^2 - \frac{x^2}{2}(\tan^{-1}x)^2 + x\tan^{-1}x$$

$$-(\tan^{-1}x)^2 - \frac{1}{2}\ln|1+x^2| + \frac{(\tan^{-1}x)^2}{2} + c$$

$$= -2(\tan^{-1}x)^2 + x\tan^{-1}x - \frac{1}{2}\ln|1+x^2| + \frac{(\tan^{-1}x)^2}{2}$$

$$= x\tan^{-1}x - \frac{3}{2}(\tan^{-1}x)^2 - \frac{1}{2}\ln|1+x^2| + c$$

---

185. $\int \tan^{-1}\sqrt{x}\,dx$

$put\ y^2 = x \quad \rightarrow 2ydy = dx$

$$= 2\int y.\tan^{-1}y\,dy$$

$u = \tan^{-1}y$	$dv = y$
$du = \frac{dy}{1+y^2}$	$v = \frac{y^2}{2}$

$$= 2\left[\frac{y^2}{2}\tan^{-1}y - \frac{1}{2}\int \frac{y^2}{1+y^2}dy\right] = y^2\tan^{-1}y - \int \frac{y^2}{1+y^2}dy$$

$$-\int \frac{y^2}{1+y^2}\,dy = -\left[\int \frac{1+y^2}{1+y^2}\,dy - \int \frac{dy}{1+y^2}\right] dy$$

$$= -\int dy + \int \frac{dy}{1+y^2}\,dy = -y + tan^{-1}\, y$$

$$\int tan^{-1}\sqrt{x}\,dx = y^2\, tan^{-1}\, y - y + tan^{-1}\, y + c$$

$$= x\, tan^{-1}\sqrt{x} - \sqrt{x} + tan^{-1}\sqrt{x} + c$$

$$= (x+1)\, tan^{-1}\sqrt{x} - \sqrt{x} + c$$

---

186. $\int e^{2x}\, sin\, e^x\, dx$ $\quad$ $put\ y = e^x \to dy = e^x dx$

$$= \int y\,.\,sin\, y\, dy$$

$u = y$	$dv = \sin y$
$du = dy$	$v = -\cos y$

$$= -y\, cos\, y + \int cos\, y\, dy$$

$$= -y\, cos\, y + sin\, y + c$$

$$= -e^x\, cos\, e^x + sin\, e^x + c$$

$$= sin\, e^x = -e^x\, cos\, e^x + c$$

---

187. $\int (x+2)\, cos(x^2 + 4x + 1)\, dx$

$$put\ y = x^2 + 4x + 1$$

$dy = 2x + 4 \quad \to dy = 2(x+2) \to \frac{1}{2}dy = (x+2)dx$

$= \frac{1}{2}\int \cos y = \frac{1}{2}\sin y + c$

$= \frac{1}{2}\sin(x^2 + 4x + 1) + c$

---

188. $\int \frac{dx}{a^2\cos^2 x + b^2\sin^2 x} = \int \frac{\frac{1}{\cos^2 x}}{a^2 + \tan^2 x\, b^2} dx$

$\int \frac{\sec^2 x}{a^2 x + \tan^2 x\, b^2} dx$

$put\ y = \tan x \quad \to dy = \sec^2 x\, dx$

$\int \frac{dy}{a^2 + y^2 b^2} dy \to put\ y = \frac{a}{b}\tan t \quad \to dy = \frac{a}{b}\sec^2 t\, dt$

$\int \frac{\frac{a}{b}\sec^2 t}{a^2 + \frac{a^2}{b^2} b^2 \tan^2 t} dt = \int \frac{\frac{a}{b}\sec^2 t}{a^2 + a^2\tan^2 t} dt$

$\frac{1}{ab}\int \frac{\sec^2 t}{\tan^2 t + 1} dt = \frac{1}{ab}\int \frac{\sec^2 t}{\sec^2 t} dt = \frac{1}{ab}\int dt = \frac{1}{ab} t + c$

$\frac{1}{ab}\tan^{-1}\left(\frac{b}{a}\tan x\right) + c$

$$189. \int \frac{dx}{\sin^2 x \cos^2 x} = \int \frac{dx}{\sin^2 x \cos^2 x \times \frac{\cos^2 x}{\cos^2 x}}$$

$$= \int \frac{\sec^2 x(1+\tan^2 x)}{\tan^2 x} dx \quad put\ y = \tan x \to dy = \sec^2 x\, dx$$

$$= \int \frac{1+y^2}{y^2} = \int \frac{1}{y^2} dy + \int dy = -\frac{1}{y} + y$$

$$= \tan x - \frac{1}{\tan x} + c = \tan x - \cot x + c$$

$$190. \int \frac{2\tan x+3}{\sin^2 x+2\cos^2 x} dx$$

$$= \int \frac{2\tan x}{\sin^2 x+2\cos^2 x} dx = i_1 + \int \frac{3}{\sin^2 x+2\cos^2 x} dx = i_2$$

$$i_1 = \int \frac{2\tan x}{\sin^2 x+2\cos^2 x \times \frac{\cos^2 x}{\cos^2 x}} dx$$

$$i_1 = \int \frac{(2\tan x)\sec^2 x}{\tan^2 x+2} dx \quad put\ y = \tan x \to dy = \sec^2 x\, dx$$

$$i_1 = \int \frac{2y}{y^2+2} dy = \ln|y^2+2| + c$$

$$i_2 = \int \frac{3}{\sin^2 x+2\cos^2 x \times \frac{\cos^2 x}{\cos^2 x}} dx$$

$$i_2 = \int \frac{3\sec^2 x}{\tan^2 x+2} dx \quad put\ y = \tan x \to dy = \sec^2 x\, dx$$

$$i_2 = \int \frac{3}{y^2+2} dy = \frac{3}{\sqrt{2}} \tan^{-1} \frac{y}{\sqrt{2}} + c$$

$$\int \frac{2\tan x+3}{\sin^2 x+2\cos^2 x} dx = i_1 + i_2$$

$$= ln\ |\ y^2 + 2\ |\ + \frac{3}{\sqrt{2}} tan^{-1} \frac{y}{\sqrt{2}} + c$$

$$= ln\ |\ tan^2 x + 2\ |\ + \frac{3}{\sqrt{2}} tan^{-1} \frac{tan\ x}{\sqrt{2}} + c$$

---

193. $\int sin^2 x\ .\ sin\ 3x\ dx$

$$= \frac{1}{2}\int (1 - cos\ 2x)\ sin\ 3x\ dx$$

$$= \frac{1}{2}[\int sin\ 3xdx - \int cos\ 2x\ sin\ 3x\ dx]$$

$$= \frac{1}{2}\left[\int sin\ 3xdx - \frac{1}{2}\int 2\ cos\ 2x\ sin\ 3x\ dx\right]$$

$$= \frac{1}{2}\left[-\frac{1}{3}cos\ 3x - \frac{1}{2}\int sin(5x) - sin(-x)\right]$$

$$= \frac{1}{2}\left[-\frac{1}{3}cos\ 3x - \frac{1}{2}\int sin(5x) + sin(x)\right]$$

$$= \frac{1}{2}\left[-\frac{1}{3}cos\ 3x - \frac{1}{2}\int -\frac{1}{5}cos\ 5x - cos\ x\right] + c$$

$$= -\frac{1}{6}cos\ 3x + \frac{1}{20}cos\ 5x + \frac{1}{4}cos\ x + c$$

---

194. $\int ln(x^2 + x)\ dx$

$u = ln(x^2 + x) \qquad dv = dx$

$du = \frac{2x+1}{x^2+x} \qquad v = x$

$$= x\,ln(x^2+x) - \int \frac{x(2x+1)dx}{x^2+x}$$

$$-\int \frac{x(2x+1)dx}{x^2+x} = -\int \frac{x(2x+1)dx}{x(x+1)}$$

$$-\int \frac{(2x+1)dx}{(x+1)} = -\int \frac{2x}{(x+1)}dx - \int \frac{dx}{(x+1)}$$

$$= ln\ |(x+1)| - 2\int \frac{(x+1)-1}{(x+1)}$$

$$= ln\ |(x+1)| - 2\int dx - \int \frac{dx}{(x+1)}$$

$$= ln\ |(x+1)| - 2x + ln\ |(x+1)| + c$$

$$= 2\,ln\ |(x+1)| - 2x + c$$

---

195. $\int \frac{xe^x}{\sqrt{1+e^x}}dx$

$u = x$	$dv = (1+e^x)^{\frac{-1}{2}}$
$du = dx$	$v = 2\sqrt{1+e^x}$

$$= 2x\sqrt{1+e^x}$$

$$-2\int \sqrt{1+e^x}\,dx = -2\int \sqrt{1+e^x} \times \frac{\sqrt{1+e^x}}{\sqrt{1+e^x}}dx$$

$$= -2\int \frac{1+e^x}{\sqrt{1+e^x}}dx = -2\int \frac{e^x}{\sqrt{1+e^x}}dx - 2\int \frac{1}{\sqrt{1+e^x}}dx$$

$$-2\int \frac{e^x}{\sqrt{1+e^x}}dx = -4\sqrt{1+e^x} + c$$

$$-2\int \frac{1}{\sqrt{1+e^x}} \times \frac{e^x}{e^x}dx = -2\int \frac{e^x}{e^x\sqrt{1+e^x}}dx$$

$put\ y = 1+e^x \quad \rightarrow dy = e^x dx \quad , \quad e^x = y-1$

$$= -2\int \frac{dy}{(y-1)\sqrt{y}} = -2\int \frac{dy}{(\sqrt{y}-1)(\sqrt{y}+1)\sqrt{y}}$$

$$put\ \ t = \sqrt{y} \quad \rightarrow dt = \frac{dy}{2\sqrt{y}} \quad \rightarrow 2dt = \frac{dy}{\sqrt{y}}$$

$$= -2\int \frac{2}{(t-1)(t+1)} dt \quad = -\int \frac{4dt}{(t-1)(t+1)}$$

$$= \frac{A}{(t-1)} + \frac{B}{(t+1)} = \frac{A(t+1)+B(t-1)}{(t-1)(t+1)}$$

$$t = 1 \rightarrow 4 = 2A \qquad \rightarrow A = 2$$

$$t = -1 \rightarrow 4 = -2B \qquad \rightarrow B = -2$$

$$= \int \frac{-2}{(t-1)} dt + \int \frac{2}{(t+1)} dt$$

$$= -[2\,ln\,|\,(t-1)\,|\, - 2\,ln\,|\,(t+1)\,|\,]$$

$$= -2\,ln\,|\,\frac{(t-1)}{(t+1)}\,|\, = 2\,ln\,|\,\frac{(\sqrt{y}-1)}{(\sqrt{y}+1)}\,|$$

$$= 2x\sqrt{1+e^x} - 4\sqrt{1+e^x} - 2\,ln\,|\,\frac{(\sqrt{1+e^x}-1)}{(\sqrt{1+e^x}+1)}\,|\, + c$$

$$= 2(x-2)\sqrt{1+e^x} - 2\,ln\,|\,\frac{(\sqrt{1+e^x}-1)}{(\sqrt{1+e^x}+1)}\,|\, + c$$

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196. $\int \frac{e^x}{(1+e^{2x})^2} dx \qquad put\ y = e^x \quad \rightarrow dy = e^x dx$

$$= \int \frac{1}{(1+y^2)^2} dy = \int \frac{1+y^2-y^2}{(1+y^2)^2} dy$$

$$= \int \frac{(1+y^2)}{(1+y^2)^2} dy + \int \frac{-y^2}{(1+y^2)^2} dy$$

$$= \int \frac{dy}{(1+y^2)} dy - \int \frac{y^2}{(1+y^2)^2} dy$$

$$= tan^{-1} y - \int \frac{y^2}{(1+y^2)^2} dy$$

$u = y$ $\qquad dv = \int y(1+y^2)^{-2}\, dy$

$du = dy$ $\qquad v = -\frac{1}{2(1+y^2)}$

$$= -\left(-\frac{y}{2(1+y^2)} + \frac{1}{2}\int \frac{dy}{(1+y^2)}\right)$$

$$= \frac{y}{2(1+y^2)} - \frac{1}{2} tan^{-1} y$$

$$= tan^{-1} y + \frac{y}{2(1+y^2)} - \frac{1}{2} tan^{-1} y + c$$

$$= \frac{y}{2(1+y^2)} + \frac{1}{2} tan^{-1} y + c$$

$$= \frac{e^x}{2(1+e^{2x})} + \frac{1}{2} tan^{-1} e^x + c$$

197. $\int \frac{x \tan^{-1} x}{\sqrt{1+x^2}} dx$

$u = \tan^{-1} x \qquad dv = x(1+x^2)^{\frac{-1}{2}}$

$du = \frac{dx}{1+x^2} \qquad v = \sqrt{1+x^2}$

$= \sqrt{1+x^2} \tan^{-1} x - \int \frac{\sqrt{1+x^2}}{1+x^2} dx$

$- \int \frac{\sqrt{1+x^2}}{1+x^2} \times \frac{\sqrt{1+x^2}}{\sqrt{1+x^2}} dx = - \int \frac{(1+x^2)}{(1+x^2)\sqrt{1+x^2}} dx$

$= - \int \frac{dx}{\sqrt{1+x^2}} dx$

قاعدة هامة/ اذا كان

$\int \frac{dx}{\sqrt{a^2+x^2}} = \ln |x + \sqrt{a^2+x^2}|$

$= - \ln |x + \sqrt{1+x^2}|$

$= \sqrt{1+x^2} \tan^{-1} x - \ln |x + \sqrt{1+x^2}| + c$

198. $\int \frac{\ln |e^x+1|}{e^x} dx$

$u = \ln(e^x+1) \qquad dv = e^{-x}$

$du = \frac{e^x}{e^x+1} \qquad v = -e^{-x}$

$$= -e^{-x}\ ln(e^{x}+1) + \int \frac{e^{x}.e^{-x}}{e^{x}+1}dx$$

$$= -e^{-x}\ ln(e^{x}+1) + \int \frac{1}{e^{x}+1} \times \frac{e^{-x}}{e^{-x}}dx$$

$$\int \frac{e^{-x}}{1+e^{-x}}dx = -\int \frac{-e^{-x}}{1+e^{-x}}dx = -\ ln(1+e^{-x})$$

$$= -e^{-x}\ ln(e^{x}+1) - ln(1+e^{-x}) + c$$

$$199. \int \frac{2e^{2x}-e^{x}-3}{e^{2x}-2e^{x}-3}dx = \int \frac{(2e^{x}-3)(e^{x}+1)}{(e^{x}-3)(e^{x}+1)}dx$$

$$= \int \frac{(2e^{x}-3)}{(e^{x}-3)}dx = \int \frac{(2e^{x}-3)}{(e^{x}-3)} \times \frac{e^{x}}{e^{x}}dx$$

$$put\ y = e^{x} \quad \rightarrow dy = e^{x}dx$$

$$= \int \frac{(2y-3)}{y(y-3)}dy = \frac{A}{y} + \frac{B}{(y-3)}$$

$$= \frac{A(y-3)+By}{y(y-3)} = \frac{Ay-3A+By}{y(y-3)}$$

$$y \rightarrow 2 = A + B \qquad , \quad y^{0} \rightarrow -3 = -3A \rightarrow A = 1$$

$$2 = A + B \rightarrow 2 = 1 + B \rightarrow B = 1$$

$$= \int \frac{1}{y}dy + \int \frac{1}{(y-3)}dy = ln\ |y| + ln\ |(y-3)| + c$$

$$= ln\ |e^{x}| + ln\ |(e^{x}-3)| = x + ln\ |(e^{x}-3)| + c$$

$$200. \int \frac{dx}{x^4+x^2} dx = \int \frac{dx}{x^2(x^2+1)} = \frac{Ax+B}{x^2} + \frac{Cx+D}{(x^2+1)}$$

$$= \frac{Ax^3+Ax+Bx^2+B+Cx^3+Dx^2}{x^2(x^2+1)}$$

$x^3 \to 0 = A + C$, $x^2 \to 0 = B + D$

$x \to 0 = A$, $C = 0$, $x^0 \to 1 = B$, $D = -1$

$$= \int \frac{dx}{x^2} + \int \frac{-1}{(x^2+1)} dx == -\frac{1}{x} - tan^{-1} x + c$$