Republic of Iraq

Ministry of Higher Education and Scientific Research

Northern Technical University

MACHINE PARTS

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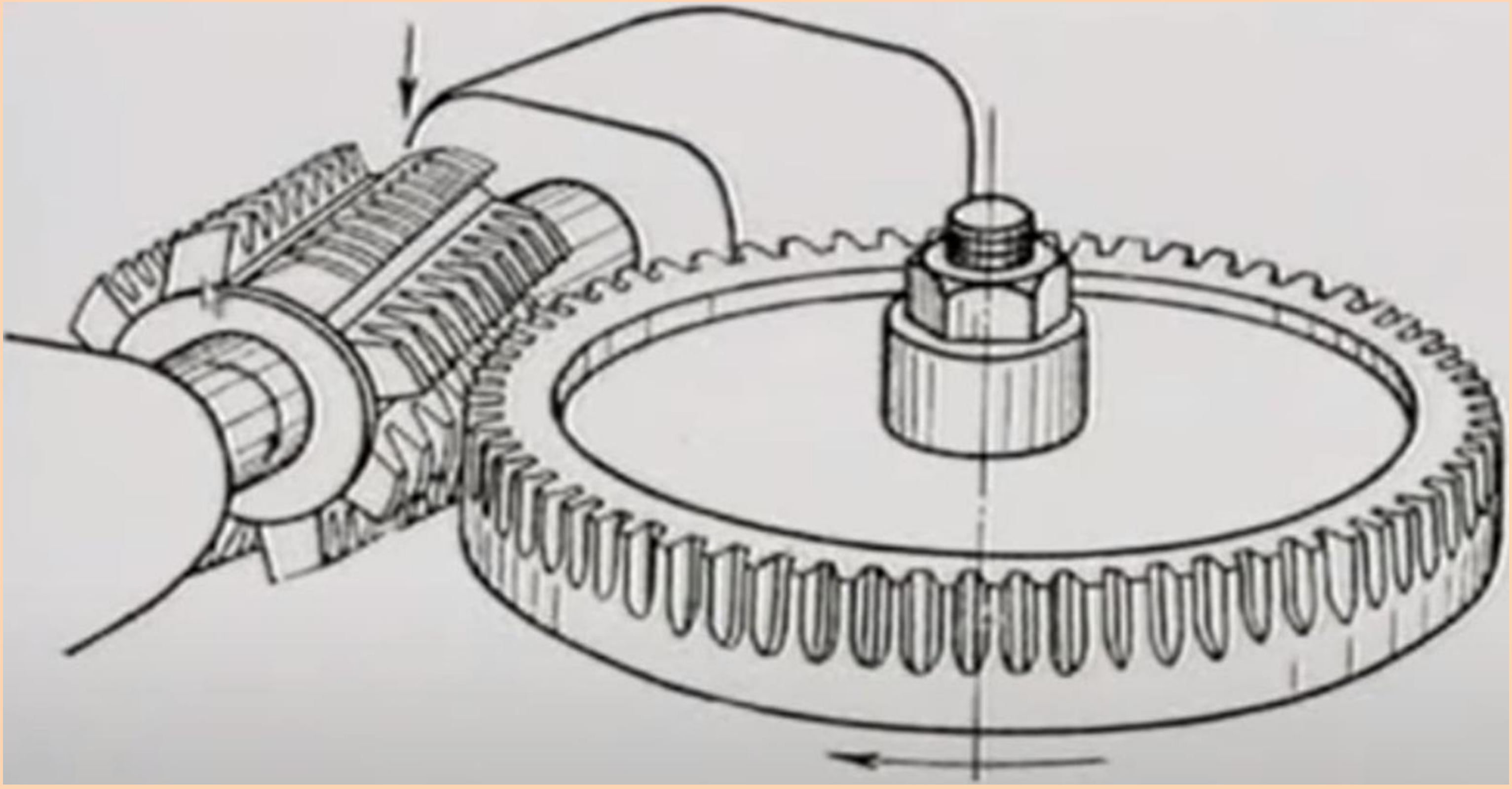

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Preface

Mechanical engineering is the basis of scientific and technological progress, and the main production and technological processes are carried out by machines or automatic lines. In this regard, mechanical engineering plays a leading role among other industries.

The use of machine parts has been known since ancient times. Simple machine parts - metal rivets, primitive gears, bolts, cranks were known before Archimedes; Rope and belt drive transmissions, charging propellers, and hinged connections were used.

Leonardo da Vinci, considered the first researcher in the field of machine parts, invented gears with cross-axles, articulated chains and rolling bearings.

This book includes comprehensive information on the parts of machinery and equipment. This book consists of sixteen chapters, each chapter contains solved questions and unsolved questions. This book benefits students of engineering colleges and institutes, as well as everyone who works in the field of designing and operating various machines and equipment. The book is for the benefit of Mosul Technical Institute - Northern Technical University - Ministry of Higher Education and Scientific Research - Republic of Iraq.

We hope that this book will be an important reference for students and those working in the field of machinery and equipment.

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Chapter 1

Review of Strength of Materials

1- Strength of Materials

1-1. Introduction

Strength of materials, also known as **Mechanics of materials**, is a subject which deals with the behavior of solid subject to stresses and strains.

Stress & Strain

When a force is applied to a structural member, that member will develop both stress and strain as a result of the force.
The applied force will cause the structural member to deform by some length, in proportion to its stiffness.

1. Stress

Stress is the force carried by the member per unit area, and typical units are $[lbf\ /\ in^2\ (psi)]$ for US Customary units and $[N\ /\ m^2\ (Pa)]$ for SI units:

$$\sigma = \frac{F}{A} \qquad (1-1)$$

Where, (F) is the applied force and (A) is the cross-sectional area over which the force acts.

2. Strain

Strain is the ratio of the deformation to the original length of the part:

$$\varepsilon = \frac{L - L_O}{L_O} = \frac{\delta}{L_O} \qquad (1-2)$$

Where (L**)** is the deformed length, (L_O**)** is the original unreformed length (ε) is the deformation, and (δ) change in length.

1-2. Types of loading

There are different types of loading which result in different types of stress.

1. **Axial Force**

Type of stress is called an Axial Stress (general case)

A. Tensile Stress (σ_t): If force is tensile as figure (1-1).

$$\sigma_t = \frac{F}{A} \qquad (1-3)$$

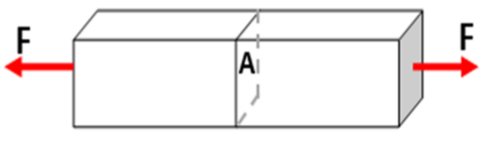


Figure 1-1: Tensile stress

B. Compressive Stress (σ_c): If force is compressive as figure (1-2).

$$\sigma_c = \frac{F}{A} \qquad (1-4)$$

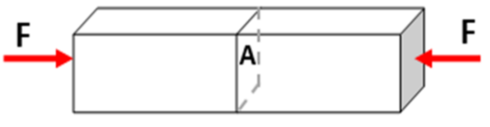


Figure 1-2: Compressive stress

2. **Shear stress (τ)**

Type of stress is called a Transverse Shear Stress as figure (1-3).

$$\tau = \frac{F}{A} \qquad (1-5)$$

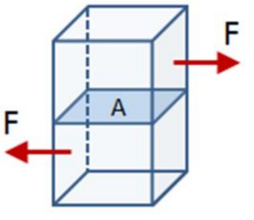


Figure 1-3: Shear stress

3. **Bending moment stress (σ_b)**

Type of stress is called a Bending Stress as figure (1-4).

$$\sigma_b = \frac{M\,.y}{I_c} \qquad (1-6)$$

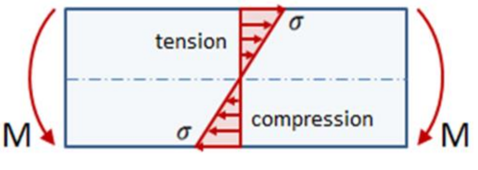


Figure 1-4: Bending stress

Where: (M) is the bending moment, (y) is the distance between the centroid axis and the outer surface, and (I_c) is the centroid moment of inertia of the cross section about the appropriate axis.

4. **Torsional stress**

Type of stress is called a Torsional Stress as figure (1-

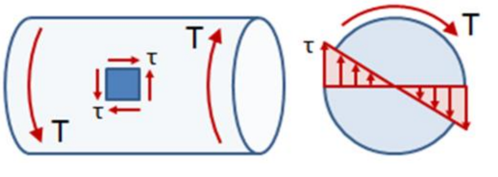


Figure 1-5: Torsional Stress

5). (**Engineer's theory of Torsion (E.T.T.)).**

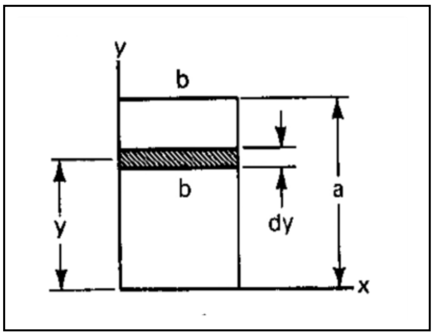
$$\frac{\tau}{r} = \frac{T}{I} = \frac{G\,.\emptyset}{L} \qquad (1-7)$$

Where: (τ) is the shear stress, (r) is the radius, (T) is the torsion torque, (I) is the polar moment of inertia of the cross section, (G) is modulus of rigidity, $(\emptyset)$ is the torsion angle, and (L) is a length of shaft.

$I = \frac{\pi d^4}{32}$ For solid circular section

$I = \frac{\pi(d_o^4 - d_i^4)}{32}$ For hollow circular section

$I = \frac{ab^3}{3}$ For solid rectangular section



1-3. Hooke's Law

Stress is proportional to strain in the elastic region of the material's stress-strain curve (below the proportionality limit, where the curve is linear), figure (1-6).

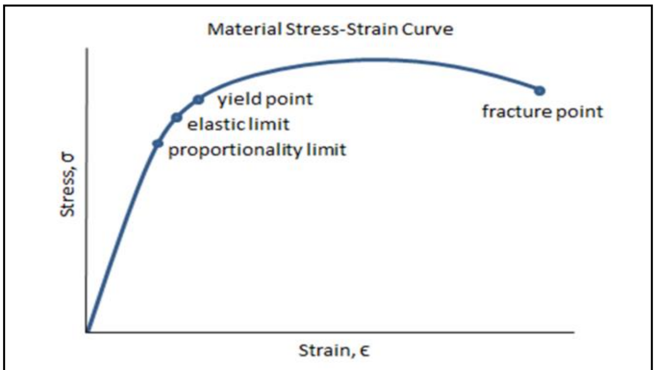


Figure 1-6: Hooke's Law

1-3-1. Engineering and True Stress

1-3-1-1. Engineering Stress (ES)

ES: is equivalent to the applied uniaxial tensile or compressive force at time, a fraction of the specimen's original cross-sectional area.

1-3-1-2. True Stress (TS)

TS: is equivalent to the applied uniaxial tensile or compressive force at time, divided

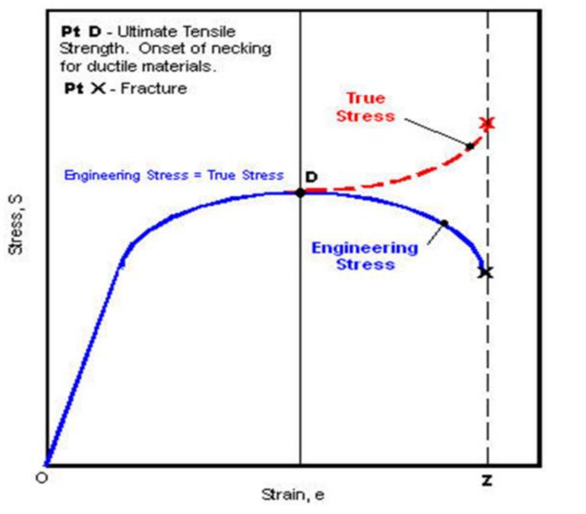


Figure 1-7: Engineering and True Stress

by the specimen's cross-sectional area at the moment.

Normal stress and strain are related by:

$$E = \frac{\sigma}{\varepsilon} \qquad (1-8)$$

Where: (E) is the elastic modulus of the material, (σ) is the normal stress, and (ε) is the normal strain.

Shear stress and strain are related by:

$$G = \frac{\tau}{\gamma} \qquad (1-9)$$

Where: (G) is the shear modulus of the material, (τ) is the shear stress, and (γ) is the shear strain. The elastic modulus and the shear modulus are related by:

$$G = \frac{E}{2(1+\mu)} \qquad (1-10)$$

Where: (μ) is Poisson's ratio.

1-4. Poisson's ratio

Poisson's ratio is the proportion of lateral (transverse) contraction strain to longitudinal extension strain in the direction of stretching force.

The value of Poisson’s ratio varies from 0.25 to 0.33. For rubber its value varies from 0.45 to 0.5. Mathematically:

$$Poisson's\ ratio = \frac{Lateral\ strain}{Longitudinal\ strain}$$

$$\mu = \frac{Lateral\ strain}{Longitudinal\ strain}$$

$$\mu = \frac{\varepsilon_{Lateral}}{\varepsilon_{Long.}} \qquad (1-10)$$

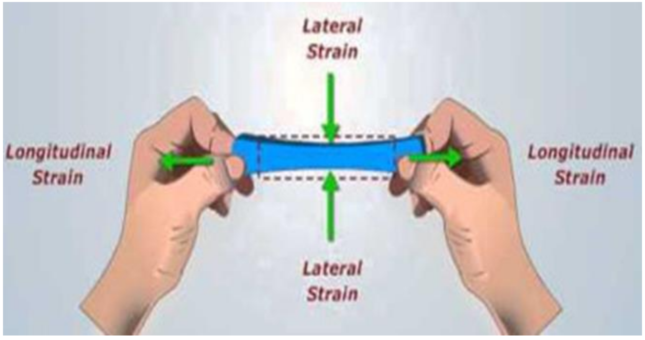
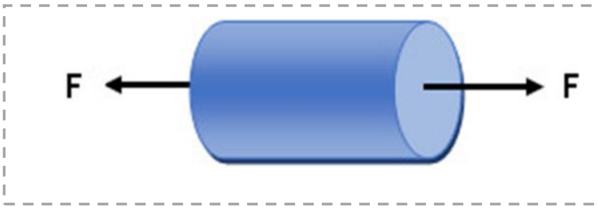


Figure 1-8: Poisson's ratio

1-5. Solve examples

Example 1

A force of *(100 KN)* is acting on a circular rod with diameter *(50 mm)*. The stress in the rod can be calculated as:



Solution:

$$\sigma_t = \frac{F}{A}$$

$$F = 100 \times 1000 = 100000 N$$

$$A = \pi . r^2$$

$$r = \frac{d}{2} = \frac{50}{2} = 25 mm = \frac{25}{1000} = 0.025 m$$

$$A = 3.14 \times (0.025)^2 = 0.0019625 m$$

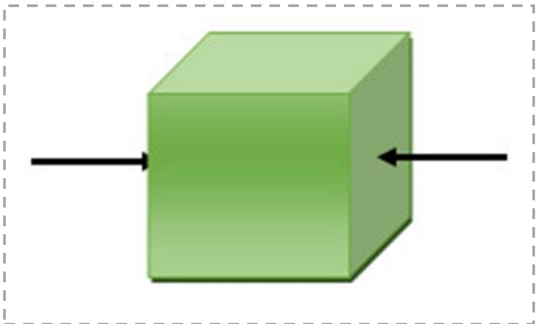
$$\sigma_t = \frac{100000 (N)}{0.0019625 (m^2)} = 50955414013 N/m^2 = \frac{50955414013}{1000000} = 50.955 MPa$$

Example 2

A compressive load of *(40 KN)* is acting on short square *(9 * 9 cm)* post of Douglas fir. The dressed size of the post is *(7 * 7 cm)* and the compressive stress can be calculated as:

Solution:

$$\sigma_c = \frac{F}{A}$$



$$F = 40 \times 1000 = 40000 N$$

$$A = a.b = 7 \times 7 = 49 cm^2 = \frac{49}{10000} = 0.0049 m^2$$

$$\sigma_t = \frac{40000 (N)}{0.0049 (m^2)} = 8163265306 \ N/m^2 = \frac{8163265306}{10000} = 8.163 MPa$$

Example 3

A metal shaft diameter *(12 mm)*, and long *(1.5 m)*. A tensile force of *(1000 N)* is applied to it and it stretches *(0.11 mm)*. Assume the material is elastic. Determine the stress and strain in the shaft?

Solution:

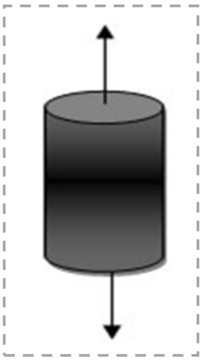
$$A = \pi . r^2 = 3.14 \times (6)^2 = 113.4\, mm$$

$$\sigma = \frac{F}{A} = \frac{1000}{113.4} = 8.818\, N/mm^2$$

$$\varepsilon = \frac{\delta}{L_o} = \frac{0.11}{1500} = 0.000073 = 73\ \mu\varepsilon$$

Example 4

A steel tensile test specimen has an across sectional area of *(120 mm²)*, and gauge length *(50 mm)*, the gradient of elastic section is *(433 KN/mm)*. Determine the modulus of elasticity?



Solution:

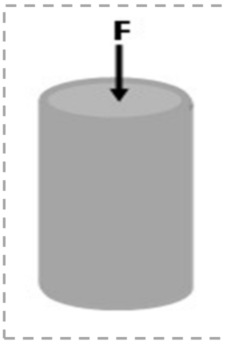
Gradient ratio (***F/δ)*** = *(433 KN/mm²)* = *433000 N/mm*

$$\mathrm{E} = \frac{\sigma}{\varepsilon} = \frac{F}{\delta} \times \frac{L}{A} = 433000 \times \frac{50}{120} = 180416.67\, N/mm^2$$

$$= 180416.67\, MPa \approx 180.42\, GPa$$

Example 5

A long of the steel column is *(4 m)*, and diameter *(50 cm)*. It carries a load of *(100 MN)*. If modulus of elasticity is (*210 GPa)*, calculate the compressive stress and strain and how much the column is compressed?



Solution:

$$A = \pi . r^2 = 3.14 \times (0.25)^2 = 0.1963 mm$$

$$\sigma = \frac{F}{A} = \frac{100 \times 10^6}{0.1963} = 509.4 \times 10^6 \ N/m^2$$

$$\because E = \frac{\sigma}{\varepsilon} \quad \Rightarrow \quad \varepsilon = \frac{\delta}{\mathrm{E}} = \frac{509.4 \times 10^6}{210 \times 10^9} = 0.00243$$

$$\because \varepsilon = \frac{\delta}{L} \quad \Rightarrow \quad \delta = \delta . L = 0.00243 \times 4000 = 9.7 \ mm$$

Example 6

Calculate the force needed to a plate of metal *(5 mm)* thick and (*0.8 m)* wide given that the ultimate shear stress *(50 MPa)*, as shown in the figure?

Solution:

The area to be cut is a rectangle

$$t = 5 mm ; w = 0.8 m = 0.8 \times 1000 = 800 mm ;$$

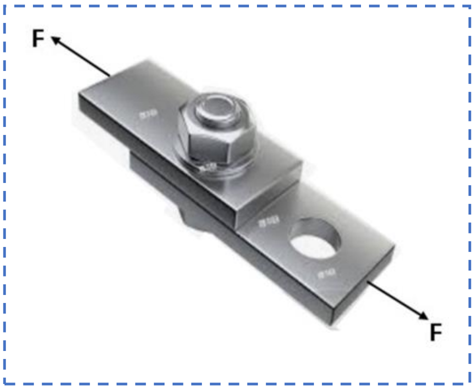
$$\tau = 50 N / mm^2$$

$$A = w.t = 5 \times 800 = 4000 \ mm^2$$

$$\because \tau = \frac{F}{A} \quad \Rightarrow \quad F = \tau . A = 50 \times 4000 = 200000 N = 200 KN$$

Example 7

Calculate the force needed to shear a Screw *(12 mm)* diameter given that the ultimate shear stress is *(90 MPa)*, as shown in the figure?



Solution:

The area to be is the circular area

$$A = \frac{\pi\, d^2}{4} = \frac{3.14 \times (12)^2}{4} = 113.04\ mm^2$$

$$\tau = \frac{F}{A}$$

$$F = \tau . A = 90 \times 113.04 = 10173.6\ N \approx 10.17\ KN$$

Example 8

A pin is used to attach a clevis to a rope. The force in the rope will be a maximum of (*60 KN)*. The maximum permitted shear stress in a pin is *(40 MPa)*. Calculate the diameter of suitable pin?

Solution

The pin is in double shear so the shear stress is ($\tau = \dfrac{F}{2A}$ *)*

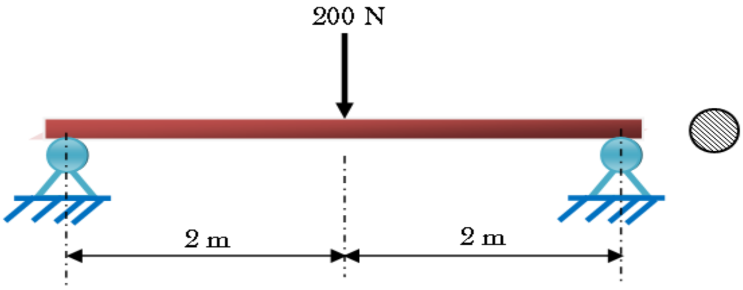
$$A = \frac{F}{2\tau} = \frac{3.14 \times (12)^2}{2 \times 40.10^6} = 750.10^{-6}\ m^2 = 750\, mm^2$$

$$\because A = \frac{\pi . d^2}{4} \qquad \Rightarrow \qquad d^2 = \frac{4A}{\pi}$$

$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4 \times 750}{\pi}} = 30.9\, mm$$

Example 9

A simply supported beam is subject a point load of (200 N) at the mid - spam of the beam as shown in the figure. The beam has a circular (50 mm) diameter . Calculate the maximum stress due to bending?



Solution:

The maximum tensile and compressive stresses due to bending are:

$$\sigma_{Max} = \frac{M \, . C}{I}$$

1 – *The maximum bending moment occurs at the mid – span of the beam.*

$$M = 100 \times 2000 = 200000 \quad N.mm$$

2 – *For the soild circule section.*

$$I = \frac{\pi \, d^4}{64} = \frac{3.14 \times 25^4}{64} = 18920.898 \, mm^4$$

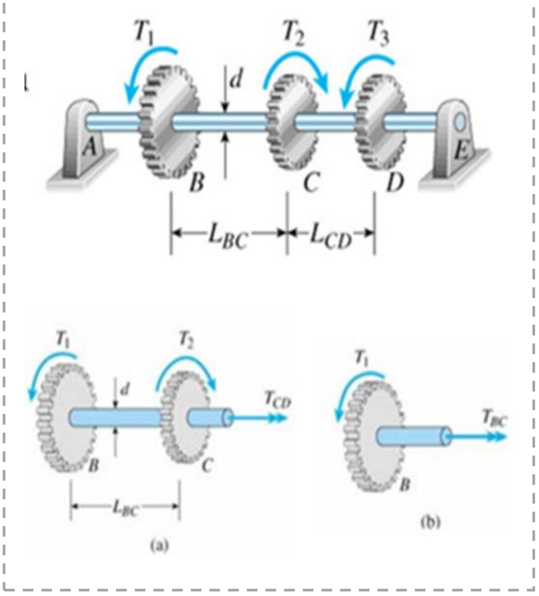
3 – *The centroid of the soild circule section is at inter section of its two axes of symmetry.*

$$C = \frac{d}{2} = \frac{50}{2} = 25 \, mm$$

$$\therefore \; \sigma_{max.} = \frac{200000 \times 25}{18920.898} = 264.26 \quad N/mm \quad \approx 264.3 \quad MPa$$

Example 10

A diameter solid steel shaft *(ABCDE)* is *(50 mm)* see in figure**.** If have torques *(T_1 = 200 N. m, T_2 = 500 N. m and T_3 = 300 N. m)*, distance between gears *(B & C)* is *(L1 = 200 mm)* and distance between gears *(C & D)* is *(L_2 = 300 mm)*, modulus of rigid is *(G = 90 GPa)*. Determine the maximum shear stress *($\tau_{max.}$)* in each part and twisting angle (ϕ_{BD})?



Solution

Given: *{d= 50 mm, T_1 = 200 N. m, T_2 = 500 N .m, T_3 = 300 N. m, L_1 = 300 mm, L_2 = 200 mm, G = 90 GPa}.*

$$I_P = \frac{\pi . d^4}{32} = \frac{3.14 \times 50^4}{32} = 613281.25 \, mm^4 \, ; \quad r = 25 \, mm$$

$$T_{BC} = - \, T_1 = -200 \; N.m$$

$$T_{CD} = T_2 - T_1 = 500 - 200 = 300 \, N.m$$

$$\because \frac{\tau}{r} = \frac{T}{I} = \frac{G\varphi}{L} \qquad \Rightarrow \qquad \therefore \ \tau = \frac{T.r}{I}$$

$$\therefore \tau_{BC} = \frac{T_{BC}.r}{I} = \frac{200.10^3 \times 25}{61328.125} = 8.15 \ MPa$$

$$\therefore \tau_{CD} = \frac{T_{CD}.r}{I} = \frac{300.10^3 \times 25}{61328.125} = 12.23 \ MPa$$

$$\phi_{BD} = \phi_{BC} - \phi_{CD}$$

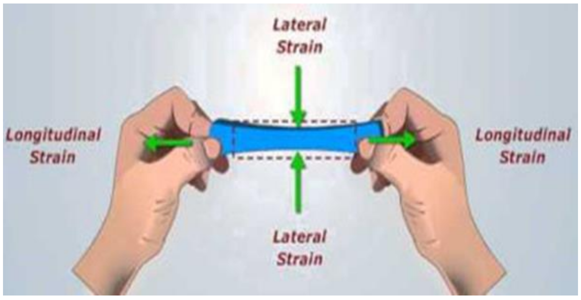
$$\because \frac{\tau}{r} = \frac{T}{I} = \frac{G\varphi}{L} \qquad \Rightarrow \qquad \therefore \varphi = \frac{T.\ L}{I.G}$$

$$\varphi_{BC} = \frac{T_{BC}.\ L_1}{I.G} = \frac{200.10^3 \times 300}{61328.125 \times 90.10^3} \approx 0.00109 \approx 0.0624^\circ$$

$$\varphi_{CD} = \frac{T_{CD}.\ L_2}{I.G} = \frac{300.10^3 \times 200}{61328.125 \times 90.10^3} \approx 0.00109 \approx 0.0624^\circ$$

Example 11

A steel wire having cross sectional area *(2 mm^2)*, see in figure. Is stretched by *(200 N)*. Find the lateral strain produced in the wire. If modulus elasticity for steel is *(210 GPA)* and Poisson's ratio is *(0.233)*?



Solution

Given: *{A= 2 mm^2, F = 200 N, μ = 0.233, G = 210 GPa}.*

$$E = \frac{\sigma}{\varepsilon_{longitudinal}} = \frac{F}{A.\varepsilon_L} \quad \Rightarrow \ \varepsilon_L = \frac{F}{A.E} = \frac{200}{2.10^{-6} \times 210.10^9} = 0.00048 \approx 4.8 \times 10^4$$

$$\because \ \mu = \frac{\varepsilon_{lateral}}{\varepsilon_{longitudinal}} \qquad \Rightarrow \ \therefore \varepsilon_{Lateral} = \mu.\varepsilon_L = 0.233 \times 0.00048 = 0.000112 \approx 1.12 \times 10^4$$

1-6 Chapter Questions

1. Young's modulus is the proportion ratio of:

a. Volumetric stress to volumetric strain.
b. Lateral stress to lateral strain.
c. Longitudinal stress to longitudinal strain.
d. The shearing stress to strain.

2. The tensile strength of a material is estimated by dividing the maximum load during the test by the:

a. Area unit at the time of fracture.
b. **Area of the original cross-section.**
c. Average area after fracture.
d. Minimum area before fracture.

3. Comparing the torque resistance of a solid shaft with another hollow shaft, having the same cross section area, the torque is:

a. Equal torque.
b. Fewer torques.
c. Higher torque.
d. Be less or more torque.

4. The stress is in the elastic limit when a tensile test for steel is:

a. No proportional to strain
b. Zero
c. Proportional to strain
d. Manimum

5. The external force affecting the body distorts the shape of the body so that the body size decreases and the length is called:

a. Bending stress
b. Tensile stress
c. Compressive stress
d. Shear stress

6. The point at which a bar with a tapering portion produces the maximum stress is at:

a. Lesser end
b. Middle of a bar
c. Greater end
d. Anywhere of a bar

7. When comparing the ultimate compressive stress and ultimate tensile stress of mild steel is:

a. Same.
b. More.
c. Less.
d. Unpredictable.

8. **When shear force at a point is zero, then bending moment is --------------- at that point.**
   a. Zero
   b. Minimum
   c. Maximum
   d. Infinity

9. **When a body experiences two equal and opposing forces, the body tends to shorten itself as a result:**

   a. A compressive strain and compressive stress are created.
   b. Tensile strain and compressive stress are generated.
   c. The production of tensile stress and strain.
   d. Compressive strain and tensile stress are generated.

10. **Modulus of rigidity represents the ratio of**
   a. Tinsel stress to transverse strain
   b. **Shear stress vs shear strain**
   c. Volumetric stress vs strain
   d. Lateral stress vs lateral strain

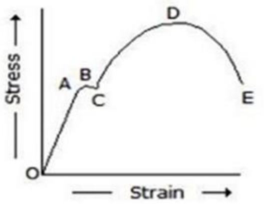
11. **When using the torsion equation ($T/J = \tau/r = G\theta/L$), the term (J/R) is called:**
   a. Modulus of Shear.
   b. Modulus of rigidity.
   c. Section modulus.
   d. Modulus of elasticity.

12. **Strain is defined the ratio between:**
   a. Change in length to half of the original length
   b. Change in length to original length
   c. Change in length to a quarter of the original length
   d. Change in length to original length weakness

13. **Within the elastic limit, the lateral strain to linear strain ratio is referred as:**
   a. Modulus of rigidity
   b. Young's modulus
   c. Bulk modulus
   d. Poisson's ratio

14. **The elastic range appears in the below-mentioned figure:**

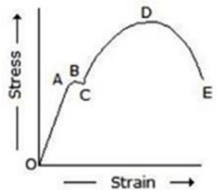
   a. After point A.
   b. After point D.
   c. Located between A and D.
   d. Located between points D and E



15. **Hook's law is still valid**
    a. Yield point
    b. Elastic limit
    c. Breaking point
    d. Limit of proportion

16. **The correct answer to which of the following statements?**
    a. In mm, the strain is expressed.
    b. The pressure per unit area is the stress.
    c. **Within the elastic limit, stress and strain are inversely proportional.**
    d. To the breaking point, Hook's law is still valid.

17. **The stress that corresponds to point "D" in the following diagram is:**



    a. Elastic limit
    b. Ultimate stress
    c. Breaking stress
    d. Yield point stress

18. **A body experiences a simple shear stress of (200 Mpa) and a direct tensile stress of (300 Mpa) in the same plane. What will be the maximum shear stress?**
    a. 150 MPa
    b. 200 MPa
    c. 300 MPa
    d. 350 MPa

19. **For steel, the Poisson's ratio is between**
    a. 0.12 - 0.17
    b. 0.18 - 0.24
    c. 0.25 - 0.33
    d. 0.34 - 0.44

20. **Materials with consistent elastic characteristics in all directions are referred to as:**
    a. Anisotropic materials
    b. Orthotropic materials
    c. Isotropic materials
    d. Uniform materials

21. **For a compound shaft, the safe twisting moment is equal to the**
    a. Maximum value determined.
    b. Extreme value.
    c. Mean value.
    d. Minimum value determined.
    e.

22. The bending formula is:

a. $\mathbf{M / I = \sigma / y = E / R}$
b. $M / R = T / J = C\theta / I$
c. $T / J = \tau / R = C\theta / I$
d. $T / l = \tau / J = R / C\theta$

23. Which materials have a Poisson's ratio greater than one?

a. Brass
b. Nickel
c. Steel
d. Aluminum

24. What material is the most elastic of the following?

a. Rubber
b. Brass
c. **Steel**
d. Plastic

25. The Young's modulus unit is:

a. Newton per square millimeter
b. Newton
c. millimeter
d. Newton per square millimeter

26. The strain unit is

a. Newton
b. millimeter
c. No unit
d. Newton per millimeter

27. An internal pressure is applied to a thin, spherical shell with dimensions of diameter (d) and thickness (t) (P). The material of the shell has a tension of:

a. $Pd / 2t$
b. $\boldsymbol{Pd / 4t}$
c. $Pd / 8t$
d. $Pd / 6t$

Chapter 2

Riveted Joints

2 - Riveted Joints

2-1. Introduction

Riveted joints: It is one of the techniques used to construct a joint between two or more metal pieces or to unite the two ends of a metal piece. Buildings, bridges, boilers, tanks, ships, aircraft hulls, car brakes, home appliances, windows, doors, metal frames, and more frequently use riveting as a rapid jointing procedure.

2-2. Types of rivet heads

There are many types of rivet heats, **f**igure (2.1).

a) Snap head.
b) Pan head.
c) Pan head with tapered neck.
d) Round counter sunk head $60^{o.}$
e) Flat counter sunk head 60^{o}.
f) Flat head.

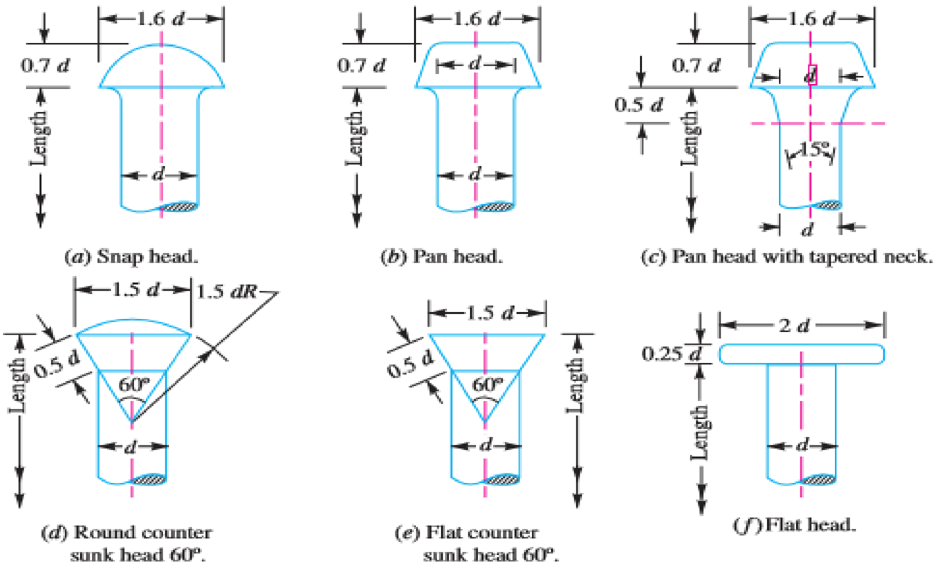


Figure 2-1: Types of Riveted Joint

2-3. Riveted joint types

The following criteria are used to classification riveted joints:

1. Depending on purpose,
2. Depending on how the connecting plates are positioned, and

3. Depending on arrangement of rivets.

Types of Riveted Joints according to arrangement of rivets:

2-3-1. Riveted Lap Joint

There are many types of riveted lap joint, figure 2.2.

a. Single.
b. Double (Chain).
c. Double (Zig Zag).

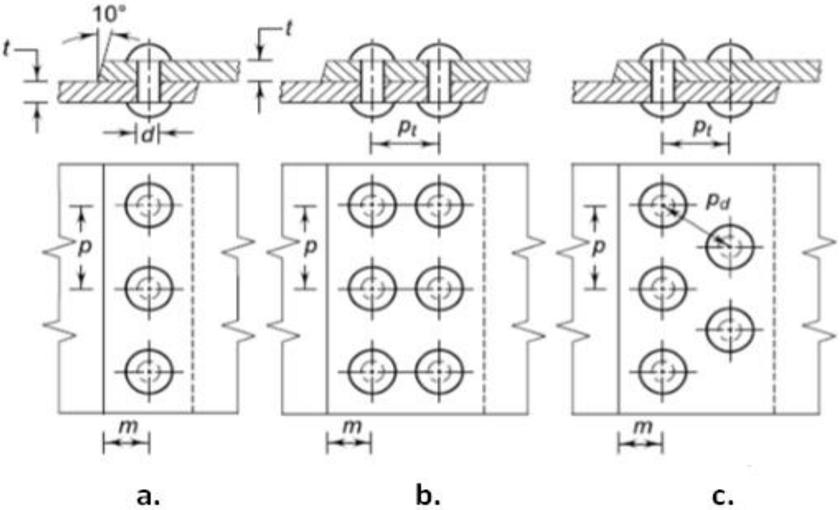


Figure 2-2: Riveted Lap Joint

2-3-2. Riveted Butt Joint (Figure 2.3).

a. Single rivet single strap.

b. Single rivet double strap.

c. Double rivet double strap (Chain).

d. Double rivet double Strap (Zig Zag).

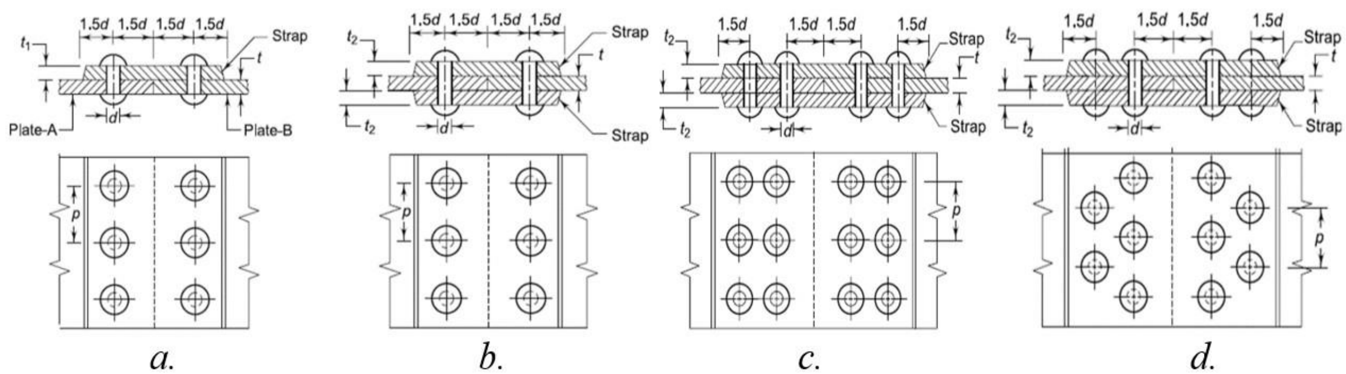


Figure 2-3: Riveted Butt Joint

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2-4. Important Terminologies

1. **Pitch (*p*):** Distance between centers of one rivet to the center of the adjacent rivet in the same row

2. **Back pitch (p_t):** or Transverse pitch is the distance between two consecutive rows of rivets in the same plate

3. **Diagonal Pitch (p_d):** Diagonal pitch is the distance between center of one rivet to the center of the adjacent rivet located in the adjacent row

4. **Margin or marginal pitch (*m*)**: Distance between the centers of the rivet hole to the nearest edge of the plat

2-5. Leak Proof Joints

A hammer and a caulking tool are used to upset the plate's edge so that it is firmly forced on the plate's surface to aid in leak proofing, figure 2.4.

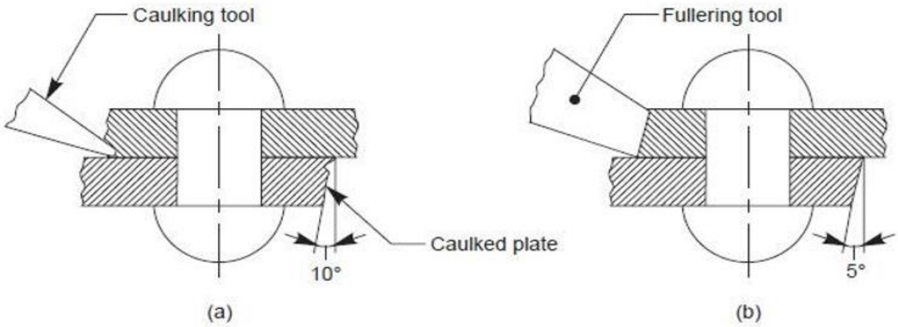


Figure 2-4: Caulking and Fullering of Riveted Joint

2-6. Design of rivet joints

The design parameters in a riveted joint are (**d, *P*** and ***m***). Diameter of the hole (***d***): When thickness of the plate (***t***) is more than (8 mm), Unwin's formula is used,

$$d = 6\sqrt{t}\,, \qquad (mm) \qquad\qquad (2-1)$$

2-6-1. Strength of riveted joint

All potential failure paths in the joint are taken into consideration when evaluating the strength of a riveted joint.

There are four types of stresses occur at riveted joints. Therefore, the failure is possible in four locations as follows:

1- Shearing stress failure in rivets.

2- Tension stress failure in plate.

3- Bearing stress failure between plate and rivet.

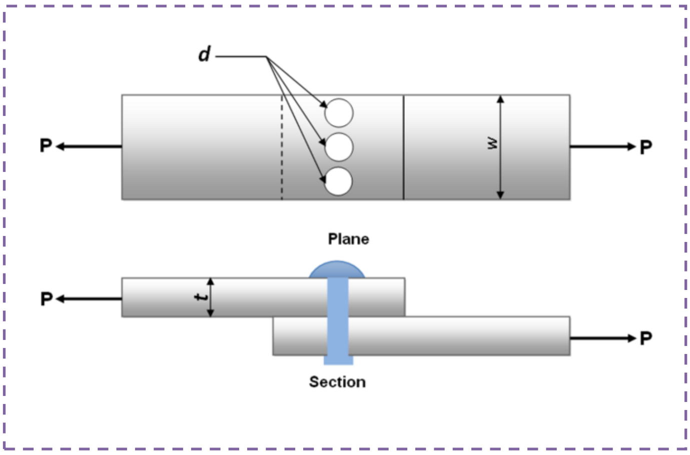
4- Shearing stress failure in plate.

2-6-1-1. Shearing stress failure in rivets:

a. All rivets in the same diameter.

$$F_r = \frac{P}{n} \qquad (2-2)$$

$$\tau_r = \frac{F_r}{A} = \frac{P}{A\,.\,n} \qquad (2-3)$$



Where:

τ_r = Shear stress failure,

n = Number of rivets,

d = Diameter of rivet,

P = Plate exerts tensile force.

b. The rivets with different diameters.

$$\tau_r = \frac{P}{A} = \frac{P}{\frac{\pi}{4}(d_1^2 + d_2^2 + d_3^2 + \cdots + d_n^2 +)} \qquad (2-3)$$

$$F_1 = \tau_{rivet} \times \frac{\pi}{4} d_1^2$$

$$F_2 = \tau_{rivet} \times \frac{\pi}{4} d_{21}^2$$

$$F_3 = \tau_{rivet} \times \frac{\pi}{4} d_3^2$$

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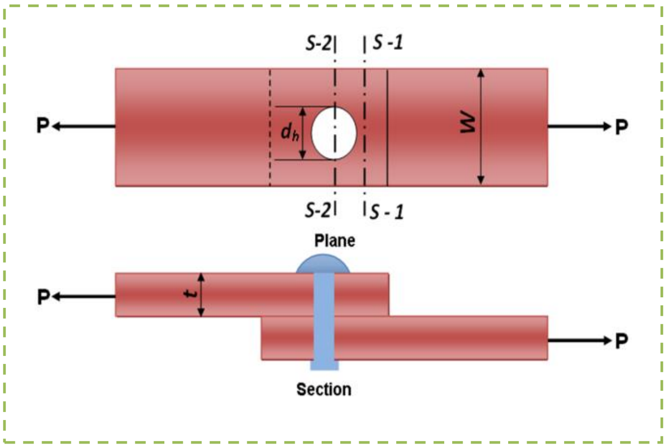
26-1-2. Tension stress failure in plate

Tearing of the plate across a row of rivets.

Due to the tensile stresses in the main plates, the main plate or cover plates may tear off across a row of rivets. In such cases, we consider only one pitch length of the plate, since every rivet is responsible for that much length of the plate only.



$$\sigma_p)_{1-1} = \frac{P}{A} = \frac{P}{w\,.\,t} \qquad (2-4)$$

$$\sigma_p)_{2-2} = \frac{P}{A} = \frac{P}{(w - n\,.\,d_h)\,.\,t} \qquad (2-5)$$

$$d_h \; = \; d_r \; + \; 3\,mm \qquad (2-6)$$

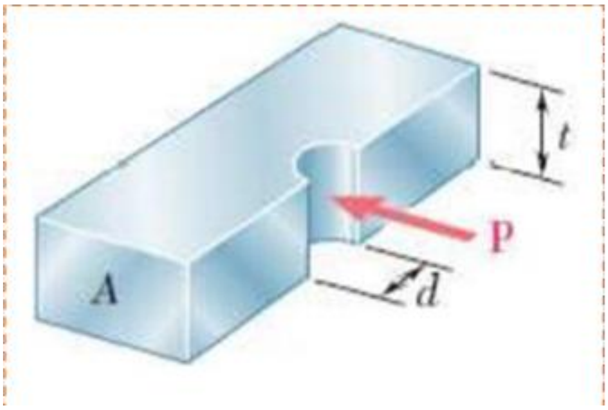
Where:

$$w = thichness\ of\ plate, t \; = \; Thickness\ of\ plate \quad , \quad d_h$$
$$= \; Diameter\ of\ hole, d_r = Diameter\,.$$

Note: Always: $[\sigma_r)_{2-2} \;>\; \sigma_r)_{1-1}]$

2-6-1-3. Bearing stress failure between plate and rivet.

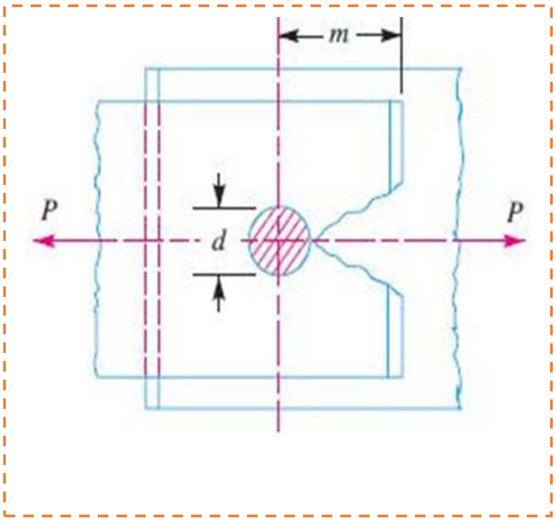
$$\sigma_b = \frac{P}{A} = \frac{\boldsymbol{P}}{\boldsymbol{d\,.\,t}} \qquad (2-7)$$



2-6-1-4 Shearing stress failure in plate

An edge of the plate tearing may cause a joint to fail. By maintaining the margin, $(m = 1.5\, d)$, where (d) is the diameter of the rivet hole, this can be prevented.

$$\tau_p = \frac{P}{A} = \frac{P}{(w - d_h)\,.\,t} \qquad (2-8)$$



2-7. Efficiency of Joint

Maximum force which a joint can transmit without causing it to fail.

1. With the rivet, the lowest value for the subsequent stresses is chosen:

$$[\tau_r\,, \sigma_p\,, \sigma_b\,, \tau_p].$$

2. Without the rivet, the normal stress is:

$$\sigma = \frac{P}{A} = \frac{\boldsymbol{P}}{w\,.\,\boldsymbol{t}} \qquad (2-9)$$

$$\eta = \frac{Least\ of\ [\tau_r\,, \sigma_p\,, \sigma_b\,, \tau_p]}{\sigma} \times 100\ \% \qquad (2-10)$$

2-8. Solve examples

Example 1

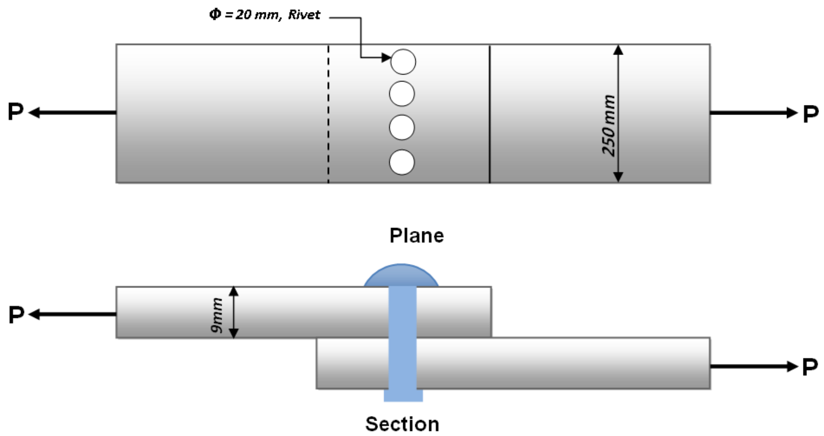
For the lap joint shown in the Figure, calculate the safe axial tensile force (P), if:

$$\sigma_{tensile} = 160\ MPa\ ,\quad \tau_{rivet} = 125\ MPa\ ,\quad \sigma_{bearing} = 375\ Mpa.$$

Assume the diameter of hole = *23 mm*.

Solution

Shear force in the rivet *(F_{rivet} = P/4).*



$$\tau_{rivet} = \frac{F}{A} = \frac{P_{rivet}/n}{\pi\left(\frac{d^2}{4}\right)} = \frac{4P}{\pi n d^2}$$

$$125*10^6 = \frac{4P}{\pi n d^2}$$

$$P_{rivet} = \frac{125.10^6(\pi n d^2)}{4} = \frac{125\times10^6\times3.14\times4\times(0.02)^2}{4} = 157000N = 157KN$$

$$\sigma_t = \frac{P_{tensil}}{(p-n.d_h)\times t}$$

$$160*10^6 = \frac{P_{tensil}}{(p-n.d_h)\times t}$$

$$P_{tensil} = 160.10^6(p-n.d_h)\times t = 160.10^6(0.250-4\times0.022)\times0.009 = 233280N = 233.3KN$$

$$\sigma_b = \frac{P_{bearing}/4}{d \times t}$$

$$375*10^6 = \frac{P_{bearing}/4}{d \times t}$$

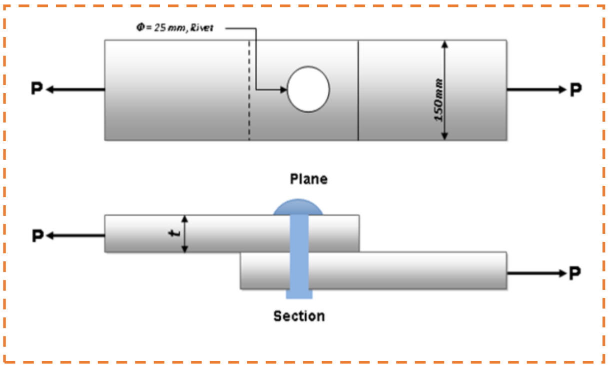
$$P_{bearing} = 4 \times 375.10^6 \times d \times t = 4 \times 375.10^6 \times 0.02 \times 0.009 = 270000N = 270KN$$

The safe force which does not cause failure neither in shear nor in tensile nor in bearing is

$$\boldsymbol{P_{safe} = 157\ kN}.$$

Example 2

In the Figure shown, assume that a *25 mm* diameter rivet joins the plates that are each *150 mm* wide. The allowable stresses are *150 MPa* for bearing in the plate material and *75 MPa* for shearing of rivet. Determine **(a)** the minimum thickness of each plate, and **(b)** the largest average tensile stress in the plates. (Assume d_h=*28 mm*).



Solution

a) The minimum thickness of each plate

1. From shearing of rivet

$$\tau_{rivet} = \frac{F}{A} = \frac{P/n}{\pi(\frac{d^2}{4})}, \quad n = 1$$

$$\tau_{rivet} = \frac{P_{rivet}}{\pi(\frac{d^2}{4})}$$

$$P_{rivet} = \tau_{rivet} \cdot \pi(\frac{d^2}{4})$$

$$= 75.10^6 \times 3.14 \times (\frac{0.025^2}{4}) = 36797\ N$$

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2. From bearing of plate (Crushing force)

$$\sigma_b = \frac{P_{bearing}}{d \times t} \quad \Rightarrow \quad P_{bearing} = \sigma_{bearing} \times d \times t$$

$$t = \frac{P_{bearing}}{\sigma_{bearing} \times d} = \frac{36797}{150.10^6 \times 0.025} = 0.0098m = 9.8\,mm$$

b) largest average tensile stress in the plate (Tearing force)

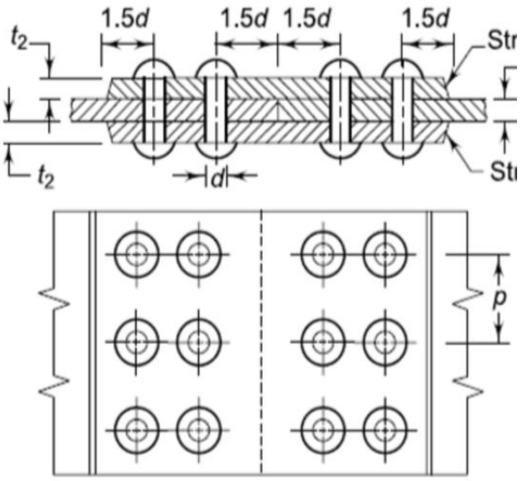
$$\sigma_t = \frac{P}{(w - n.d_h) \times t} = \frac{36797}{(0.15 - 0.028) * 0.0098} = 30767000Pa = 30.08MPa$$

Example 3

In plates (28 mm) thick, The use of (25 mm) diameter rivets spaced (100 mm) pitch allows for the construction of a double riveted double cover butt joint. The permitted stress levels are:

Tearing stress (tension stress) in plate (150 Mpa), shearing stress in rivet (125 Mpa) and crushing stress (bearing stress) (170 Mpa).

Find the efficiency of joint, taking the strength of the rivet in double shear as twice as that of single shear?



Solution

$$Given: t = 20mm; d = 25mm; p = 100mm; \sigma_t = 150MPa$$
$$\tau_r = 125MPa; \sigma_b = 180MPa; d_h = d + 3 = 25 + 3 = 28mm.$$

1. Tearing resistance of the plate

$$\sigma_t = \frac{P}{(p - n.d_h) \times t}$$

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$$P_T = \sigma_t \times (p - nd_h) \times t = 150 \times (100 - 1 \times 28) \times 28$$
$$\therefore P_T = 150 \times 62 \times 28 = 260400 N = 260.4 KN$$

2. Shearing resistance of the rivets

$$\tau_r = \frac{P_C / n}{\pi \times \frac{d^4}{4}}$$

$$P_S = \frac{2 \tau_r \times n \times \pi \times d^2}{4} = \frac{2 \times 125 \times 2 \times 3.14 \times (25)^2}{4}$$
$$\therefore P_S = \frac{981.25}{4} = 245.313 \ N = 245.3 \ KN$$

3. Crushing resistance of the rivets

$$\sigma_b = \frac{P_C / n}{d \times t}$$

$$P_C = \sigma_p \times n \times d \times t = 170 \times 2 \times 25 \times 28$$
$$\therefore P_C = 238000 N = 238 KN$$

$$\therefore Stength of\ the\ joint = Least of\ (P_T \ , \ P_S \ , \ P_C) = 238\ KN$$

Efficiency of the rivet joint

That the unriveted or solid plate's strength:

$$P_{soild} = \sigma_t \times p \times t = 150 \times (100 - 1 \times 28) \times 28$$
$$\therefore P_{soild} = 150 \times 100 \times 28 = 420000 N = 420 KN$$

$$\therefore \eta = \frac{Least of (P_T, P_S, P_C)}{P_{soild}} = \frac{238}{420} \times 100\% = 56.67\%$$

Example 4

Between two (30 mm) thick plates, a double riveted lap joint is created. The diameter and pitch of the rivets are (50 and 150) millimeters, respectively. Find the minimal force per pitch that will cause the joint to fail, if the ultimate stresses are (600 Mpa) in tension, (480 Mpa) in shear in the rivet, and (960 Mpa) in crushing (bearing). Find out the actual stresses created in the plates and the rivets if the above joint is subjected to a load with a factor of safety of (4).

Solution

$Given:$

$$t = 30mm; d = 50mm; p = 150mm; w = 200mm;$$

$$\sigma_t = 600MPa; \tau_r = 480MPa; \sigma_b = 960MPa; d_h = d + 3 = 50 + 3 = 53mm.$$

1. Tearing resistance of the plate

$$\sigma_t = \frac{P}{(p - n.d_h) \times t}$$

$$P_T = \sigma_t \times (p - nd_h) \times t = 600 \times (150 - 1 \times 53) \times 30$$

$$\therefore P_T = 600 \times 97 \times 90 = 5238000N = 5238KN$$

2. Shearing resistance of the rivets

$$\tau_r = \frac{P_C / n}{\pi \times \frac{d^4}{4}}$$

$$P_S = \frac{\tau_r \times n \times \pi \times d^2}{4} = \frac{480 \times 2 \times 3.14 \times (50)^2}{4}$$

$$\therefore P_S = \frac{7536000}{4} = 1884000\ N = 1884\ KN$$

3. Crushing resistance of the rivets

$$\sigma_b = \frac{P_C / n}{d \times t}$$

$$P_C = \sigma_p \times n \times d \times t = 960 \times 2 \times 50 \times 30$$
$$\therefore P_C = 2880000N = 2880KN$$

$$\therefore Stengthof\ the\ joint = Leastof\ (P_T\ ,\ P_S\ ,\ P_C) = 1884\ KN$$

The minimal force per pitch required to tear the joint, as seen from above, is (1884 KN).

Example 5

a single riveted lap junction made of plates that are (6 mm) thick and (20 mm) in diameter with a pitch of (50 mm). Determine the efficiency of the following:

Assume

Permissible tensile stress in plate = 120 MPa

Permissible shearing stress in rivets = 90 MPa

Permissible crushing stress in rivets = 180 MPa

Solution

$$Given: t = 6mm; d = 20mm;\ P = 50mm\ ;\ \sigma_t = 120MPa$$
$$\tau_r = 90MPa; \sigma_b = 180MPa; d_h = d + 3 = 20 + 3 = 23mm.$$

1. **Tearing resistance of the plate**

$$\sigma_t = \frac{P_T}{(p - n.d_h) \times t}$$

$$P_T = \sigma_t \times (p - nd_h) \times t = 120 \times (50 - 1 \times 23) \times 6$$
$$\therefore P_T = 120 \times 27 \times 6 = 19440N = 19.44KN$$

2. Shearing resistance of the rivets

$$\tau_r = \frac{P_C / n}{\pi \times \frac{d^4}{4}}$$

$$P_S = \frac{\tau_r \times n \times \pi \times d^2}{4} = \frac{90 \times 1 \times 2 \times 3.14 \times (20)^2}{4}$$

$$\therefore P_S = \frac{226080}{4} = 56520 \ N = 56.52 \ KN$$

3. Crushing resistance of the rivets

$$\sigma_b = \frac{P_C / n}{d \times t}$$

$$P_C = \sigma_p \times n \times d \times t = 180 \times 1 \times 20 \times 6$$

$$\therefore P_C = 21600 N = 21.6 KN$$

$$\therefore Stength of\ the\ joint = Least of\ (P_T\ ,\ P_S\ ,\ P_C) = 19.44\ KN$$

Efficiency of the joint

We know that the strength of the unriveted or solid plate

$$P_{soild} = \sigma_t \times p \times t = 120 \times 50 \times 6$$

$$\therefore P_{soild} = 36000 N = 36 KN$$

$$\therefore \eta = \frac{Least of (P_T, P_S, P_C)}{P_{soild}} = \frac{19.44}{36} \times 100\% = 54\%$$

2-9. Chapter Questions

1. Typically, rivets are constructed of

a. A hard Conformable material
b. A conformable material
c. A ductile material
d. A brittle material

2. The distance between a plate's edge and the centerline of the closest row of rivets is referred to as:

a. Transverse pitch
b. Diagonal pitch
c. Margin
d. Pitch

3. The rivet in a single riveted lap joint is subjected to

a. A compressive stress
b. A double shear
c. A tensile stress
d. A single shear

4. A boiler and pressure vessel's rivet head is

a. Half countersunk.
b. Snap head.
c. Flat head.
d. Countersunk head.

5. The goal of caulking and fullering involves creating the riveted junction

a. Without any remaining strains
b. Strong
c. Leak proof
d. Permanent

6. The distance between a rivet's center and an adjacent rivet's center in the same row is referred to as:

a. A transverse pitch
b. A diagonal pitch
c. A Pitch
d. A Margin

7. A lap joint is constantly subjected to

a. A bending moment
b. A torsional moment
c. A compressive force
d. A tensile force

8. The joint efficiency with the lowest value is assumed when:

a. Double riveted butt joint
b. Single riveted lap joint
c. Single riveted butt joint
d. Single riveted lap joint

9. The link between the diameter of the rivet (d) and the thickness of the cylinder wall (t) is given by Unwin's formula:

a. $d = \sqrt{t}$
b. $d = 4\sqrt{t}$
c. $\boldsymbol{d = 6\sqrt{t}}$
d. $d = 8\sqrt{t}$

10. Which of the following isn't a main part of rivet?

a. Head
b. Shank
c. Point
d. Thread

11. A rivet is specified as a (50 mm) rivet. What does it mean?

a. Hole plate diameter is (50 mm).
b. Shank diameter is (50 mm).
c. Head diameter is (50 mm).
d. Both head and shank diameter are (50 mm).

12. Riveted joints is suitable for a

a. High temperature services
b. Medium temperature services
c. Low temperature services
d. None of these

13. Riveted joints is used for joining of

a. Structure
b. Pipe
c. Pressure vessel
d. Tank

14. In riveted joints main plate fails due to

a. Shearing
b. Compressing
c. crushing
d. Tearing

15. When thickness of plate is 25 mm, then diameter of rivet is used in joints is

a. 20 mm
b. 32 mm
c. 24 mm
d. 30 mm

16. Failure in rivet occurs by which mode?

a. Shear
b. Compression
c. Tensile
d. Each of the mentioned

17. A strap is used in a lap joint which is riveted to each of the two plates.

a. TRUE
b. FALSE
c. Can be true or false
d. Cannot say

18. Transverse pitch is the distance between two consecutive rows of rivets in the same plate

a. **Back pitch**
b. Pitch
c. Transverse pitch
d. Diagonal pitch

Chapter 3

Welded Joints

3. Welded Joints

3-1. Introduction

Welding joint is a manufacturing process in which two materials typically metals or thermoplastics are linked by melting their ends together under heat. External pressure may also be used to help the melting process, and after the materials have cooled and hardened, the weld joint is permanent, figure (3-1).

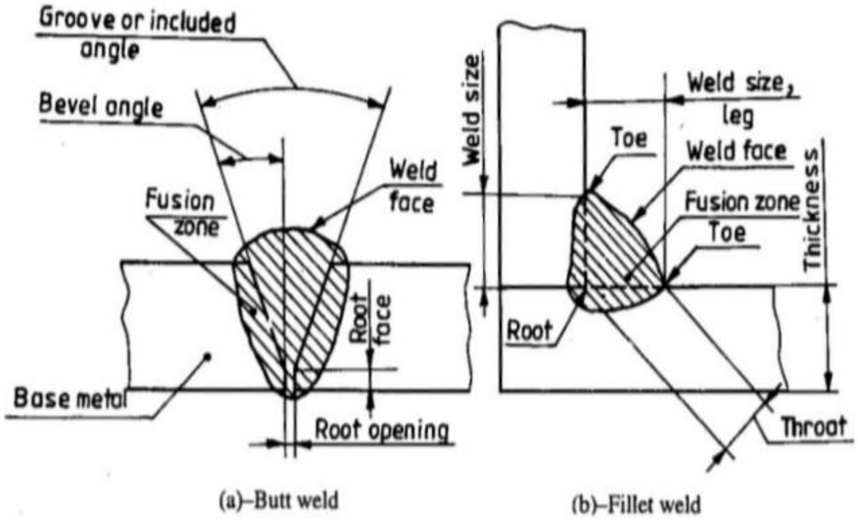


Figure 3-1: Parts joint welding

3-2. Symbol of welding

Figure (3-2) it shows all the symbols used in welding

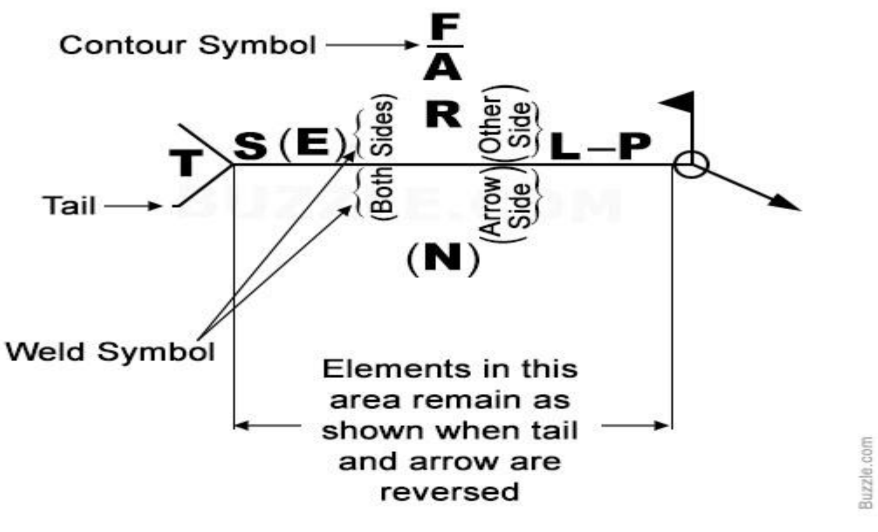


Figure 3-2: Symbol of welding

- **Reference Line**: It is placed near the joint it describes.
- **Arrow**: It points to the location or joint or spot that is to be welded.
- **Weld Symbol:** Distinguishes sides of the joint by using arrow and spaces above and below the reference line. The side where arrow points are known as arrow side whiles the opposite side is known as other side.
- **Tail:** It indicates the welding or cutting process along with welding specification, procedures, or supplementary information related to the weld.
- **L:** It indicates length of weld.
- **P (Pitch symbol)**: It is the distance between two consecutive welds, measured from the center of both welds.
- **F (Finish symbol)**: It indicates the need for finishing processes like grinding, brushing, or machining.
- **Contour**: It indicates the shape of the finished weld bead.
- **A (Groove angle)**: It mentions the angle of the opening between the two welded parts.
- **R (Root Opening):** It denotes the distance between the root edges of two metals that need to be joint.
- **E (Groove Weld Size):** It indicates the size of the groove weld.
- **S (Depth):** It indicates the size or penetration (strength) of certain type of welds.
- **N:** It indicates the number of spots, seams, studs, plugs, slots, or projection welds

3-3. Types of welding joint

There are five types of welding joints which are given below, (figure 3-3).

1. Edge Joint,
2. Butt Joint,
3. Corner Joint,
4. Lap Joint,
5. Tee Joint.

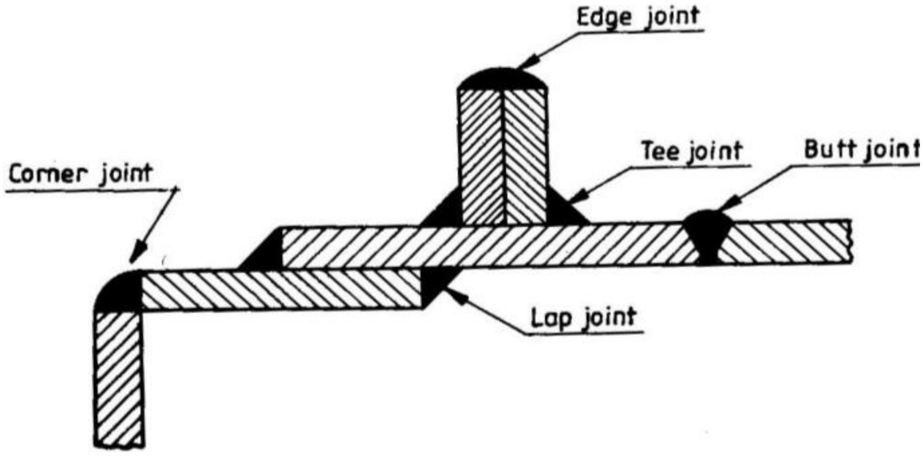
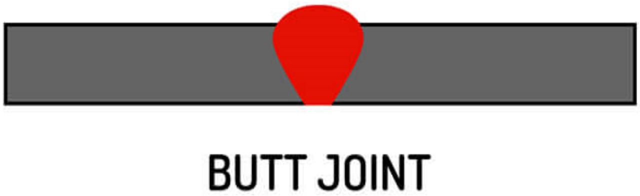


Figure 3-3: Types of welding joint

3-3-1. Butt Joint welding

Butt joints are a form of junction used to solder thin metal sheets where two metal components are linked in the same plane, figure (3-4).



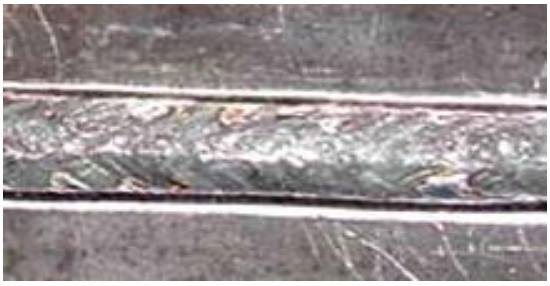


Figure 3-4: Butt joint welding

Any butt-welded joint is weak behind it for the reasons listed below.

1. Cracking,
2. Slag entrapment,
3. Excessive porosity,

3-3-1-1. Various welding butt joint types

1. Single welded,
2. Double welded,
3. Open welded,
4. Closed welded.

3-3-1-2. Advantages and disadvantages of Butt-welding joint

Table (3-1) shown Butt-Welding joint advantages and disadvantages.

Table 3-1: Butt-Welding joint advantages and disadvantages

NO.	Advantages	Disadvantages
1	Universally recognized technique.	Chances of porosity in butt welds.
2	Easiest welding method.	
3	Most common method.	It's necessary to prepare the edges of thick metal components while working with them.
4	Very affordable (cheap method).	

3-3-1-3. Applications of Butt-Welding Joint

1. Pipes,
2. Valves,
3. Flanges,
4. Fittings.

3-3-2. Corner Joint welding

One of the most common ways to join metal sheets is using a corner junction, which is made by positioning two portions' corners at a right angle to one another. This method is utilized on the sheet's outside edge., figure (3-5).

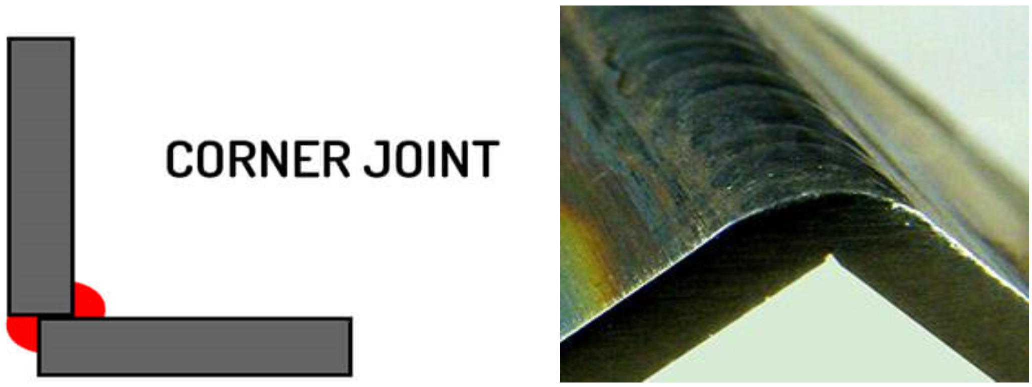


Figure 3-5: Corner joint welding

3-3-2-1. Corner joints welding types

1. Flush welded corner joint,
2. Full open welded corner joint,
3. Half open welded corner joint.

3-3-2-2. Styles of welding corner joints

Corner Joints are made using several welding techniques.

1. Spot weld,
2. Bevel Grooved weld,
3. Corner Flange weld,
4. Edge weld,
5. Flare V groove weld,
6. Butt weld,
7. Square groove weld,
8. Fillet weld,
9. J- Groove weld,
10. U-groove weld,
11. V-groove weld.

3-3-2-3. Advantages and disadvantages of Corner - welding joint

Table (3-2) shown Corner -Welding joint advantages and disadvantages.

Table 3-2: Corner - welding joint advantages & disadvantages

NO.	Advantages	Disadvantages
1	It is possible to make strong welds.	There is a possibility of increased Wear and Tear in a corner joint.
2	A variety of welds, including thinner and thicker ones, are feasible.	
3	The most often used method of joining metal sheets.	

3-3-2-4. Applications of Corner Welding Joint

1. When joining pieces of sheet metal together to form various shapes, corner joints welding is utilized.
2. A near corner joint may be used to solder thin metal sheets where strength is not required.
3. Welding is done on one side of heavier metal sheets to create a half corner junction.
4. This method can be used to build boxes, box frames, and other related types of fabrication.

3-3-3. Edge Joint Welding

Edge joints are the kind of joints created by joining the edges of two separate components by welding, figure (3-6).

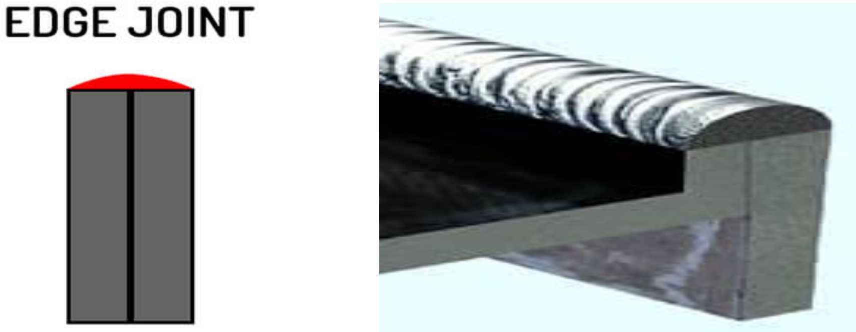


Figure 3-6: Edge joint welding

3-3-3-1. Welding edge joint styles

Edge Joints are made using several welding techniques.

1. V-Groove edge joint welded,
2. J- Groove edge joint welded,
3. U- Groove edge joint welded,
4. Bevel Groove edge joint welded,
5. Edge-flange edge joint welded,
6. Corner flange edge joint welded,

7. Square-groove weld/butt edge joint welded.

3-3-3-2. Advantages and disadvantages of edge welding joint

Table (3-3) shown edge -welding joint advantages and disadvantages.

Table 3-3: Edge -welding joint advantages and disadvantages.

NO.	Advantages	Disadvantages
1	Useful for sheets that are no thicker than 3 mm.	At the thickness of the connection, the weld does not penetrate entirely.
2	This type weld joint doesn't need to be prepared.	Applications involving stress and pressure cannot employ this type of welding joint.
3	The sheets can be joined together without filler material.	Edge welds are less typical than other kinds of weld joints.
4		Due to build up amassing on the edges, these joints are frequently replaced with new joints.
5		Extremely uncommon joint method

3-3-3-3. Applications of Edge Welding Joint

1. This welding joint is typically employed when sheet edges are close together and nearly parallel at the welding spot.
2. These joints are most frequently employed for metal components with flanging up edges or when a weld is required to connect two nearby sections.
3. Edge welds are most frequently utilized for materials whose sheets are no thicker than 3 mm.
4. Automotive gas tanks & assembly housing also use flanged joints with edge welding.
5. The world of aerospace repair frequently uses this welded junction.

3-3-4. Lap Joint Welding

The type of joint known as a lap joint is When elements are arranged in overlapping places, two work pieces are positioned next to one another, forming one above the other, figure (3-7).

Lap joint could mean:

1. One-sided,
2. Double Sided.

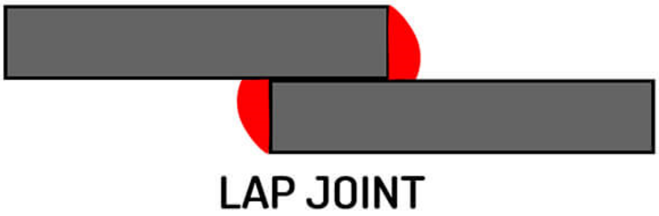


Figure 3-7: Lap joint welding

3-3-4-1. Styles of lap joint welding

Types of welding used to create lap joints

1. Fillet lap joint welding,
2. Bevel-groove lap joint welding,
3. Slot lap joint welding,
4. Plug lap joint welding,
5. Spot lap joint welding,
6. Flare bevel groove lap joint welding,
7. J-groove lap joint welding.

3-3-4-2. Advantages & Disadvantages of Lap Welding Joint

Table (3-4) shown lap -welding joint advantages and disadvantages.

Figure 3-4: Lap -welding joint advantages and disadvantages

NO.	Advantages	Disadvantages
1	A satisfactory lap weld can be produced with little difficulty.	One sided lap welding is unsupportable on sheets carrying heavy loads.
2	Because the second sheet is present on the opposite side of the joint, there is no chance of blowing through the weld.	
3	Lap joints may be fabricated more quickly and easily.	
4	A double-sided weld results in stronger welded joints.	

3-3-4-3. Applications of lap welding joint

1. Gas tungsten arc welding, gas metal arc welding, and resistance spot welding are the most prevalent applications for lap weld joints. Lap welding rarely employs high intensity welding techniques like electron or laser beam welding.
2. Wood and plastic can both use lap joints.
3. Lap welds are frequently employed in procedures involving automation. They are employed in:
   A. Temporary framing,
   B. Tabling,
   C. Frame assembly in cabinet making.

3-3-5. Tee Joint Welding

Two parts coming together at a right angle to form a joint is known as a tee junction, or (90 degrees), and one part is positioned above the other in the middle, as seen in the figure (3.8).

Figure 3-8: Tee joint welding

3-3-5-1. Styles of Tee welding joint

Used welding techniques to make Tee Joints

1. Fillet Tee welding joint,
2. Flare-bevel Tee welding joint,
3. Plug Tee welding joint,
4. J-groove weld,
5. Slot Tee welding joint,
6. Bevel-groove Tee welding joint,
7. Melt-through Tee welding joint.

3-3-5-2. Advantages and Disadvantages of Tee Welding Joint

Table (3-5) shown Tee -welding joint advantages and disadvantages.

Figure 3-5: Tee -welding joint advantages and disadvantages

NO.	Advantages	Disadvantages
1	For a plain Tee welding joint, there is no preparation	To ensure effective penetration on the weld roof, extra caution is required.

2	necessary.	In Tee welded joints, corrosion fatigue is commonplace.
3		Tee welded joints may have increased cracking, moisture entrapment, excessive porosity, or corrosion because of the geometry they create.

3-3-5-3. Applications of Tee Welding Joint

1. Tee welded connectors are used to join metal pieces to bases of various types.
2. Machine and structural applications have used tee welded joints,
3. When attaching thin plates, a single beveled junction that can be welded from one side is typically employed,
4. To weld hefty plates from both sides, a double beveled junction is required.

3-4. Design of welded joints

3-4-1. Design of a Butt Joint

The butt joints can be used in either compression or tension. Average tensile stress in a butt welded joint subjected to tensile load [P**,** (N/mm^2)], figure (3-9), is given by,

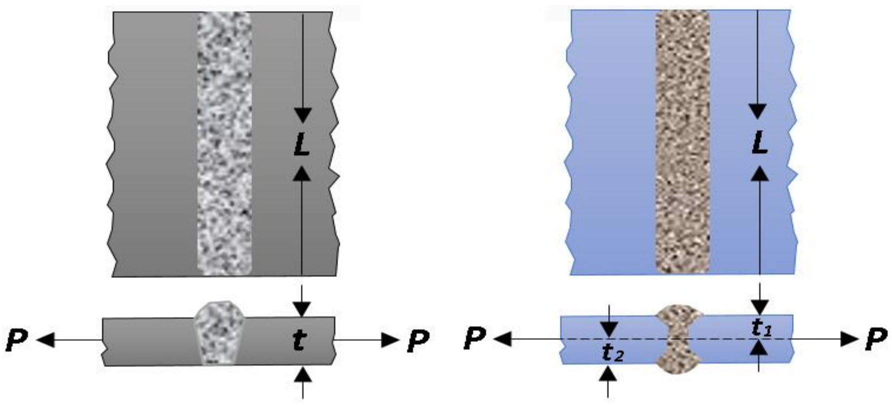
$$\sigma_t = \frac{P}{A} = \frac{P}{t\,.\,L} \qquad (3-1)$$

Whereas, (A**)** is throat area (mm^2), (**t)** is throat thickness(mm) and (L) is length of the weld(mm). **(**$\boldsymbol{\sigma_t}$) must be ≤ [$\boldsymbol{\sigma_t}$] for the joint to be safe.

Also, Average Compressive Stress in a butt welded joint subjected to compressive load, (**P)** is given by the following formula,

$$\sigma_c = \frac{P}{A} = \frac{P}{t\,.\,L} \qquad (3-2)$$

Which must be ≤ [$\boldsymbol{\sigma_c}$]



a. *b.*

Figure 3-9: Butt Joint, a. Single V-Butt Joint, b. Double V-Butt Joint

The throat area of a double V-butt joint is equal to $(t_1 + t_2)$, where $(t_1 \ \& \ t_2)$ are the top and bottom throat thicknesses.

Codes for unfired pressure vessels, for example, propose reducing the strength of the butt welded joint by a quantity known as the joint's efficiency. where a reduction in strength is desired, the following modification and rewriting of the equation:

$$\eta = \frac{Output}{Input} = \frac{\sigma_t}{P.t.L} \times 100\% \qquad (3-3)$$

3-4-2. Design of a Fillet Joint

3-4-2-1. Transverse Fillet Weld

Tensile strength is the goal while designing transverse fillet welds. The fillet section is assumed to be a right-angled triangle for the sake of strength calculations, with the hypotenuse making equal angles with the two sides as illustrated in figure (3-10).

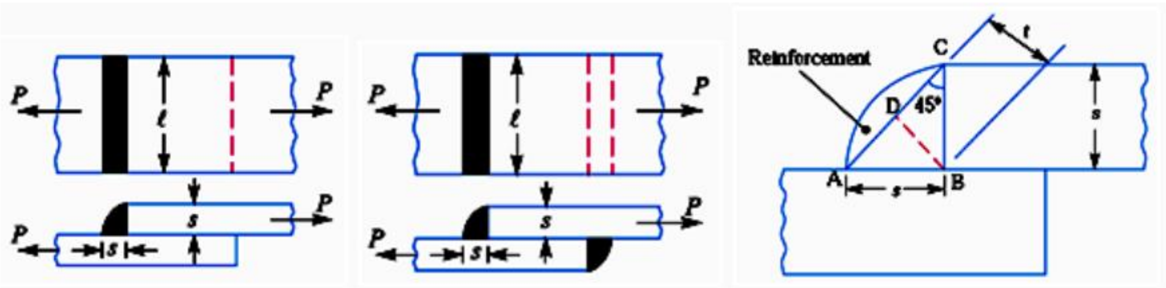


Figure 3-10: Shown transverse single & double fillet weld

Size or leg of the weld (s) is defined as the length of each side (AB = BC), and throat thickness (t) is defined as the distance between the hypotenuse and the intersection of two legs (BD). At the throat, a minimum area is reached, if (L) is the weld's length.

Throat area, $A = \boldsymbol{t . L = s. sin\, 45°. L = 0.707\, s . L}$

When a single transverse fillet weld is subjected to a tensile load (P), the tensile stress (σ_t) is calculated as follows:

$$\sigma_t = \frac{P}{A} = \frac{P}{0.707\, s\, .\, L} \leq [\sigma_t] \qquad (3-4)$$

Also, the formula below provides that for a double transverse fillet weld:

$$\sigma_t = \frac{P}{2A} = \frac{P}{1.414\, s\, .\, L} \leq [\sigma_t] \qquad (3-5)$$

3-4-2-2. Parallel Fillet Weld

For shear strength, parallel fillet welds are used. Consider about a parallel fillet weld as shown in figure 3-11.

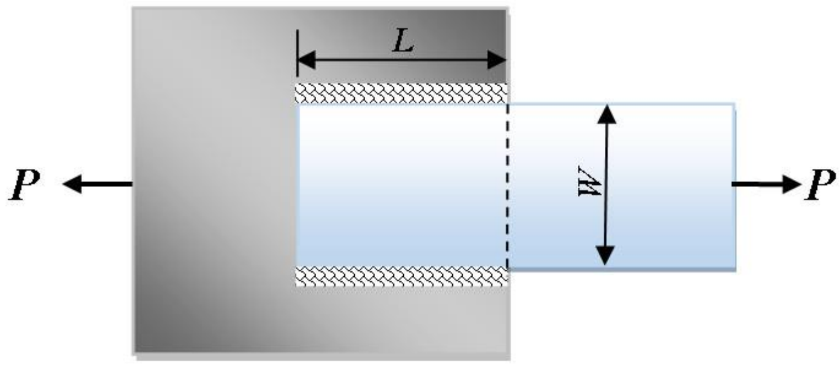


Figure 3-11: Parallel Fillet Weld

Throat Area ($A = 0.707\, s . L$), where (s & L) are size and length of the weld. For a parallel fillet weld subjected to tensile load (P), shear stress (σ_t) is given by the following:

$$\sigma_t = \frac{P}{2A} = \frac{P}{1.414\, s\, .\, L} \leq [\tau] \qquad (3-6)$$

3-4-3. Axially loaded unsymmetrical welded joints

In some applications, welding unsymmetrical pieces like an angle or a (T) to the beams or steel plates is necessary. Figure (3-12) shows two parallel fillet welds being used to join an angle section to a vertical beam (1 & 2). The external force pressing on the joint passes via (G), which represents the angle section's center of gravity (G). Assume that the opposing forces (P1 & P2) are configured in the welds (1 & 2), respectively. in the equation (6).

$$P_1 = 0.707\, h\,.\; l_1\,.\;\; \tau \qquad (3-7)$$

$$P_2 = 0.707\, h\,.\; l_2\,.\;\; \tau \qquad (3-8)$$

$$P = P_1 + P_2 \qquad (3-9)$$

Since the moment of forces about the center of gravity is equal to zero,

$$P_1\,.\; y_1 = P_2\,.\; y_2 \qquad (3-10)$$

Substituting expressions equations (7 & 8) in the expressions (9),

$$l_1\,.\; y_1 = l_2\,.\; y_2 \qquad (3-11)$$

Assuming total length of welds as (l),

$$l = l_1 + l_2 \qquad (3-12)$$

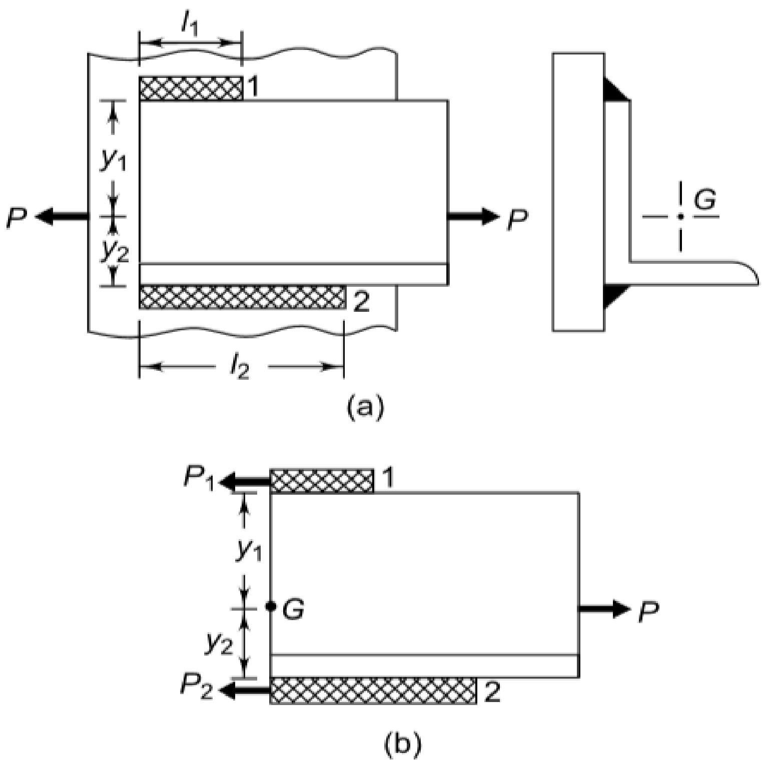


Figure 3-12: Axially loaded unsymmetrical welded joints

3-5. Solve Examples

Example 1

Figure (3-13) of a gas tank has an inner diameter of (5 meters). As depicted in the figure, it is encased by hemispherical shells using a butt - welded junction. The hemispherical cover and the cylindrical shell both have a thickness of (16 mm). If the acceptable tensile stress in the weld is (73 N/mm^2), determine the maximum internal pressure to which the tank may be exposed. Assume that the welded junction is 66% efficient.

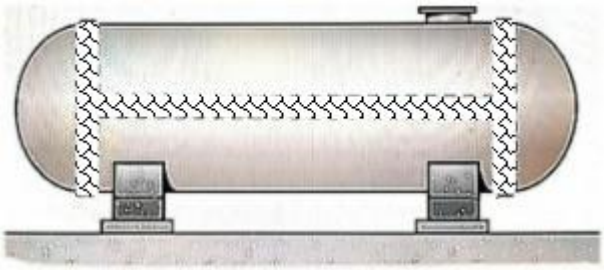


Figure 3-13: A gas tank

Solution:

Given:

$$\boldsymbol{D} = 5\,m = 5000\,mm, t = 16\,mm, \sigma_t = 73\frac{N}{mm^2}, \eta = 66\,\%$$

1- Tensil force in plate (F):

$$Circumference\ of\ the\ shell\ (L) = \pi.D = 3.14 \times 5000 = 15700\,mm$$

$$\text{Tensil force in plate } (F) = \sigma_t.t.L.\eta$$

$$\text{Tensil force in plate } (F) = 73 \times 16 \times 15700 \times 0.66 = 12102816\,N \quad \approx 12.1\,MN$$

2- Allowable internal pressure (P):

$$Area\ (A) = \pi.\frac{D^2}{4} = 3.14 \times \frac{5000^2}{4} = 19625000\,mm^2$$

$$\text{Allowable internal pressure (P)} = \frac{F}{A}$$

$$\text{Allowable internal pressure (P)} = \frac{12102816}{19625000} = 0.617\,N/mm^2$$

Example 2

A steel plate as in figure (3-14), have (*85 mm*) wide (*18 mm*) thick, is welded to another steel plate by means of double parallel fillet welds as shown in figure. The plates are subjected to a static tensile force of (*133 KN*). Determine the required length of the welds if the permissible shear stress in the weld is (*83 N/mm²*).

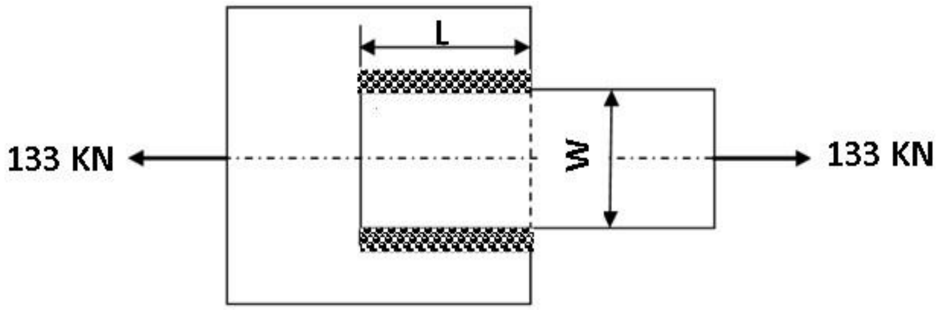


Figure 3-14: A steel plate

Solution:

$\boldsymbol{Given}$:

$$P = 133\,KN = 133000\,N, \tau = 83\,\frac{N}{mm^2}, h = 18\,mm$$

$$P = 1.414\,h.L.\tau$$

$$\therefore\ Leng\ of\ Weld\ \ (L) = \frac{P}{1.414\,h.\tau} = \frac{133000}{1.414 \times 18 \times 83} = 62.96\,mm$$

$$\text{Adding (15 mm)}\ of\ length\ for\ strting\ and\ ending\ of\ the\ weld$$

$$\therefore\ Leng\ of\ Weld\ \ (L) = 62.96 + 15 = 77.96 \approx 78\,mm$$

Example 3

Two steel plates as in figure (3-15), (*215mm*) wide and (*16 mm*) thick, are jointed together by means of double transverse fillet welds as shown in figure. The maximum tensile stress for the plates and the welding material should not exceed (*133 N/mm²*). Find the required length of the weld, if the strength of weld is equal to the strength of the plates.

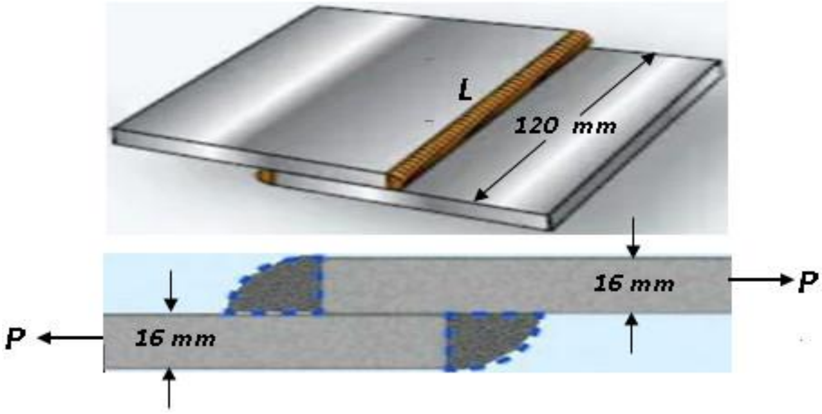


Figure 3-15: A two steel plates

Solution

$\boldsymbol{Given}$:

$$W = 120\,mm, t = 16\,mm, h = 16\,mm, \sigma_t = 133\,N/mm^2$$

$$Tensil\ force\ in\ plate\ (P) = A.\sigma_t = W \times t \times \sigma_t$$

$$\therefore P = 120 \times 16 \times 133 = 255360\,N$$

$$P = 1.414\,h.L.\sigma_t$$

$$\therefore\ Length\ of\ the\ weld\ (L) = \frac{P}{1.414\,h.\sigma_t}$$

$$L = \frac{255360}{1.414 \times 16 \times 133} = 84.87\,mm$$

$$\text{Adding (15 mm)}\ of\ length\ for\ strting\ and\ ending\ of\ the\ weld$$

$$\therefore\ Leng\ of\ Weld\ \ (L) = 84.87 + 15 = 99.87 \approx 100\,mm$$

Example 4

A steel plate as in figure (3-16), (*66 mm*) wide and (*14 mm*) thick, is joined with another steel plate by means of single transverse and double parallel fillet welds, as shown in figure. The joint is subjected to a maximum tensile force (*83 KN*). The permissible tensile and shear stresses in the weld material are (*88 & 62 N/mm*^{2}) respectively. Determine the required length of each parallel fillet weld.

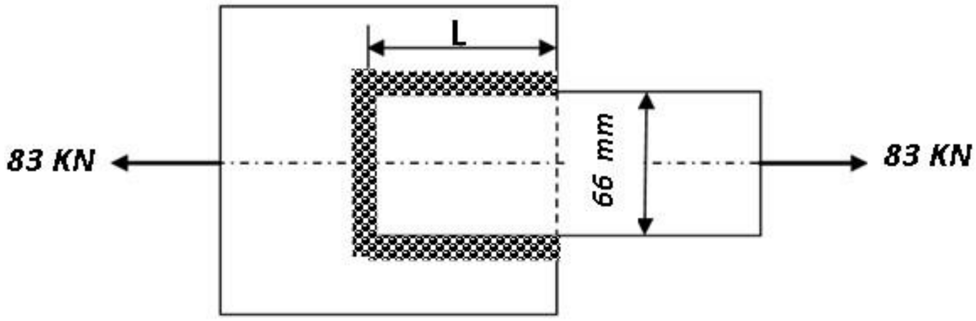


Figure 3-16: A steel plate

Solution

$\boldsymbol{Given}$:

$$P = 83\ KN = 83000\ N, \sigma_t = 88 \frac{N}{mm^2}, \tau = 62 \frac{N}{mm^2}, h = 14\ mm$$

$$The\ strength\ of\ transverse\ fillet\ weld\ (P_1) = 0.707\ h. L. \sigma_t$$

$$P_1 = 0.707\ \times 14 \times L \times 88 = 871.024\ L \qquad (1)$$

$$The\ strength\ of\ double\ fillet\ fillet\ weld\ (P_2) = 1.414\ h. L. \sigma_t$$

$$P_2 = 1.414\ \times 14 \times L \times 62 = 1227.352\ L \qquad (2)$$

Adding (15 mm) $of\ length\ for\ strting\ and\ ending\ of\ the\ weld$

$$\therefore\ Leng\ of\ Weld\ \ (L) = 62.96 + 15 = 77.96 \approx 78\ mm$$

$$\because P = P_1 + P_2 \qquad (3)$$

$$Substituting\ the\ first\ and\ second\ equations\ into\ the\ third\ equation\ produces:$$

$$83000 = 871.024\ L + 1227.352\ L$$

$$83000 = 2098.375\ L$$

$$\because L = \frac{83000}{2098.375} = 39.55\ mm$$

Adding (15 mm) $of\ length\ for\ strting\ and\ ending\ of\ the\ weld$

$$\therefore\ Leng\ of\ Weld\ \ (L) = 39.55 + 15 = 54.55 \approx 55\ mm$$

Example 5

To create an angle in the side welds while welding an ISA angle (180 * 66 * 14) to a steel plate, as shown in figure (3-16). The angle section's center of gravity, which is (73 mm) from the short side, is subject to a static load of (233 KN). The maximum load that can be applied to a weld length in a millimeter is (733 N). How much length of a (14 mm) fillet weld is needed?

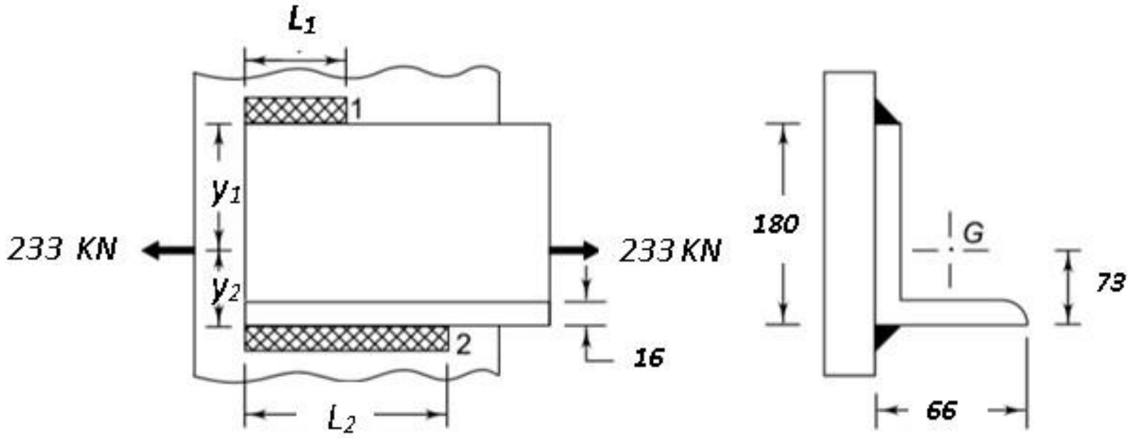


Figure 3-17: A steel plate with side welds angle

Solution

Given:

$$P = 233\ KN = 233000\ N, Allowable\ Load = 733\ Neton\ per\ Milmeter\ of\ weld$$

$$\therefore\ Total\ length\ of\ the\ weld\ (L) = \frac{P}{P_{all}} = \frac{233000}{733} = 317.87\ mm$$

$$L = L_1 + L_2 \qquad \Rightarrow \quad L_1 + L_2 = 317.87 \ldots\ldots\ldots\ldots (1)$$

$$y = y_1 + y_2 \ \Rightarrow \ y_1 = 180 - 73 = 107\ mm$$

$$y_{1.}L_1 = y_2.L_2 \quad \Rightarrow \quad L_1 = \frac{y_{2.}L_2}{y_1} = \frac{73\ L_2}{107} = 0.682\ L_2 \ldots\ldots\ldots (2)$$

$$Substituting\ the\ second\ equation\ into\ the\ first\ equation, we\ get\ (L_1):$$

$$0.682\ L_2 + L_2 = 317.87\ mm$$

$$L_2 = \frac{317.87}{1.682} = 188.98\ mm$$

$$L_1 = 1.466\ L_2 = 0.682\ \times 188.98 = 128.89\ mm$$

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3-6 Chapter Questions

1. The purpose of the transverse fillet welds is

e. Bending strength
f. Shear strength
g. Tensile strength
h. Compressive strength

2. Arc welding is also known as

a. pressure welding
b. Plastic welding
c. non-pressure welding
d. None of these

3. It mentions the angle of the opening between the two welded parts.

a. Groove angle
b. Root Opening
c. Pitch symbol
d. Weld Symbol

4. It denotes the distance between the root edges of two metals that need to be joint.

a. Groove angle
b. **Root Opening**
c. Pitch symbol
d. Weld Symbol

5. It is the distance between two consecutive welds, measured from the center of both welds.

a. Groove angle
b. Root Opening
c**. Pitch symbol**
d. Weld Symbol

6. Distinguishes sides of the joint by using arrow and spaces above and below the reference line.

a. Groove angle
b. Root Opening
c. Pitch symbol
d. **Weld Symbol**

7. Welding joint design is based on -------- strength.

a. Tension
b. Shearing
c. Compressive
d. Bending

8. Which joint is designed for shear strength?

a. Parallel fillet welding joint
b. Transverse fillet welding joint

c. Both A and B
d. None

9. Welding is removed by ----------.

a. Polishing
b. Grinding
c. Cutting
d. None of these

10. Which of the following welding joint based on shear strength?

a. Butt Welded joints
b. Transverse fillet Welded joint
c. Parallel fillet Welded joint
d. All of these

11. Which of the following welding joint based on tensile strength?

a. Butt Welded joints
b. Transverse fillet Welded joint
c. Parallel fillet Welded joint
d. Both A and B

12. Which of the following types is not fillet weld?

a. Butt joint
b. T - joint
c. Lap joint
d. Corner joint

13. Welded joint made by overlapping the plate is called -------.

a. Butt welded joint
b. Fillet welded joint
c. Fillet butt welded joint
d. None

13. The term for a welded junction created by positioning the plate edge to edge is ------.

a. Butt welded joint
b. Fillet welded joint
c. Fillet butt welded joint
d. None

14. The advantage of a welded joint over a riveted joint is ----------.

a. Introduce residual stresses
b. Requires highly skilled labor and supervision
c. Easy process
d. Lighter in weighed

15. Parallel fillet welds are under ----------.

- **a. Shear stress**
- b. Bending stress
- c. Compressive stress
- d. Tensile stress

16. Butt welds are under ----------.

- a. Shear stress
- **b. Tensile and compressive stress**
- c. Compressive stress
- d. Bending stress

17. Corner welding joint types are:

- a. Flush Corner Joint and Half Open Corner Joint.
- b. Half Open Corner Joint and Full Open Corner Joint.
- c. Full Open Corner Joint and Flush Corner Joint.
- d. Full Open Corner Joint, Flush Corner Joint, and Half Open Corner Joint.

18. Why are butt welded joints longitudinal joints?

- **a. high strength requirements**
- b. low strength requirements
- c. low as well as high strength requirements
- d. none

19. Lap welded joints used in circumferential joints because of?

- a. High strength requirements.
- **b. Low strength requirements.**
- c. Low as well as high strength requirements.
- d. None.

20. Longitudinal welded joint fails by

- a. **Hoop stress.**
- b. Radial stress.
- c. Hoop Longitudinal stress.
- d. Tensile stress.

21. Circumferential welded joint fails by

- **a. Longitudinal stress.**
- b. Tensile stress.
- c. Radial stress.
- d. Hoop stress.

22. In which case, molten metal is used to do welding?

a. Groove type
b. spot type
c. Plug type
d. None

23. Joint which does not fall under lap joint category?

a. Transverse fillet
b. Parallel fillet
c. Circular fillet
d. None

24. Choose which is not a lap joint?

a. Single V
b. Double V
c. Single U
d. Single S

25. Area under tension in a single transverse fillet lap weld is

a. $\mathbf{0.707\ t\ .L}$
b. $0.807\ t\ .L$
c. $1.404\ t\ .L$
d. $0.107\ t\ .L$

26. The advantage of a welded joint over a riveted joint is:

a. Introduce residual stresses
b. Requires highly skilled labor and supervision
c. Lighter in weight
d. Heavy in weight

27. Welded joint used in the longitudinal joint of a cylindrical pressure vessel is;

a. Butt joint
b. Lap joint
c. corner joint
d. Tee joint

28. Efficiency of a welded joint with respect to a riveted joint is:

a. Smaller
b. Equal
c. Greater
d. Weakens

29. Which of the following welded joints is the stronger?

a. Butt joint
b. Lap joint

c. Corner joint
d. Edge joint

30. The strength of a welded joint with respect to a riveted joint is

a. Greater
b. Smaller
c. Equal
d. None

31. Welded joints are

a. Hinged joint
b. Rigid joint
c. Freely supported joint
d. None

32. Lap welded joints are

a. Rectangular
b. Square
c. Triangular
d. None

33. Welding requires

a. Non-skilled worker
b. Skilled worker
c. Any worker
d. None

34. Figure illustrates how to join two plates that are each (75 mm) wide and (15 mm) thick using a single transverse weld and a double parallel fillet weld. If (75 Mpa & 60 Mpa) are the maximum tensile and shear stresses, respectively, as in figure (3-18).

If the joint is being loaded statically, the length of each parallel fillet weld should be determined.

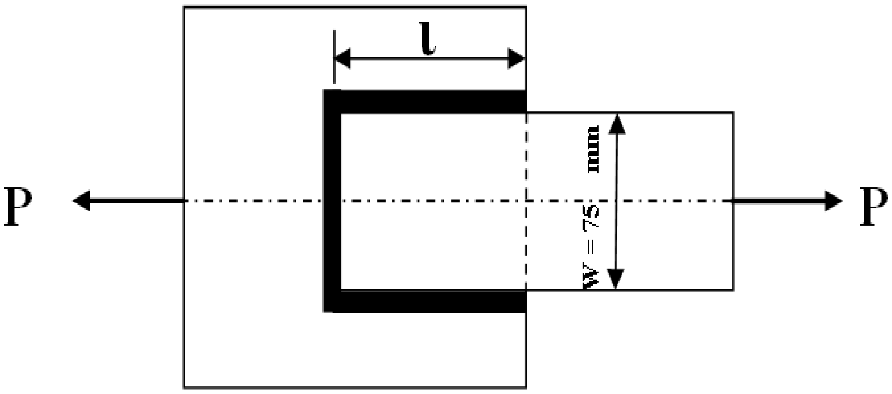


Figure 3-18: A steel plate

Dr. Emad T. Karash Dr. Jamal N. Sultan Ahmed M. Mosa Iman Z. Alshih Yahya

35. According to the illustration, a steel plate with fillet welds has an ISA (200*100*10) angle welded on it. (*150 KN*) of static force is applied to the angle, and (*70 N/mm^2*) of shear stress is permitted for the weld as in figure (3-19). Calculate the length of the top and bottom welds?

Answer: [*l_1 = 108.81 mm, l_2 = 194.28 mm*].

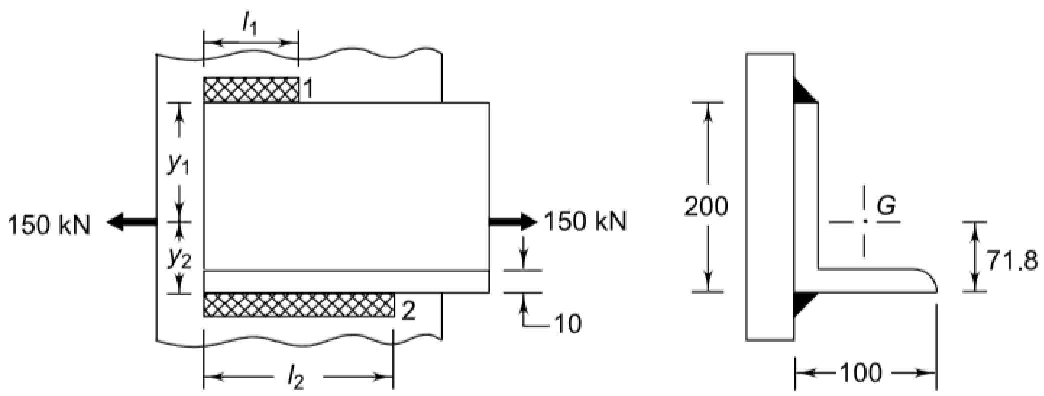


Figure 3-19: A steel plate with side welds angle

36. A circumferential fillet weld is used to join a circular shaft as in figure (3-20), with a support that has a (*50 mm*) diameter, as indicated in the image. it experiences a (2500 N/mm) torsional moment, if the permitted shear stress in the weld is restricted to (140 *N/mm^2*). Determine the size of the weld?

37. Answer: [*t = 4.55 mm, h = 6.43 mm*].

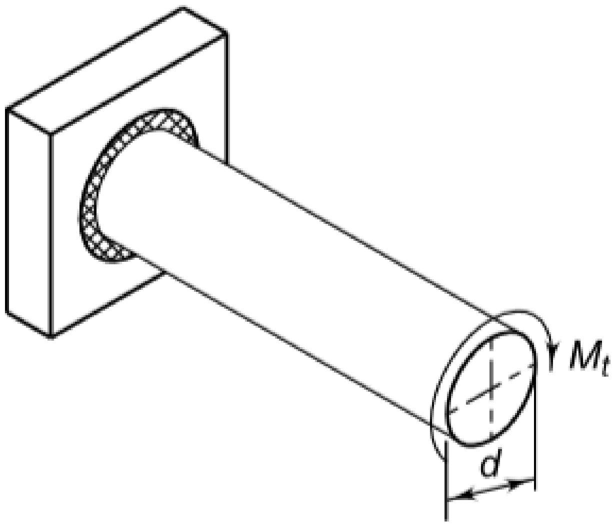


Figure 3-20: A steel bar

Chapter 4

Screwed Joints

4. Screwed Joints

4-1. Introduction

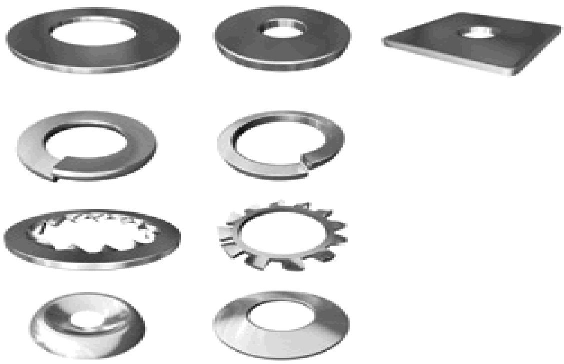
Typically, fasteners like screws, bolts, nuts, washers, and cotter joints are used to make non-permanent couplings, and some permanent joints through the process which includes welding, riveting, soldering, brazing etc.

4-2. Fasteners type

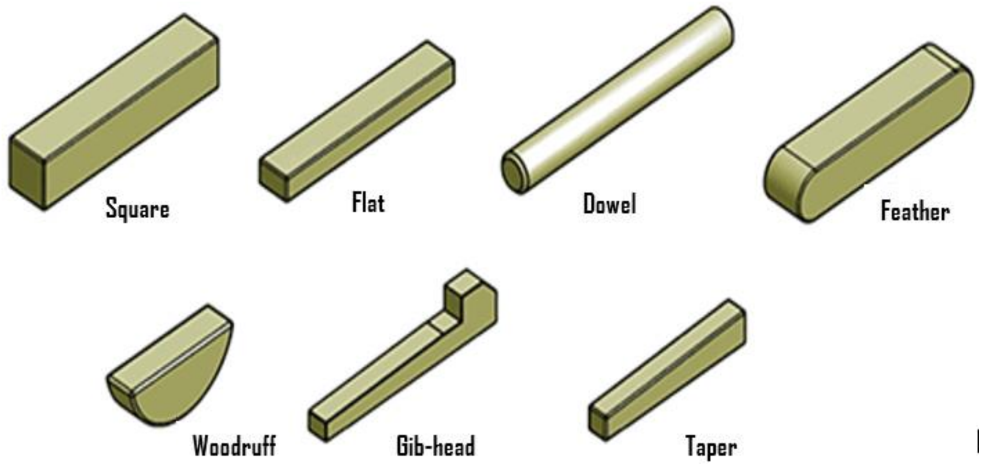
1. **bolts and nuts (threaded)**

2. **set screws (threaded)**

3. **washers**



4. **keys**



5. **Pins and rings**

Pins **Rings**

4-3. Screwed Joints

A continuous helical groove is cut onto a cylindrical surface to create a screw thread, or a screw thread is a ridge with a helix-shaped, consistently sized part.

External thread: External threads are on the outside of a member and chamfer on the end of the screw thread makes it easier to engage the nut.

1. **Internal Thread:** Internal threads are on the inside of a member and an internal thread is cut using a tap.

A single helical groove on the cylinder is cut to create a single thread (also known as a single-start screw), then a second thread is cut to create a double thread (also known as a double-start screw) in the area between the first thread's grooves. It is also possible to establish triple and quadruple (or multiple-start) threads. Figurers can use either their right or left hand to cut the helical grooves (4-1 & 4-2).

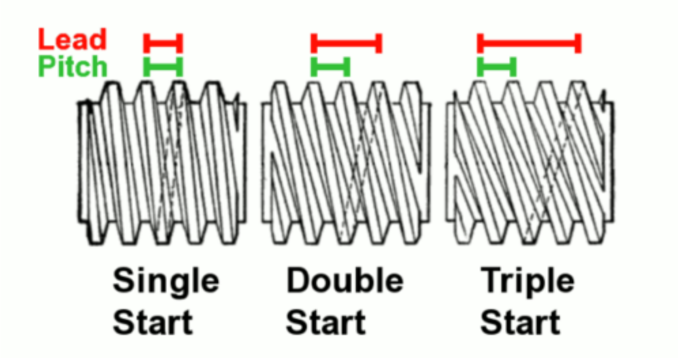


Figure 4-1: Types of start

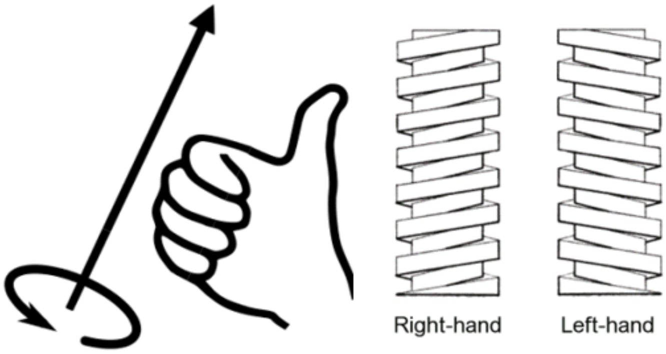


Figure 4- 2 : The helical grooves right and left hand

A bolt and a nut are the two components that make up a screwed joint. When connecting or disconnecting machine parts quickly and safely without endangering the machine or the fastening, screwed joints are frequently employed.

4-4. Types of threads used in power screws

Threads come in a variety of styles, figure 4-3.

1. Shape V threads
2. Whitworth threads
3. Buttress thread
4. Square threads
5. ACME threads

6. Worm’s threads

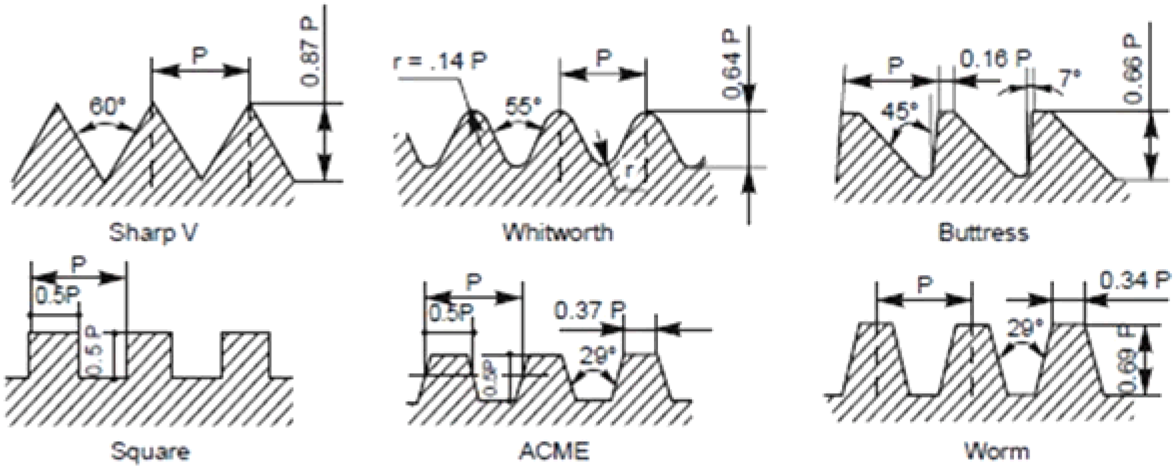


Figure 4-3: Types of threads

4-5. Applications of power screws

1. It is used to raise the load, for example, screw jack.
2. It is used to obtain a precise motion, for example, lead screw of lathe.
3. It is used to load a specimen, for example, on a universal testing machine.
4. It is used to clamp a work piece, for example, vice.

4-6. Parts of power screws

A power screw has following three parts.

1. It consists a Screw,
2. It consists a Nut,
3. It consists a part which holds either nut or bolt in place.

4-7. Advantages and disadvantages of screwed joints

The advantages and disadvantages of screwed joints are as follows.

1. **Advantages**
    a. Screwed joints are extremely reliable in operation,
    b. Screwed joints are easy to assemble and disassemble,

c. A wide variety of screwed joints can be adapted to various operating conditions,

d. Screws are relatively inexpensive to produce due to standardization and highly efficient manufacturing processes.

2. Disadvantages

The fundamental drawback of screwed joints is the stress concentration in the threaded parts, which are weak spots under varying load circumstances.

The strength of screwed joints should be considered inferior to that of riveted or welded ones.

4-8. Important screw thread terminology

The following screw thread terms, as shown in figure (4-4), are relevant to the subject:

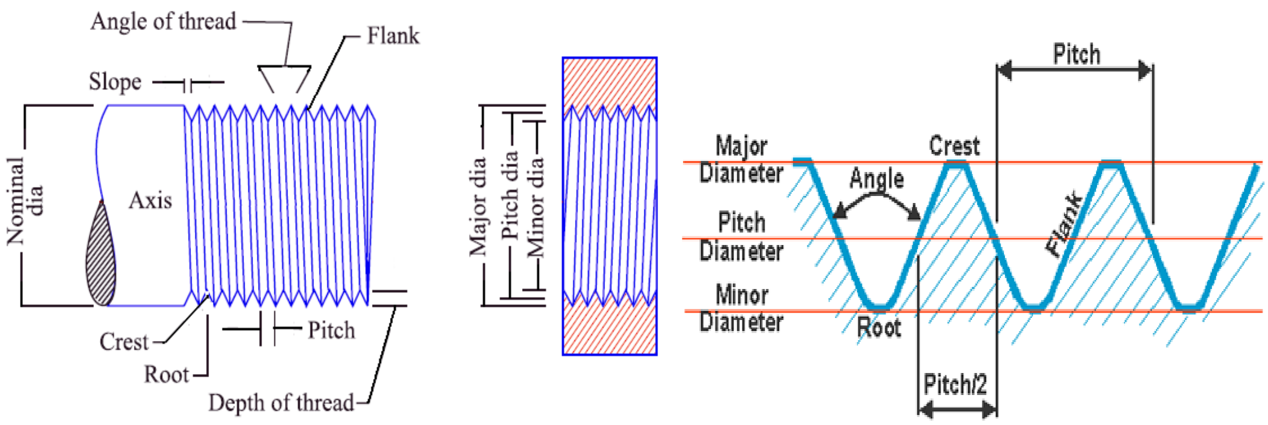


Figure 4-4: Terms used in screw threads

1. **Major Diameter-** The diameter of the thread that would touch the crests is its largest.
2. **Minor Diameter** - It is the smallest diameter of the thread which would touch the roots.
3. **Pitch Diameter** - It is a middle diameter between the major and minor diameter of screw threads.
4. **Root** - Root of the deepest part of the grove that corresponds with the minor diameter.
5. **Flank Angle** -This is the angle made by the intersection of the two thread flanks.
6. **Pitch** - Pitch of a thread is the distance between 2 crests.

Mathematically،

$$Pitch\ (P) = \frac{1}{Number\ of\ threads\ per\ unit\ length\ of\ screw} \qquad (4-1)$$

7. **Angle** - The thread angle of a screw is the angle between the threads.
8. **Crest** - Crest of the thread is the top part of the grove that corresponds with the major diameter.
9. **Flank** - The flank is the angle at which the helix is raised to form a crest.
10. **Slope**. It is half the pitch of the thread.

4-9. ISO metric screw threads

Vee threads are commonly used as fastening threads. They provide the following advantage:

1. Vee threads produce more friction, which reduces the possibility of loosening,
2. Vee threads are stronger because they have a thicker thread at the core diameter,
3. ISO metric screw threads with vee threads are easier to manufacture.

A threaded equilateral triangle in Figure 4-5 has a thread angle of (60^o). The pitch of this triangle is equal to its base. Tables (4-1 & 4-2) show the dimensions of the standard profile

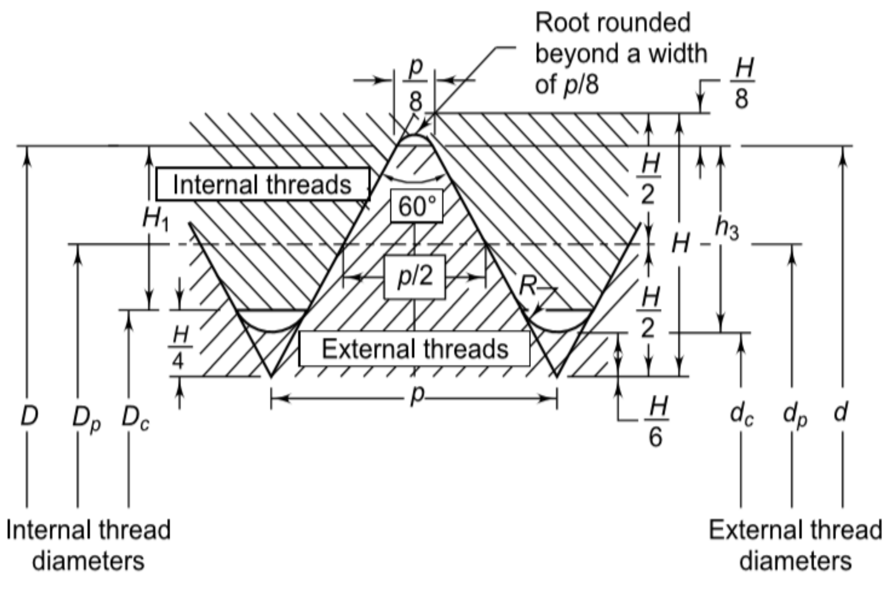


Figure 4-5: Profile of the external and internal threads

A coarse screw thread is identified by the letter "M" and the nominal diameter in millimeters (mm), for instance (M12).

The letter "M," the nominal diameter and pitch in millimeters (mm), and the sign "x" are used to identify fine scribe screw threads, as in the following example: (M 12 x 1.5).

Table 4-1: The fundamental dimension for ISO metric screw threads (Coarse series)

Designation	*Nominal or major dia d/D (mm)*	*Pitch (p) (mm)*	*Pitch diameter d_p/D_p (mm)*	*Minor diameter (mm)*		*Tensile stress area (mm^2)*
				d_c	*D_c*	
M 4	4	0.70	3.545	3.141	3.242	8.78
M 5	5	0.80	4.480	4.019	4.134	14.20
M 6	6	1.00	5.350	4.773	4.917	20.10
M 8	8	1.25	7.188	6.466	6.647	36.60
M 10	10	1.50	9.026	8.160	8.376	58.00
M 12	12	1.75	10.863	9.853	10.106	84.30
M 16	16	2.00	14.701	13.546	13.835	157
M 20	20	2.50	18.376	16.933	17.294	245
M 24	24	3.00	22.051	20.319	20.752	353
M 30	30	3.50	27.727	25.706	26.211	561
M 36	36	4.00	33.402	31.093	31.670	817
M 42	42	4.50	39.077	36.479	37.129	1120
M 48	48	5.00	44.752	41.866	42.587	1470
M 56	56	5.50	52.428	49.252	50.046	2030
M 64	64	6.00	60.103	56.639	57.505	2680
M 72	72	6.00	68.103	64.639	65.505	3460
M 80	80	6.00	76.103	72.639	73.505	4340
M 90	90	6.00	86.103	82.639	83.505	5590
M 100	100	6.00	96.103	92.639	93.505	7000

Table 4-2: The fundamental dimension for ISO metric screw threads (Fine series).

Designation	Nominal or major dia d/D (mm)	Pitch (p) (mm)	Pitch diameter d_p/D_p (mm)	Minor diameter (mm) d_c	D_c	Tensile stress area (mm^2)
M 6 × 1	6	1.00	5.350	4.773	4.917	20.1
M 6 × 0.75	6	0.75	5.513	5.080	5.188	22.0
M 8 × 1.25	8	1.25	7.188	6.466	6.647	36.6
M 8 × 1	8	1.00	7.350	6.773	6.917	39.2
M 10 × 1.25	10	1.25	9.188	8.466	8.647	61.2
M 10 × 1	10	1.00	9.350	8.773	8.917	64.5
M 12 × 1.5	12	1.50	11.026	10.160	10.376	88.1
M 12 × 1.25	12	1.25	11.188	10.466	10.647	92.1
M 16 × 1.5	16	1.50	15.026	14.160	14.376	167
M 16 × 1	16	1.00	15.350	14.773	14.917	178
M 20 × 2	20	2.00	18.701	17.546	17.835	258
M 20 × 1.5	20	1.50	19.026	18.160	18.376	272
M 24 × 2	24	2.00	22.701	21.546	21.835	384
M 24 × 1.5	24	1.50	23.026	22.160	22.376	401
M 30 × 3	30	3.00	28.051	26.319	26.752	581
M 30 × 2	30	2.00	28.701	27.546	27.835	621
M 36 × 3	36	3.00	34.051	32.319	32.752	865
M 36 × 2	36	2.00	34.701	33.546	33.835	915
M 42 × 4	42	4.00	39.402	37.093	37.670	1150
M 42 × 3	42	3.00	40.051	38.319	38.752	1210
M 48 × 4	48	4.00	45.402	43.093	43.670	1540
M 48 × 3	48	3.00	46.051	44.319	44.752	1600

4-10. Bolted Joint, Design Procedure

1. Initial stresses due to screwing up forces (Tensile).

$$F_i = 2805\, d \qquad (4-2)$$

$$F_i = intial\ force$$

2. Maximum tensile stress due to external forces.

Figure 4-6 shown dimension of set screw.

$$\sigma_t = \frac{P}{A} = \frac{P}{\pi.d^2/4} = \frac{4P}{\pi.d_1^2} \qquad (4-3)$$

$$d = \frac{d_1}{0.8} \qquad (4-4)$$

$$\sigma_t = \frac{S_{ty}}{n}$$

$$S_{ty} = \frac{S}{n}$$

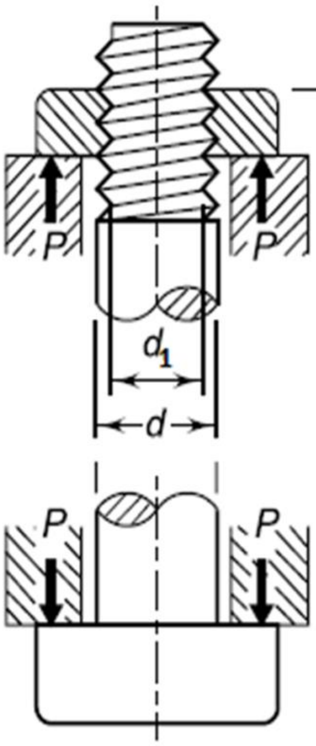


Figure 4-6: Dimension set screw

Where:

$\sigma_t = Maximum\ tensile\ stress,$

$P = External\ force,$

$d = Outer\ diameter\ of\ the\ bolt,$

$d_1 = Inner\ diameter\ of\ the\ bolt,$

$n = Safty\ factor,$

$S_{ty} = Yield\ strength\ in\ tension,$

$S_{sy} = Yield\ strength\ in\ shear.$

3. Shear stress

$$F_i = \pi\ .\ \ n\ .\ d_1 .\ h_\tau \qquad (4-7)$$

$F_i = Intial\ force\ ; h_\tau = Shear\ hieght$

4. Combined tension and shear stress

$$\tau_{max} = \frac{1}{2}\sqrt{\sigma_t^2 + 4\tau^2} \qquad (4-8)$$

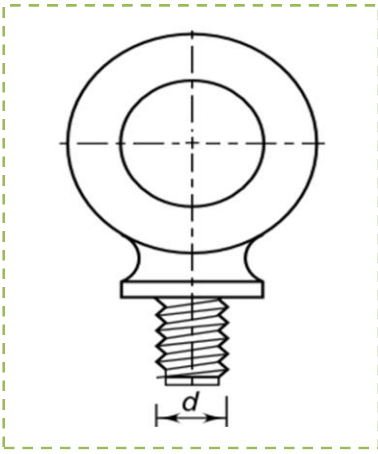
$$\sigma_{max} = \frac{\sigma_t}{2} + \frac{1}{2}\sqrt{\sigma_t^2 + 4\tau^2} \qquad (4-9)$$

$\tau_{max} = Maximum\ shear\ stress\ ; \sigma_{max} = Maximum\ normal\ stress$

4-11. Solve Examples

Example 1:

As depicted in the illustration, an eye bolt is to be used to lift a load of (P = 10 KN). The motor's frame is fastened down to receive the eye bolt. The eye bolt has coarse threads. It is composed of plain carbon steel 30C8, which has a safety factor of 3 (F.S = 3), and a strength of (250 N/mm^2). Find out the bolt's size?



Solution

Given data:

P = 15 KN = 15000 N, S_{sy} *= 250 Mpa ,* *F.S = 3*

$$S_{ty} = \frac{S_{sy}}{0.5} = \frac{250}{0.5} = 500\ MPa$$

$$\sigma_t = \frac{S_{ty}}{F.S} = \frac{500}{3} = 166.67\ MPa$$

$$\sigma_t = \frac{P}{A} = \frac{P}{\frac{\pi}{4} d_1^2} = \frac{4P}{\pi . d_1^2}$$

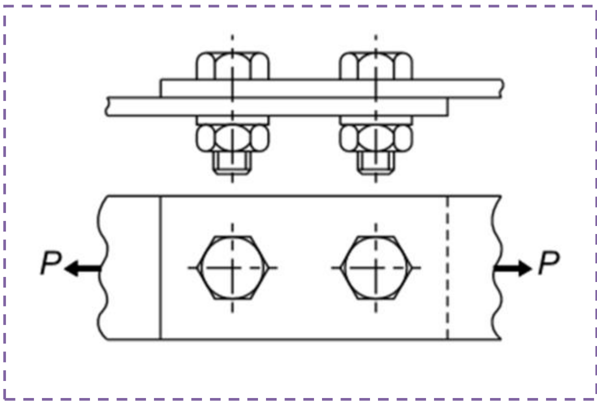
$$\therefore d_1 = \sqrt{\frac{4p}{\pi . \sigma_t}} = \sqrt{\frac{4 \times 15000}{3.14 \times 166.67}} = \sqrt{114.65} = 11.9\ mm$$

$$d = \frac{d_1}{0.8} = \frac{11.9}{0.8} = 14.88 \approx 15\ mm$$

According to Table (4-1) The typical size of the bolt is (**M16)**.

Example 2:

Two bolts are used to secure two plates, as shown in Figure. The bolts are made of plain carbon steel 30C8 ($S_{sy} = 200\,MPa$) with a safety factor of 3 (F.S = 3). Determine the bolt size if (P = 10 kN).



<u>Solution</u>

Given data:

$$P = 10\,KN = 10000\,N\ ,\ Ssy = 200\,Mpa\ ,\quad F.S = 3$$

$$\sigma_t = \frac{S_{sy}}{F.S} = \frac{200}{3} = 66.67\ MPa$$

$$Shear\ area\ of\ two\ bolts = 2A = 2\left(\frac{\pi}{4}\right).d^2\ (mm^2)$$

$$\tau_t = \frac{P}{2A} = \frac{P}{2\times\frac{\pi}{4}d_1^2} = \frac{2P}{\pi.d_1^2}$$

$$\therefore d_1 = \sqrt{\frac{2p}{\pi.\sigma_t}} = \sqrt{\frac{2\times10000}{3.14\times66.67}} = \sqrt{95.54} = 9.77 \approx 10mm$$

$$d = \frac{d_1}{0.8} = \frac{9.77}{0.8} = 21.21 \approx 13mm$$

From Table (4-1), the standard size of the bolt is **M16**.

4-12. Power Screws and Ball Screws

Objectives

a. Recognize and understand advantages and disadvantages of different types of power screws.
b. Determine the power necessary for driving power screws at different speeds and torques.
c. Understand principles of operation of ball screws and how they differ from friction-type power screws.
d. Understand and calculate torque and efficiencies of power screws and ball screws.
e. Understand and envision how power screws and ball screws can be used in different designs.

A screw and nut to transmit power or motion the axial movement of the nut is used to drive a load, figure 4-6.

Power screw used to change angular motion into linear motion, usually transmits power. Examples include vises, presses, jacks, lead screw on lathe.

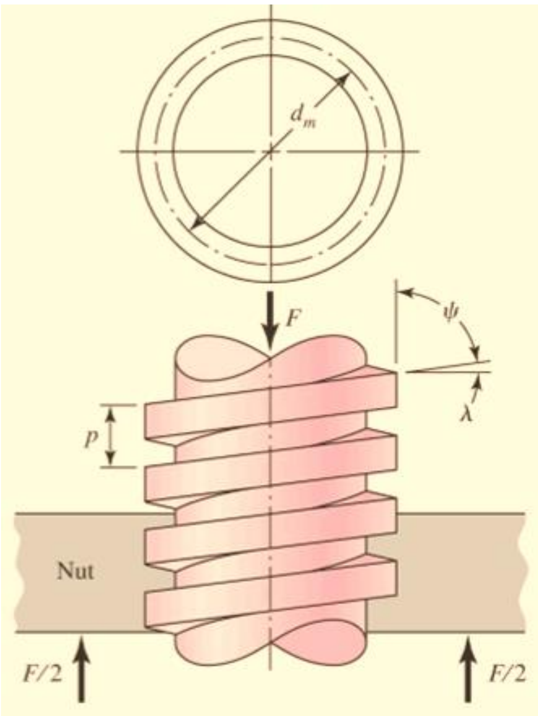


Figure 4-7: Power screw.

4-13. Tooth profiles

a. **Square thread** – Most efficient for transferring torque to linear motion.

b. **Acme thread** – Easier to make – Good when well lubricated – Efficiency slightly lower than square.

c. **Buttress thread** – More efficient than Acme – Closer to square than Acme – Used when force is transmitted in only direction.

Force required to push a box up or down an inclined figure 4-8.

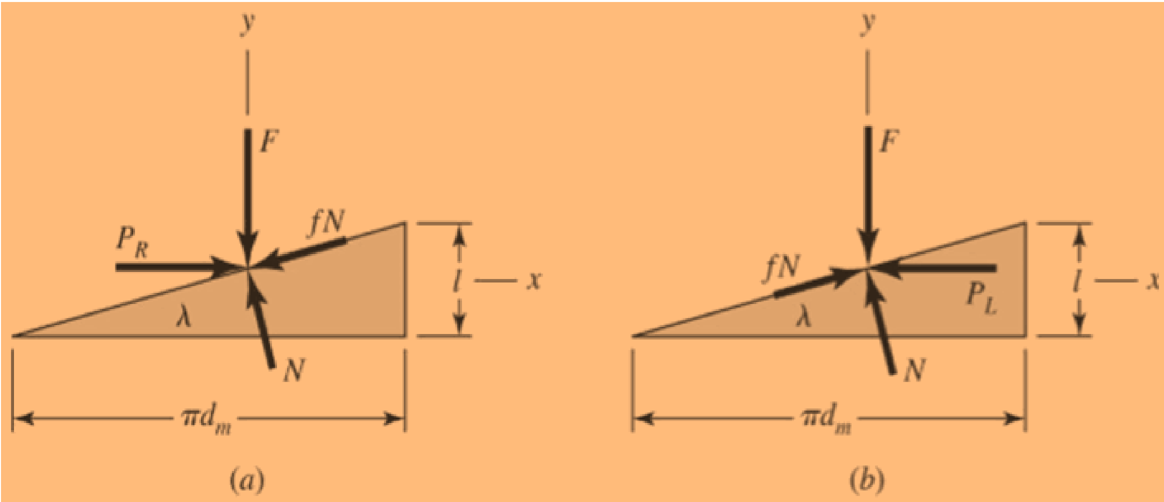


Figure 4-8: Force exerted, a- Up the plane, b- Down the plane.

$P = Force\, required\, to\, move\, load,$

$f = Cofficient\, of\, friction,$

$N = Normal\, force,$

$fN = Friction\, force,$

$D_P = \pi\, d_m = Pitch\, diameter,$

$L = Lead\, angle\, of\, the\, thread,$

$\lambda = Lead\, angle,$

$d_m = Meand\, diameter,$

$\varphi = Helix\, angle$

1. **For raising the load**

$$\sum F_H = P_R - N\sin\lambda - f.N\cos\lambda = 0 \qquad (4-10)$$

$$\sum F_V = F - N \ \sin\lambda + f.N\cos\lambda = 0 \qquad (4-11)$$

2. For lowering the load

$$\sum F_H = -P_L - N \ \sin\lambda + f.N\cos\lambda = 0 \qquad (4-12)$$

$$\sum F_V = F - N \ \sin\lambda - f.N\cos\lambda = 0 \qquad (4-13)$$

- Eliminate (N) and solve for (P) to raise and lower the load.

$$P_R = \frac{F(\sin\lambda + f.\ N\cos\lambda)}{\cos\lambda - f\ .\ N\ \sin\lambda} \qquad (4-15)$$

$$P_L = \frac{F(f\ .\ N\cos\lambda - \sin\lambda)}{\cos\lambda + f\ .\ N\ \sin\lambda} \qquad (4-16)$$

Where:

$$P_R = Raising\ load$$

$$P_L = Raising\ load$$

- Divide numerator and denominator by $(cos\lambda)$ and use relation:

$$\lambda = \tan^{-1}\frac{L}{\pi\ .\ d_m} \qquad (4-17)$$

$$P_R = \frac{F\ \left[\left(\frac{L}{\pi\ .\ d_m}\right) + f\right]}{1 - (\frac{f\ .\ L}{\pi\ .\ d_m})} \qquad (4-18)$$

$$P_L = \frac{F\ \left[\left(\frac{L}{\pi\ .\ d_m}\right) + f\right]}{1 + (\frac{f\ .\ L}{\pi\ .\ d_m})} \qquad (4-19)$$

4-14. Torque

The torque is the product of the force ***P*** and the mean radius.

1. **Raising Torque (T_R)**

$$T_R = \frac{F \,.\, d_m}{2} \left(\frac{\pi \,.\, f \,.\, d_m + L}{\pi \,.\, d_m - f \,.\, L} \right) + \left(\frac{F \,.\, f_c \,.\, d_c}{2} \right) \qquad (4-19)$$

2. **Lowering Torque (T_L)**

$$T_L = \frac{F \,.\, d_m}{2} \left(\frac{\pi \,.\, f \,.\, d_m - L}{\pi \,.\, d_m - f \,.\, L} \right) + \left(\frac{F \,.\, f_c \,.\, d_c}{2} \right) \qquad (4-20)$$

Where:

$$T_R = Raising\ torque$$

$$T_L = raising\ torque$$

4-15. Power Screw Efficiency

If $(\boldsymbol{f} = \mathbf{0})$ in Equation (19), then obtain:

$$T_0 = \frac{F \,.\, L}{2\pi} \qquad (4-21)$$

Which, is the torque required to raise the load .

The efficiency is therefore:

$$\eta = \frac{T_0}{T_R} \times 100\% = \frac{F \,.\, L}{2\pi \,.\, T_R} \times 100\% \qquad (4-22)$$

4-15-1. Power Screw Stress Analysis

The following stresses should be checked on both nut and screw:

1. Shearing stress in screw body.

$$\tau = \frac{16\, T}{\pi \,.\, d_m^3} \qquad (4-23)$$

2. Axial stress in screw body.

$$\sigma = \frac{F}{A} = \frac{4\, F}{\pi \,.\, d_m^2} \qquad (4-24)$$

3. Thread bearing stress.

$$\sigma_B = \frac{F}{A} = \frac{4\,F}{\pi\,.\,d_m\,.\,n_t\,(\frac{P}{2})} = \frac{2\,F}{\pi\,.\,d_m\,.\,n_t\,.\,P} \qquad (4-25)$$

Where:

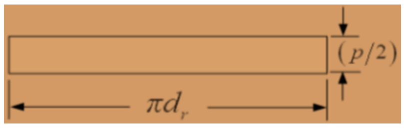
n_m = Number of engaged threads

4. Thread bending stress.

$$\sigma_b = \frac{M\,.\,c}{I} \qquad (4-26)$$

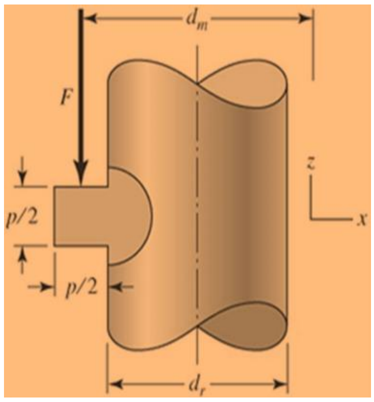
Were,

$$M = \frac{F\,.P}{4} \qquad (4-27)$$



$$I = \frac{\pi\,.\,d_m\,.\,n_t\,.\,(\frac{P}{2})^3}{12} \qquad (4-28)$$

$$c = \frac{(\frac{P}{2})}{2} = \frac{P}{4} \qquad (4-29)$$



5. Transverse shear stress at the center of the thread root.

$$\tau = \frac{3\,V}{2\,A} = \frac{3\,F}{\pi\,.\,d_m\,.\,n_t\,.\,P} \qquad (4-30)$$

4-16. Solve Examples

Example 3

A square thread power screw has a major diameter of (*40 mm)* and a pitch of (*6 mm)* with double threads and it is to be used in an application similar to that of the figure. Applicable data are thread and collar coefficient of friction equal to (0.13), collar diameter of (*40 mm)*, and a load of (15 KN) per screw. Determine:

1. Thread depth, thread width, mean or pitch diameter, minor diameter, and lead.
2. Torque required to rotate the screw "against" the load.
3. Torque required to rotate the screw "with" the load.
4. Overall efficiency.

Solution:

1. From the square thread figure above, it can be seen the thread depth and width are the same and equal to half the pitch, or 3 mm. Also

$$d_m = d - \frac{P}{2} = 40 - \frac{6}{2} = 37\,mm = 0.037\,m$$

$$d_r = d - P = 40 - 6 = 34\,mm = 0.034\,m$$

$$L = n_t\,P = 2 \times 6 = 12\,mm = 0.03\,m$$

2. For a square thread, the torque required to raise the load is:

$$T_R = \frac{Fd_m}{2}\left[\frac{L + f\pi d_m}{\pi d_m - f\,L}\right] + \frac{F\,f_c\,d_c}{2}$$

Where:

f = fc = 0.13, dc = 40 mm = 0.04 mm

$$T_R = \frac{15000 \times 0.037}{2}\left[\frac{0.03 + 0.13 \times 3.14 \times 0.037}{3.14 \times 0.037 - 0.13 \times 0.03}\right] + \frac{15000 \times 0.13 \times 0.04}{2}$$

$$T_R = 277.5 \times [\frac{0.0451}{0.1162 - 0.0039}] + 39$$

$$T_R = 277.5 \times [0.5842] + 39 = 201.114\, N.m$$

3. For a square thread, the torque required to lower the load is:

$$T_L = \frac{Fd_m}{2}[\frac{f\pi d_m - L}{\pi d_m - f\, L}] + \frac{F\, f_c\, d_c}{2}$$

$$T_L = \frac{14000 \times 0.037}{2}[\frac{0.13 \times 3.14 \times 0.037 - 0.03}{3.14 \times 0.037 + 0.15 \times 0.03}] + \frac{14000 \times 0.13 \times 0.04}{2}$$

$$T_L = 277.5 \times [\frac{0.0145}{0.1162 + 0.0045}] + 39$$

$$T_L = 277.5 \times [\frac{0.0145}{0.1207}] + 39$$

$$T_L = 14100 \times [0.06718] = 72.337\, N.m$$

4. Overall efficiency is:

$$Efficiency(\eta) = \frac{T_0}{T_R} = \frac{FL}{2\pi\, T_R}$$

$$Efficiency(\eta) = \frac{14000 \times 0.03}{2 \times 3.14 \times 201.114} \times 100\%$$

$$Efficiency(\eta) = 33.25\,\%$$

Example 4:

A double-threaded power screw with ISO metric trapezoidal threads is used to lift a weight of (300 KN) with a pitch of (12 mm). The screw threads' friction coefficient is (0.15). Find the following while ignoring collar friction:

1. Thread depth, thread width, mean or pitch diameter, minor diameter, and lead,
2. Torque to raise the load,

3. Torque lowers the load,
4. Efficiency of the screw.

Solution:

Given

$[F = 300000\,N, \quad for\ screw,$

$d = 0.1\,m, P = 0.012\,m, \mu = 0.15, Number\ of\ starts = 2]$

1. From the trapezoidal thread figure above, it can be seen the thread depth and width are the same and equal to half the pitch, or($3\,mm$).

Also

$$d_m = d - \frac{P}{2} = 100 - \frac{12}{2} = 94mm = 0.094m$$

$$d_r = d - P = 100 - 12 = 88mm = 0.088m$$

$$L = n_t\, P = 2 \times 12 = 24mm = 0.024m$$

2. For a trapezoidal thread, the torque required to raise the load is:

$$T_R = \frac{Fd_m}{2}\left[\frac{L + f\pi d_m}{\pi d_m - fl}\right]$$

$$T_R = \frac{300000 \times 0.094}{2}\left[\frac{0.024 + 0.15 \times 3.14 \times 0.094}{3.14 \times 0.094 - 0.15 \times 0.024}\right]$$

$$T_R = 14100 \times \left[\frac{0.0683}{0.2983 - 0.0036}\right]$$

$$T_R = 14100 \times [0.2318] = 3268.38\,N.m$$

3. For a trapezoidal thread, the torque required to lower the load is:

$$T_L = \frac{Fd_m}{2}[\frac{f\pi d_m - L}{\pi d_m + fl}]$$

$$T_L = \frac{300000 \times 0.094}{2}[\frac{0.15 \times 3.14 \times 0.094 - 0.024}{3.14 \times 0.094 + 0.15 \times 0.024}]$$

$$T_L = 14100 \times [\frac{0.02028}{0.2983 + 0.0036}]$$

$$T_L = 14100 \times [\frac{0.02028}{0.3019}]$$

$$T_L = 14100 \times [0.06718] = 947.24\, N.m$$

4. Overall efficiency is:

$$Efficiency(\eta) = \frac{T_0}{T_R} = \frac{FL}{2\pi\, T_R}$$

$$Efficiency(\eta) = \frac{300000 \times 0.024}{2 \times 3.14 \times 3268.38} \times 100\%$$

$$Efficiency(\eta) = 35.08\ \%$$

4-17. Chapter Questions

1. The bolt shank is put under the following stress when a nut is tightened by putting a washer below it:

a. Torsional shear stress.
a. Compressive stress.
b. Direct shear stress.
c. Tensile stress.

2. The axial force applied to the bolt as a result relies on:

a. Initial tension, Stiffness of bolt and parts held by bolt, and External applied load.
b. Stiffness of bolt and parts held by bolt.
c. Initial tension.
d. External applied load.

3. Setscrews are

a. Almost identical to tap bolts, but with a wider range of head shapes.
b. Typically used with a nut and slotted for a screwdriver.
c. Similar to studs.
d. **is a screw that is used to secure an object.**

4. The washer's inner diameter is:

a. Less than the size of a nut.
b. Regardless of the nut's size.
c. More than the size of a nut.
d. Equal than the size of a nut.

5. The designation M 36 × 2 means

a. Metric fine threads of 36 mm outside diameter and 2 mm pitch
b. Metric coarse threads of 36 mm outside diameter and 2 mm pitch
c. Metric threads of 36 mm pitch diameter and 2 mm pitch
d. Metric threads of 36 mm core diameter and 2 mm pitch

6. The designation M 20 means

a. Metric coarse threads of 20 mm outside diameter
b. Metric fine threads of 20 mm outside diameter
c. Metric threads of 20 mm core diameter
d. Metric threads of 20 mm pitch diameter

7. The largest diameter of external or internal screw thread is called

a. Major diameter
b. Minor diameter
c. Pitch diameter
d. None of the above

8. The formula of a shearing stress in screw body is:

a. $\tau = \frac{32\,T}{\pi\,.\,d_m^3}$

b. $\boldsymbol{\tau = \frac{16\,T}{\pi\,.\,d_m^3}}$

c. $\tau = \frac{64\,T}{\pi\,.\,d_m^3}$

d. $\tau = \frac{8\,T}{\pi\,.\,d_m^3}$

9. A screw is specified by ----------- diameter.

a. Mean
b. Major
c. Minor
d. Pitch

10. Most efficient for transferring torque to linear motion.

a. **Square thread**
b. Acme thread
c. Buttress thread
a. Worm thread

11. Easier to make – Good when well lubricated – Efficiency slightly

a. Square thread
b. Acme thread
c. Buttress thread
b. Worm thread

12. More efficient than Acme – Closer to square than Acme – Used when force is transmitted in only direction.

a. Square thread
b. Acme thread
c. Buttress thread
d. Worm thread

13. The following factors determine square threaded power's maximum efficiency:

a. Screw Pitch.
b. Lead angle of screw
c. Nominal diameter of screw
d. Friction angle

14. Which of the screw threads from the list below is the strongest thread?

a. ACME screw threads.
b. Square screw threads.
c. V-threads screw.
d. Buttress screw threads.

15. There are multiple threads utilized for:

a. High load carrying capacity.
b. High efficiency.
c. Low efficiency for self-locking.
d. High mechanical advantage.

16. Which of the following screw threads is utilized for power transmission both ways?

a. Trapezoidal threads and square thread
b. Buttress threads
c. Trapezoidal threads
d. square thread

17. Initial stresses due to screwing up forces (Tensile).

a. $\boldsymbol{F_i = 2805\ d}$
b. $F_i = 2800\ d$
c. $F_i = 2810\ d$
d. $F_i = 2815\ d$

18. It is used to raise the load, for example,

a. Vice
b. Screw jack
c. Universal testing machine
d. Lead screw of lathe vice

19. In a single start thread

a. Lead and pitch are equal
b. Lead is double the pitch
c. Pitch is double the lead
d. Lead is half the pitch

20. What type of thread are suitable for lead screw of machine tools

a. V-Shape thread
b. Whitworth screw thread
c. Acme threads
d. Square threads

21. What type of threads is suitable for small precision components and measuring gauge

a. V-Shape thread
b. Whitworth screw thread
c. Acme threads
d. Square threads

22. The pitch of three start thread is the lead divided by

a. One
b. Two
c. Three
d. Four

23. The distance through which a screw thread advances axially in one turn is called

- **a. Lead of thread**
- b. Pitch of thread
- c. Diameter of thread
- d. Depth of thread

23. Which of these thread types is used on mechanical jacks?

- a. Acme thread
- **b. Square thread**
- c. Buttress thread
- d. Worm thread

24. A screw thread is formed on a cylindrical surface by cutting

- **a. Helical grooves**
- b. V- grooves
- c. Square grooves
- d. Half round grooves

25. Spring washers are used under nuts to prevent

- a. Damage to the bolt
- b. Damage to the nut
- **c. Damage to the job vibration**
- d. Slackness of nuts due to

26. Which one of the following thread forms on bolts and nuts is meant for general fastening purposes

- **a. V-Shape thread**
- b. Whitworth screw thread
- c. Acme threads
- d. Square threads

27. Which of the following fastens permanently?

- a. Screw fastening
- b. Fastening with bolt and nut
- **c. Welding**
- d. Rivet joints

28. In a threaded assembly the contact between the male and female threads takes place on the

- a. Pitches
- **b. Flanks**
- c. Crests
- d. Roots

29. While threading on lathe, the carriage is moved by means of

- **a. Hand wheel**

b. Lead screw
c. Feed rod
d. Gear train on a rack

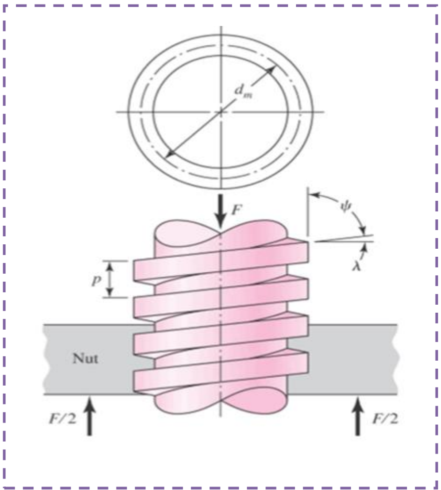
30. Which one of the following are advantages the screwed joints.

a. Screwed joints are lightly reliable in operation.
b. Screwed joints are inconvenient to assemble and disassemble.
c. A wide range of screwed joints may be adapted to different operating conditions.
d. Screws are relatively expensive · to produce due to standardization and highly efficient manufacturing processes.

31. Power screw used to change ---------- in to linear motion, usually transmits power. Examples include vises, presses, jacks, lead screw on lathe.

a. angular motion
b. uniform motion
c. spiral motion
a. Liner motion

32. A square thread power screw has a major diameter of (*32 mm)* and a pitch of (*4 mm)* with double threads and it is to be used in an application similar to that of the figure. Applicable data are thread and collar coefficient of friction equal to (μ = *0.08*), collar diameter of (*40 mm*), and a load of (*6.4 KN*) per screw. Determine:

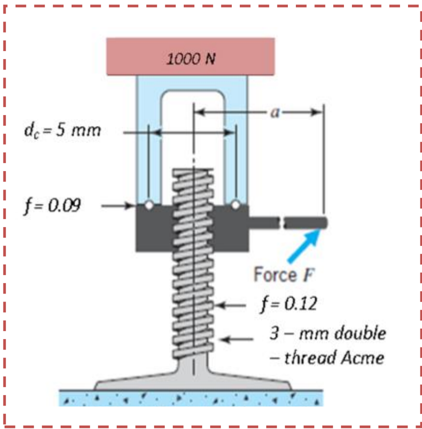


1. Thread depth, thread width, mean or pitch diameter, minor diameter, and lead.
2. Torque required to rotate the screw "against" the load.
3. Torque required to rotate the screw "with" the load.
4. Overall efficiency.

<u>**Answer**</u>

$$[\boldsymbol{dm = 30\,mm, dr = 28\,mm, L = 8\,mm, TR = 26.2\,N.m, TL = 9.8\,N.m, \eta = 31}\,\%]$$

33. A screw jack shown in figure with a (3 mm), double-thread. Acme screw is used to raise a load of (1000 N). A plain thrust collar of mean (5 mm) diameter is used. Coefficients of running friction are estimated as (0.12 & 0.09) for f and fc respectively.



a. Determine the screw pitch, lead, thread depth, mean pitch diameter, and helix angle.
b. Estimate the starting torque for raising and for lowering the load.
c. Estimate the efficiency of the jack when raising the load. Assume that the starting friction is about one-third higher than running friction.

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Chapter 5

Keyed Joint system

5- Keys Joint

5-1. Introduction

The key is a machine element that connects the transmission shaft to the machine's rotating element, such as a pulley, gear, sprocket, or flywheel.

It is always inserted with the shaft's axis parallel. Temporary fasteners like the keys experience a lot of crushing and shearing stress. The key is an opening or recess in the pulley's hub and shaft that enables the key's insertion. A keyed joint made up of the shaft and shaft hub is shown in Figure 5-1.

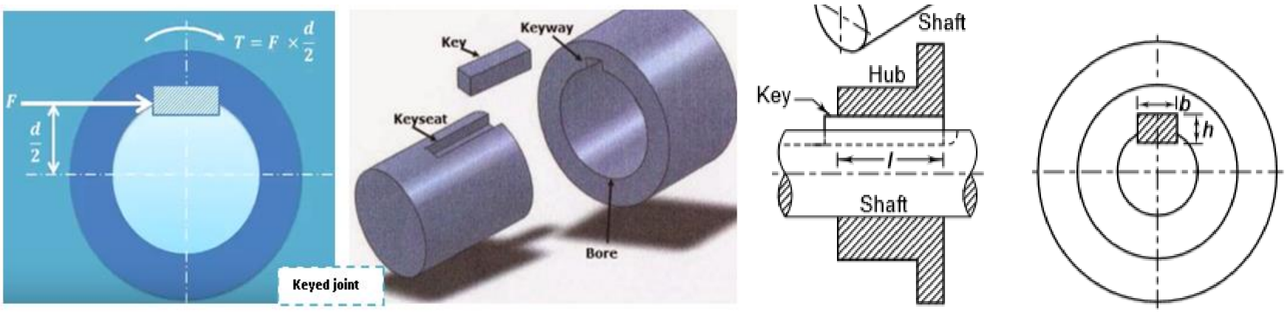

Figure 5-1: Keyed joint

5-2. Function of shaft key

1. The key transfers torque from the hub of the mating member to the shaft and vice versa,
2. It is also employed to stop relative rotational motion between the joint machine component, such as a gear or a pulley, and the shaft. Axial motion between two elements is likewise avoided in this key,
3. Plain carbon steel keys, such as (48c8 or 50c8), are used,
4. Vertical or horizontal milling cutters are typically used to cut keyways. The keyways cause stress concentration in the shaft, causing the part to fail. This is the primary disadvantage of the key-ways joint,
5. The tensile strength of the key-way material should not be less than (600 N/mm^2).

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5-3. Types of shaft key

There are following types of shaft key used in machines figure 5-2.

1. Sunk keys,
2. Saddle keys,
3. Tangent keys,
4. Round keys,
5. Splines keys,
6. Kennedy keys.

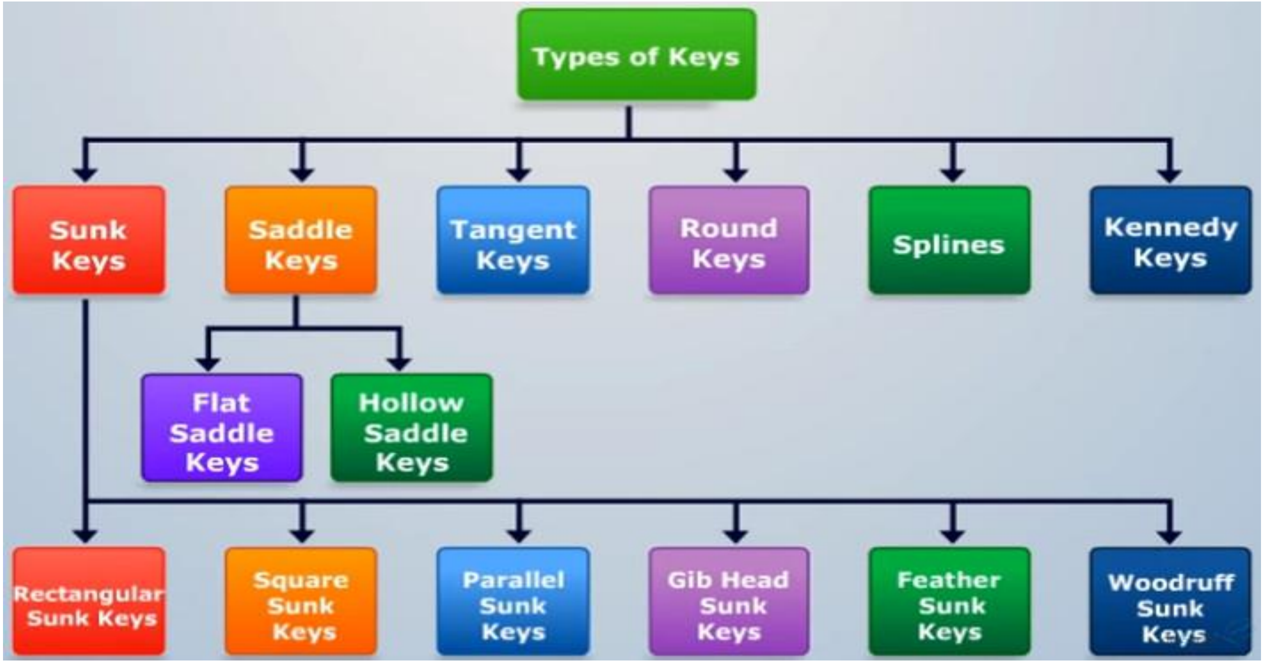


Figure 5- 2: Types of Keys

The keys are made of drawn steel with a tensile strength of about (700 Mpa). They must be stronger and harder than the machine parts to be connected so that they do not deform when driven in.

5-3-1. Taper sunk keys

Figure (5-3) depicts long bodies with a rectangular cross-section, an inclined back surface, and a plane or rounded front surface.

The inclination is (1:100), which means that the taper is (1 mm per 100 mm).

1. **Round-ended sunk keys (**Laid-in keys) - They are pressed (or inserted) into the shaft's snugly fitting groove, after which the hub is forced onto the key's sunk hole. If a key cannot be driven in or out, these keys are used.

2. **Straight-ended sunk keys** (Tapered driving keys) - In this instance, the shaft and hub (or the relevant machine parts) are mounted as usual, and the sinking key is then driven in. If there is enough room to drive them in and out from either side, they are used.

3. **Tapered driving keys -** are the thicker ends of which have a nose. They are used when driving in or out can only be done from one side.

4. **Woodruff keys -** Can also perform the same duties as taper-sunk keys because, thanks to their rotatable positioning in the keyway, they can adjust to the taper in a hub keyway.

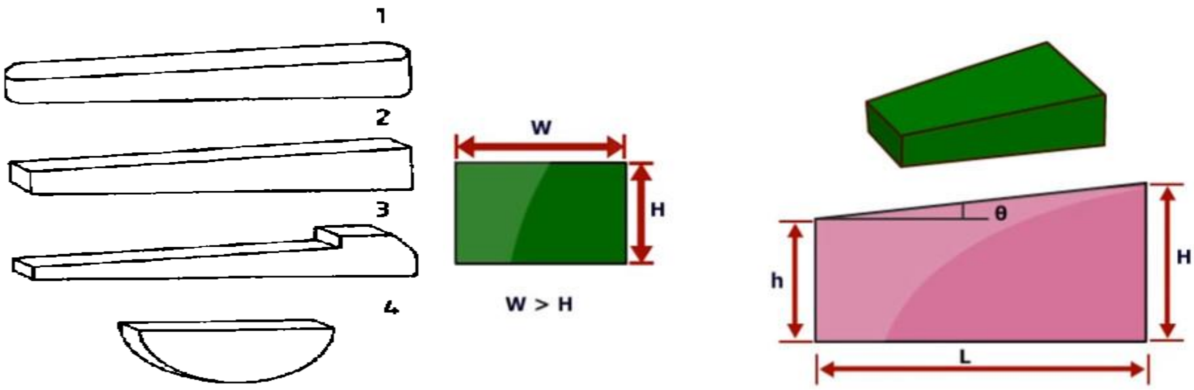


Figure 5-3: Taper sunk keys

e-Taper sleeves

These are truncated cone-shaped bodies with internal and external tapers that serve as direct connections between machine parts. In general, they are employed with machine spindles where tools with taper shanks are applied. Cotters are inserted into the taper sleeves through lateral oblong holes in order to disengage the connection. The clamping sleeve is a unique variety of taper sleeve that serves as an intermediary in the joints of machine parts. On shafts, clamping sleeves are installed, allowing anti-friction bearings, toothed gears, and other components to be mounted. As a result of a taper between 1 in 10 and 1 in 20, they have homogeneous circumferential tension, which ensures precise true running. Figure (5-4) shows the taper sleeves and clamping sleeves.

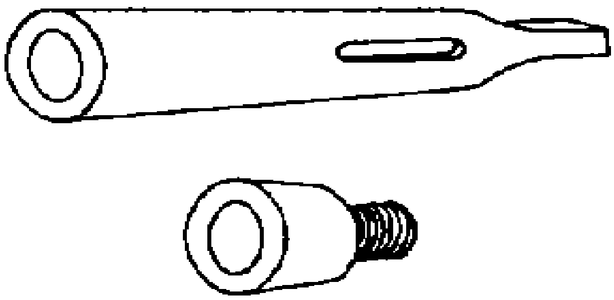


Figure 5-4: Taper sleeves and clamping sleeves

f- Cotters

Cotter joint is a type of mechanical joint which is used to join two axial rods or bars. It is also known as spigot and socket joint. This joint doesn’t allow any angular movements of rods which it connects figure 5-6. This joint is applicable for tensile loads as well as compressive loads. It is a detachable joint. It consists of mainly three parts:

1. Spigot
2. Socket
3. Cotter.

Spigot is the male part of the joint and socket is the female part of the joint. Typically, wrought iron or mild steel are used to create the cotter. For connecting spinning shafts with torque, it is not appropriate.

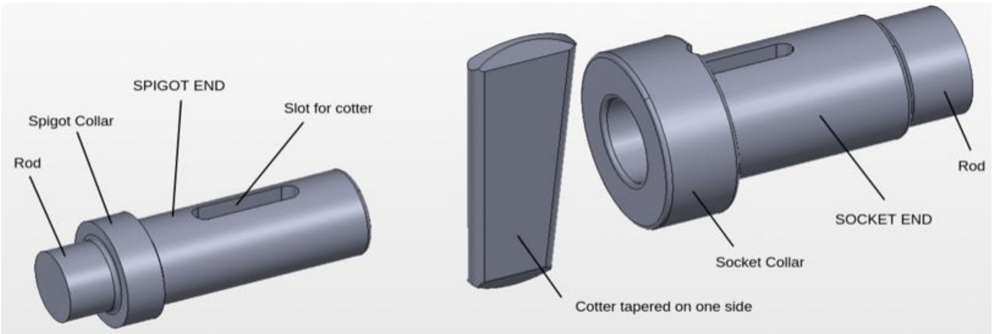


Figure 5-6: Cotter joint parts

There are three types of cotter joints figures 5-6, 5-7, 5-8.

i. Socket and Spigot joints

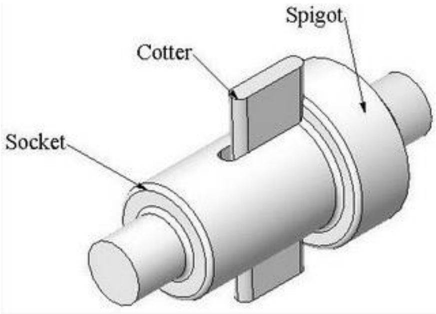


Figure 5-6: Socket and Spigot joints parts

ii. Sleeve and Cotter joints

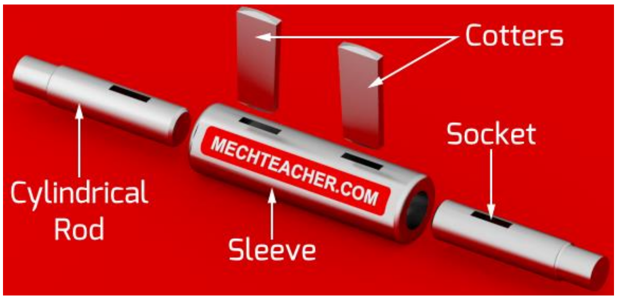


Figure 5-7: Sleeve cotter joint parts

iii. Gib and Cotter joints

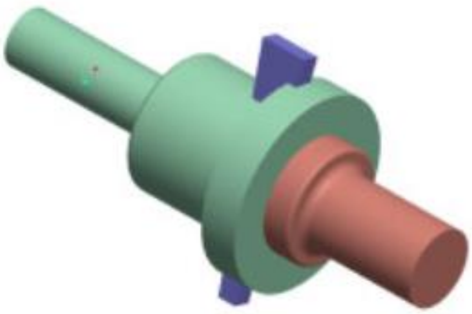
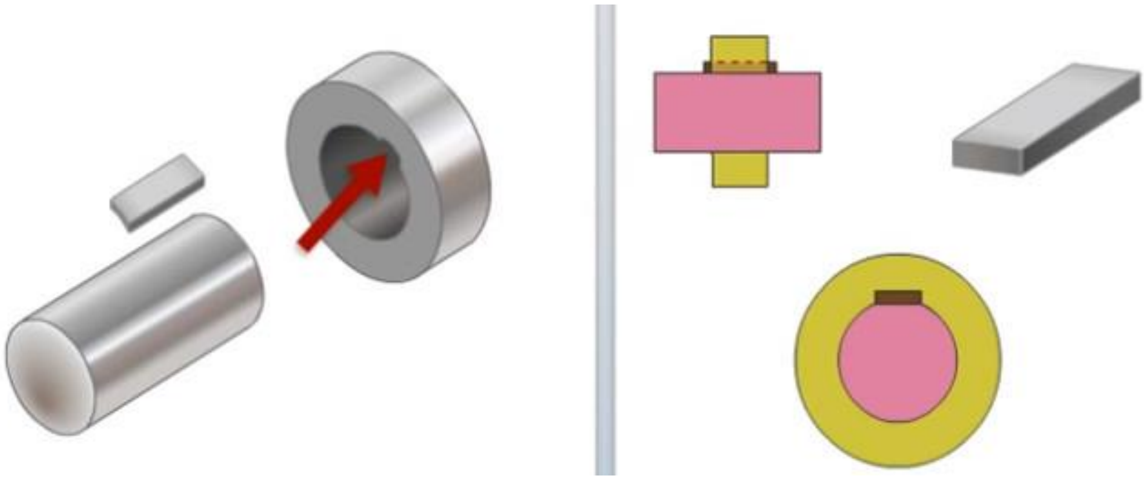


Figure 5-8: Gib and Cotter joints parts

5-3-2. Hollow and flat saddle keys

These have lengthy bodies with a rectangular cross-section, a modest taper, and an inclined back surface. They only serve to convey weak rotary forces. No keyway needs to be created for these, figure (5-9).

- ❖ Hollow keys have a concave bottom in the longitudinal direction. These keys have edges that touch the shaft and mimic cutting edges,
- ❖ The shaft must be flattened in the area where the flat key will be applied in order to ensure a proper fit between the flat key and the shaft.



a. **b.**

Figure 5-9: a- Hollow and **b-** flat saddle keys

5-3-3. Tangent keys

As shown in figure (5-10), the tangent keys, also known as tangential keys, are fitted as a pair at right angles, with each key withstanding torsion only in one direction. In huge, heavy-duty shafts, they are utilized.

- ❖ High torque is transmitted via them,
- ❖ Both a single key and a pair at right angles can be used with them,
- ❖ Torque can only be sent in one direction by a single tangent key.

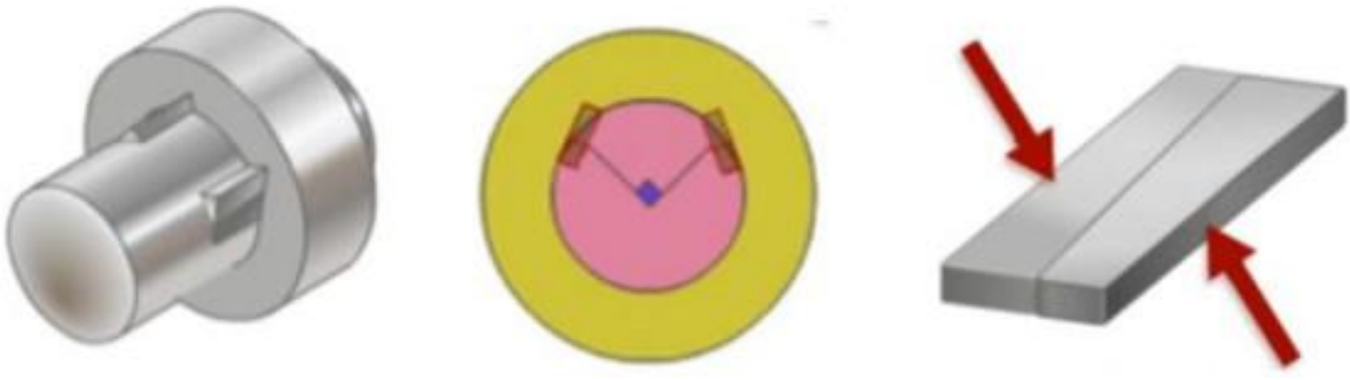


Figure 5-10: Tangent keys

5-3-4. Round key

The round key is derived from the key and adding it is what makes the algorithm a block cipher rather than just a permutation figure (5-11).

- The round cross section of the round keys allows them to fit into holes drilled partially in the shaft and partially in the hub,
- After the assembly, a slot is drilled to allow for appropriate shaft alignment,
- These are employed in transmissions with little torque.

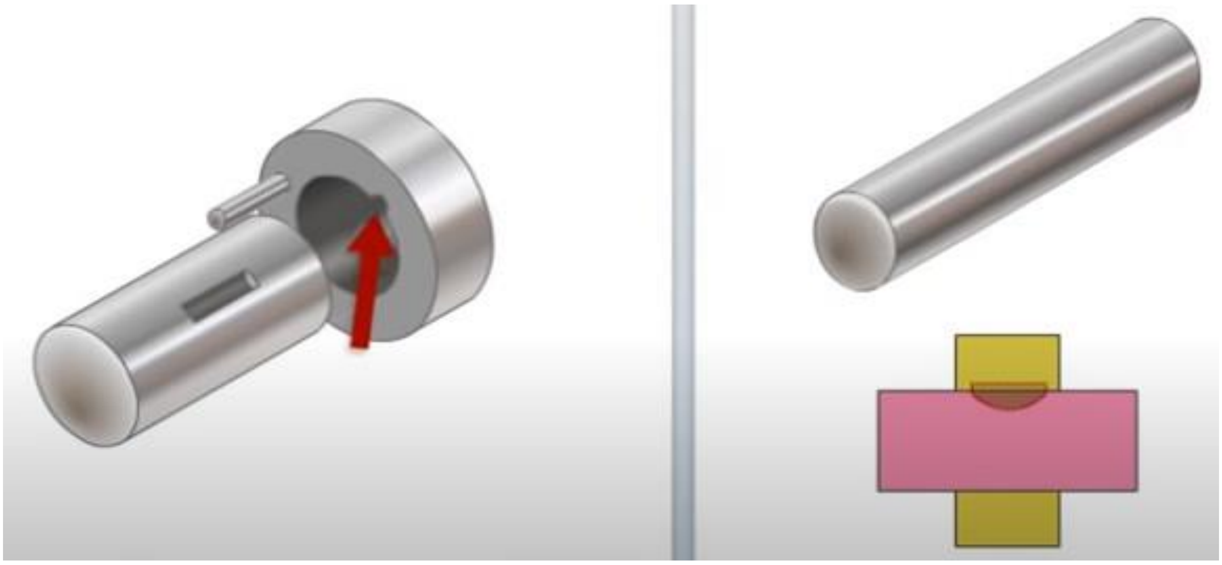


Figure 5-11: Round key

5-3-5. Splines

Splines are used commonly in high power transmission systems for coupling two rotating components such as a shaft and its gear figure (5-12). They provide higher load carrying capacity over keyed shafts, and hence, represent better durability performance.

- ❖ Splines are a collection of keys that are manufactured as a single unit with the shaft,
- ❖ There are keyways available in the hub,
- ❖ These are employed in high-torque transmission, such as that seen in auto transmissions,
- ❖ Splines are also capable of axial movement.

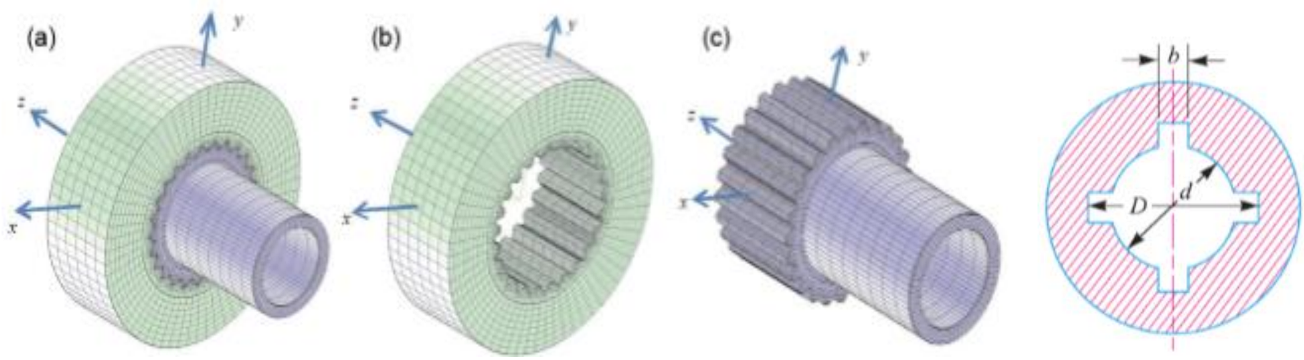


Figure 5-12: Spline joint parts

5-3-6. Kennedy keys

Kennedy key is square taper key fitted into a key way of square section and driven from opposite ends of the hub and used in pairs 90° apart. Kennedy keys transmit torque in two directions but Tangent keys pairs of taper keys set that can withstand torque only in one direction figure (5-13).

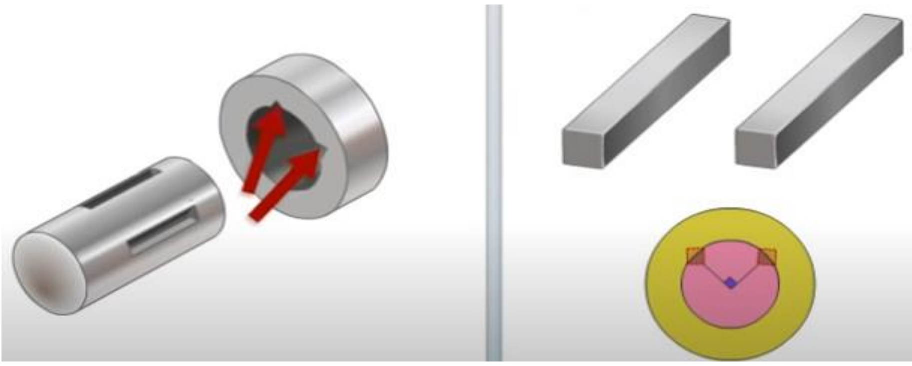


Figure 13: Kennedy key parts

5-4. Selection type of the key

They take into account the following considerations while choosing the type of key for a particular application:

1. Transmission power,
2. Fit tightness,
3. Connection stability,
4. Cost.

5-5. Design of sunk key

The sunk keys are divided equally between the keyways of the shaft and the hub, boss, or gear, respectively. The various kinds of sinking keys include:

Rectangular sunk key, figure (5-14) depicts a rectangular buried key. This key's standard dimensions are:

$$Width\ of\ key:\ \ w = \frac{d}{4} \qquad (5-1)$$

$$Thickness\ of\ key:\ \ t = \frac{2\,w}{3} = \frac{d}{6} \qquad (5-2)$$

Where:

$$d\ =\ Diameter\ of\ the\ shaft\ or\ diameter\ of\ the\ hole\ in\ the\ hub.$$

Only the top side of the key has a taper (1 in 100).

5-5-1. Strength in the sunk key

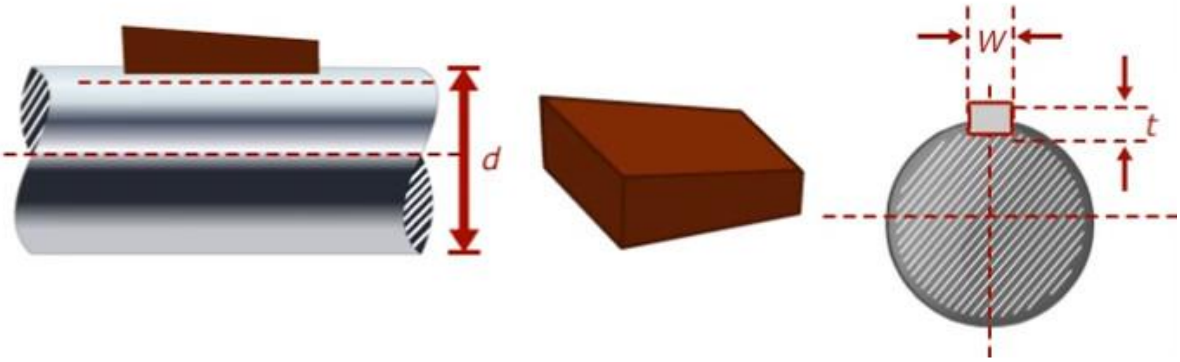


Figure 5-14: Rectangular Sunk Key

1- Consider shearing of the key

The following describes the tangential shearing force at the shaft's circumference:

$$F = \tau .\ A = \tau . L . w \qquad (5-3)$$

Torque transmitted by shaft is:

$$T_s = F\ .\ \frac{d}{2} = \tau . L . w .\ \frac{d}{2} = \frac{1}{2}(\tau\ . L . w .\ d) \qquad (5-4)$$

Were,

T = *Torque transmitted by the shaft,*

F = *Tangential force acting at the circumference of the shaft,*

d = *Diameter of shaft,*

L = *Length of key,*

w = *Width of key,*

t = *Thickness of key,*

τ & σ_c = *Shear and crushing stresses for the material of key, and*

τ_1 = *Shear stress in shaft.*

T_s = *Torque transmitted under acting* tangential shearing force.

2. Consider crushing of the key

The shaft's diameter is being crushed tangentially by a force that is:

$$F = \sigma_c .\ A = \sigma_c\ .\ \frac{t}{2}\ .w = \frac{1}{2}(\sigma_c\ .t\ .w) \qquad (5-5)$$

Torque transmitted by shaft is:

$$T_c = F\ .\ \frac{d}{2} = \sigma_c\ .\frac{t}{2}\ .L\ .\ \frac{d}{2} = \frac{1}{4}(\sigma_c\ .t\ .L\ .d) \qquad (5-6)$$

$T_c = Torque\ transmitted\ under\ acting$ tangential crushing force.

If the key is equally effective at shearing and crushing, then:

$$Curshing\ torque = shearing\ torque$$

Substituting in the two equations (5-4 & 5-6) we get the following:

$$\frac{1}{2}(\tau\ .L\ .w\ .\ d) = \frac{1}{4}(\sigma_c\ .t\ .L\ .d)$$

$$\sigma_c = \frac{2\,\tau\ .\ w}{t} \qquad (5-7)$$

For typical critical materials, the permitted crushing stress is twice as high as the permissible shearing stress. The shearing strength of the key must be equal to the torsional shear strength of the shaft in order to determine the length of the key needed to transmit the shaft's full power.

Torsional shear strength of the shaft is:

$$T_t = \frac{\pi\ .\tau_1\ .d^3}{16} \qquad (5-8)$$

$T_t = Torque\ transmitted\ under\ acting$ torsional shear strength .

Were,

$$\tau_1 = Shearsstress\ for the shaft material$$

From the two equations (5-4 & 5-8) being equal, we get the following:

$$T_s = T_t$$

$$\frac{1}{2}(\tau\ .L.w.\ d) = \frac{\pi\ .\tau_1\ .d^3}{16}$$

$$L = \frac{\pi\ .\tau_1\ .d^2}{8\,w\ .\tau} \qquad (5-9)$$

$$Taking \quad (w) = \frac{d}{4}$$

$$\therefore \quad L = \frac{\pi \times \tau_1 \times d^2}{8 \times \frac{d}{4} \times \tau} = \frac{3.14 \times \tau_1 \times d}{2 \times \tau} = 1.57\,d \times \frac{\tau_1}{\tau}$$

We know that the metal of key is the same as the metal of the shaft, and therefore:

$$\tau = \tau_1$$

$$L = \frac{1.57\ d\ .\tau_1}{\tau} = \quad 1.57\ d \qquad (5-10)$$

5-6. Effect of Keyways

A little thought will reveal that the shaft's keyway cut affects the shaft's ability to support loads. This is brought on by a buildup of stress close to the keyway's corners and a reduction in the shaft's cross-sectional area. In other words, the shaft's torsional strength is decreased. The following relationship regarding the keyway's weakening effect. Is based on H. F. Moore's experimental findings.

$$k_e \ = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\ \left(\frac{h}{d}\right) \qquad (5-11)$$

Were,

k_e = Factor for reducing shaft strength. It represents the strength of the shaft with the keyway divided by the strength of the same shaft without the keyway.

$$w = Width\ of\ keyway, \quad d = Diameter\ of\ shaft, and$$

$$h = Depth\ of\ keyway = \frac{Thickness\ of\ key\ (t)}{2}$$

It is usually assumed that the strength of the keyed shaft is 75% of the solid shaft, which is somewhat higher than the value obtained by the above relation. In case the keyway is too long and the key is of sliding type, then the angle of twist is increased in the ratio (k_θ) as given by the following relation:

$$k_\theta = 1 + 0.4\left(\frac{w}{d}\right) - 0.7\left(\frac{h}{d}\right) \qquad (5-12)$$

Were,

k_θ = Reduction factor for angular twist.

5- 7. Solve examples

Example 1

Design a rectangular key for a (50 mm) diameter shaft. The key material's shearing and crushing stresses are ($\tau = 63\ MPa, \sigma = 105\ MPa$).

Solution

Given: $\{d = 50\ mm, \tau = 63\ MPa\ , \sigma = 105\ MPa\}$.

The rectangular key is designed for a shaft of (50 mm) diameter ،

$$Width of\ key,\ \ w = \frac{d}{4} = 12.5\,mm$$

$$and\,thickness of\ key, t = \frac{2w}{3} = \frac{d}{6} = 8.3\,mm$$

The length of key is obtained by considering the key in shearing and crushing.

Let: L = Length of key.

Considering the shaft's torsional shearing strength (or transmitted torque),

$$T = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 63 \times (50)^3 = 154546875 \qquad N.mm$$

Additionally, we are aware that the key's shearing strength (or torque communicated)،

$$T = L \times w \times \tau \times \frac{d}{2} = L \times 17 \times 63 \times \frac{50}{2} = 13387.5\ \ L$$

$$\therefore L = \frac{T}{13387.5} = \frac{154546875}{13387.5} = 115.44\ mm$$

Now considering crushing of the key, we know that shearing strength (or torque transmitted) of the key,

$$T = L \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} = L \times \frac{8.3}{2} \times 105 \times \frac{50}{2} = 5446.875\ \ L$$

$$\therefore L = \frac{T}{5446.875} = \frac{154546875}{5446.875} = 283.73\ mm$$

The length of the key is the bigger of the two values.

L = 283.73 say 284 mm.

Example 2

A steel shaft with a 30 mm diameter and a yield strength of (300 MPa). It is necessary to utilize a parallel key with dimensions of (14 mm) width by (9 mm) thickness manufactured of steel with a yield strength of (250 MPa). If the shaft is loaded to transfer the maximum allowable torque, determine the length of key that is needed. Utilize the idea of maximum shear stress and a factor of safety of (2).

Solution

Given:

$$\{d = 45\,mm, \sigma_y \;\; for\ shaft = 300\,MPa, w = 14\,mm, t = 9\,mm, \sigma yk \;\; for\ key = 250\,MPa\}.$$

Let: L = Length of key.

The maximum shear stress in the shaft, according to the maximum shear stress theory is:

$$\tau_1 = \frac{\sigma_y}{2 \times F.S} = \frac{300}{2 \times 2} = 75\ N/mm^2$$

And the key's maximum shear stress is:

$$\tau = \frac{\sigma_y}{2 \times F.S} = \frac{250}{2 \times 2} = 62.5\ N/mm^2$$

(**Note:** Yield strength for shaft and key materials is different). The maximum torque transmitted by the shaft and key is:

$$T = \frac{\pi}{16} \times \tau_1 \times d^3 = \frac{\pi}{16} \times 75 \times (30)^3 \;=\; 397406.25\ N/mm^2$$

Let's start by thinking about key failure caused by shearing. We are aware that the maximum transmitted torque (T),

$$T = L \times w \times \tau \times \frac{d}{2} = L \times 14 \times 62.5 \times \frac{30}{2} = 13125\ L$$

$$\therefore \quad L = \frac{T}{13125} = \frac{39740625}{13387.5} = 29.68\ mm$$

Now, determine the maximum torque (T) communicated by the shaft and key using the following equation, given that the key failed due to crushing:

$$T = L \times \frac{t}{2} \times \tau_{ck} \times \frac{d}{2} = L \times \frac{9}{2} \times \frac{250}{2} \times \frac{30}{2} = 8437.5\ L$$

$$Taking \quad \tau_{ck} = \frac{\sigma_{y_k}}{F.S.}$$

$$\therefore \quad L = \frac{T}{8437.5} = \frac{39740625}{8437.5} = 47.1\ mm$$

Using the greater of the two values, we get:

L = 47.1 say 48 mm.

Example 3

A mild steel shaft with a (50 mm) diameter and an extension of (L = 85 mm) is attached to a (33 kW) and (733 rpm) motor. Design the keyway in the motor shaft extension bearing in mind the mild steel key's allowed shear and crushing loads of (65 MPa and 130 Mpa), respectively. Compare the key's shear strength to the shaft's normal strength.

<u>Solution</u>

Given:

$\{P = 33\ KW = 33000\ W, N = 733\ rpm, d = 50\ mm, L = 85\ mm, \sigma c\ for\ key = 65\ MPa\ and\ \tau = 133\ MPa\}.$

The formula of a torque transmitted by the motor is:

$$T = \frac{60P}{2\pi N} = \frac{60 \times 33000}{2 \times 3.14 \times 733} = 430131\ \ N.mm$$

The equation of a torque in shearing is:

$$\because \ T = L \times w \times \tau \times \frac{d}{2}$$

$$\therefore \quad w = \frac{2T}{L \times \tau \times d} = \frac{2 \times 430131}{85 \times 65 \times 50} = 3.11 \ \ mm$$

This width of keyway is too small. The width of keyway should be at least (*d/4*).

$$\therefore \quad w = \frac{d}{4} = \frac{50}{4} = 12.5 \ \ mm$$

Since ($\sigma c \ = \ 2\,\tau$), therefore, a square of ($w \ = \ 12.5$ & $t \ = \ 12.5\, mm$).

According to H.F. Moore, the shaft strength factor,

$$K_e = 1 - 0.2\left(\frac{w}{d}\right) \ -1.1\left(\frac{h}{d}\right)$$

$$\because \qquad h = \frac{t}{2}$$

$$k_e = 1 + 0.2\left(\frac{w}{d}\right) \ -1.1\left(\frac{t}{2d}\right) = 1 + 0.2(\frac{12.5}{50}) - 1.1(\frac{12.5}{2 \times 50})$$

$$k_e = 1 + 0.05 - 0.1375 = 0.9125$$

The formula of a strength of the shaft with keyway is:

$$F_{Normal\,strength\,of\;the\,shaft} = \frac{\pi}{16} \times \tau \times d^3 \times k_e$$

$$= \frac{3.14}{16} \times 65 \times (50)^3 \times 0.9125 = 1455009.8 \ N$$

Also, the formula of a shear strength of the key is:

$$F_{Shear\;srength\,t\,of\;the\,key} = L \times w \times \tau \times \frac{d}{2}$$

$$= 85 \times 12.5 \times 65 \times \frac{50}{2} = 1726562.5 \ N$$

$$\therefore \frac{F_{Shear\;srength\,t\,of\;the\,key}}{F_{Normal\,strength\,of\;shaft}} = \frac{1726562.5}{1455009.8} = 1.187$$

5-8. Chapter Questions

1. Using the Kennedy key?

a. Applications with heavy duty.
b. Applications with light duty.
c. Applications with high speed.
d. Equipment that is precise.

2. The key that only fits in the hub's keyway is known as,

a. Feather key
b. Kennedy key
c. Saddle key
d. Woodruff key

3. Splines are employed when,

a. The speed being transmitted is high.
b. High power must be transmitted.
c. The shaft and hub are moving relative to one another.
d. High torque must be imparted.

4. When the gear must slide on the shaft, the type of key used is:

a. Kennedy key.
b. Feather key.
c. Sunk key.
d. Woodruff key.

5. The keyway,

a. They are increases stress concentration, and reduces strength and rigidity of shaft.
b. It is increasing stress concentration.
c. Increase strength and rigidity of shaft
d. Increase rigidity of shaft

5. The key is referred to as a semi-circular disk of uniform thickness.

a. Saddle key
b. Sunk key
c. Woodruff key
d. Feather key

7. Splines are frequently used in:

a. Gearbox for machine tools.
b. Gear box for automobile.
c. Gearbox OF Hoist and crane.
d. Bicycle

8. In the case of a sunk key,

a. **Both the shaft and the hub have keyways cut into them.**
b. The keyway is only cut in the shaft.
c. The keyway is only cut in the hub.
d. The shaft and hub do not have keyways cut into them.

9. The compressive stress induced in a square key is:

a. Bigger than shear stress

b. Less than shear stress
c. **Shear stress is applied twice.**
d. Equal to shear stress

10. When designing a shaft, key, and hub, care is taken to ensure that
a. The key is the strong link.
b. The hub is the weakest link.
c. The key is the weakest link.
d. The shaft is the weakest link.

11. The function of key is:
a. To attach a transmission shaft to gears or other spinning machine parts.
b. To transfer torque from the shaft to the hub and the other way around.
c. To stop the connected element's shaft from rotating relative to it.
d. Each of the previous three actions.

12. Sunk key taper is standard to:
a. 1 in 10
b. 1 in 25
c. 1 in 50
d. 1 in 100

13. In terms of shaft diameter (D), the standard width for a square or flat key is:
a. *d*/2
b. *d*/4
c. *d/8*
d. *d*

14. Sunk key only fits in the keyway of the ------.
a. Hub
b. Sleeve
c. Neither the sleeve nor the hub
d. Both the sleeve and the hub

15. Taper is generally given on key?
a. Both sides
b. Only the top side
c. Whichever side
d. Only the bottom side

16. The shaft strength factor, according to H.F. Moore, as in the following equation:
a. $\boldsymbol{K_e = 1 - 0.2\left(\frac{w}{d}\right) - 1 - 1.1\left(\frac{h}{d}\right)}$
b. $K_e = 1 - 0.3\left(\frac{w}{d}\right) - 1.2\left(\frac{h}{d}\right)$
c. $K_e = 1 - 1.1\left(\frac{w}{d}\right) - 0.2\left(\frac{h}{d}\right)$
d. $K_e = 1 - 0.3\left(\frac{w}{d}\right) - 0.2\left(\frac{h}{d}\right)$

17. The keyway width should be at least equal to ——————.
a. d/4
b. d
c. d/2
d. d/3

18. Sunk key has a ———— drive, which is its main advantage.

a. Negative
b. Positive
c. Negative
d. The listed none
e. Neutral

19. Equation applied to find a torsional shearing strength (or torque transmitted) of the shaft is:

a. $\boldsymbol{T = \frac{\pi}{16} \times \tau \times d^3}$
b. $T = \frac{\pi}{16} \times \tau \times d^4$
c. $T = \frac{\pi}{16} \times \tau \times d^2$
d. $T = \frac{\pi}{16} \times \tau \times d$

20. Permits for Woodruff Key ————- motion between the shaft and the hub.

a. Eccentric
b. Circular
c. Axial
d. Radial

21. Find the Kennedy key's length needed to transmit 1200 N-m, and the key's permitted shear is 40 N/mm2. The shaft's diameter and key's width can be assumed to be (40 mm) and (10 mm), respectively.

a. 36 mm
b. 49 mm
c. 46 mm
d. 53 mm

22. Stub teeth on involute splines have a pressure angle of ——————-.

a. 90 ^{o}
b. 35 ^{o}
c. 45 ^{o}
d. 60 ^{o}

23. ——————- are employed if there isn't space for the key to be driven in or out.

a. Woodruff keys
b. Tapered driving keys
c. Taper sleeves
d. Round keys

24. ---------- is the male part of the joint and socket is the female part of the joint.

a. **Spigot**
b. Cotter
c. Spline
d. Socket

25. The bottom of --------- is concave in longitudinal direction.

a. **Hollow keys**
b. Taper keys
c. Round keys
d. Tangent keys

26. ----------- used for low torque transmission.

a. Round key
b. Tangent key
c. Spline
d. Woodruff key

27. A thickness of the rectangular sunk key is -------------.

a. d / 6
b. d / 2
c. d / 4
d. d / 8

28. If the key is sunk

a. Only the shaft has the keyway cut into it.
b. The keyway has only been cut in the hub.
c. Both the shaft and the hub have keyways cut into them.
d. Neither the shaft nor the hub has the keyway drilled.

29. Designing a shaft, key, and hub requires consideration of the following:

a. The shaft is the most fragile part.
b. The strongest element is the key.
c. The weakest element is the key.
d. The weakest part is the hub.

30. In a square key, the compressive stress is either:

a. equal to the shear stress.
b. Two times that shear stress.
c. double that shear stress.
d. half that shears stress.

31. When using a saddle key, power is transferred using,

a. Key's crushing resistance.
b. Key's sheer resistance.
c. Tensile strength.
d. friction strength.

32. Design the rectangular key to fit a 50 mm shaft. The main material has shearing and crushing stresses of (42 MP and 70 Mpa), respectively.
[$\boldsymbol{Ans.\ Ts = 13.125L\ KN.mm, Tt = 1030\ KN.Mm, L = 79.25\ mm,\ T = 7.2625L\ KN.mm,\quad L = 141.8\ mm}$]

33. A steel shaft with a diameter of (45 mm) and a yield strength of (400 Mpa) is used. A parallel key of size (14 mm) width and (9 mm) thickness made of steel with yield strength of (340 Mpa) is to be used. If the shaft is loaded to transfer the maximum allowable torque, determine the length of key that is needed.

$[Ans. L = 104.6\ mm]$

Chapter 6

Frictional Clutches

6. Frictional Clutches

6-1. Introduction

A clutch connects the two shafts of a device, so they can spin in unison or spin at different speeds, depending on the situational needs. In a car, one shaft, the flywheel, is connected to the engine while the other, the clutch plate, is connected to the transmission. Using friction between a clutch plate and a flywheel, the clutch either keeps the wheels spinning in sync with the engine, or it disconnects the wheels from the engine, so the car can stop.

6-2. Types of clutches according to the method of operation

1. Mechanical clutches
2. Pneumatic clutches
3. Hydraulic clutches
4. Electromagnetic clutches.

6-3. Main Part of a Clutch

The main parts of a clutch are classified into three groups

1. Driving members
2. Driven members
3. Operating members.

6-3-1. Driving member

The driving member has a flywheel mounted on the crankshaft of the engine. The flywheel is fixed to a cover which supports a pressure plate or driving disc, pressure springs and releasing levers.

The whole assembly of the flywheel and the cover rotate all the times. The clutch housing and the cover provided with an opening. From this opening, the heat is evaporated generated by the friction during the clutch operation.

6-3-2. Driven member

The driven member has a disc or plate, called the clutch plate. It is free to slide alongside on the splines of the clutch shaft. Driven member carries friction materials on both of its surface. When a driven member is held between the flywheel and the pressure plate, it helps to rotate the clutch shaft through the splines.

6-3-3. Operating member

The operating members have a foot pedal, linkage, release or throw-out bearing, release leavers and the springs essential to ensure the proper operation of the clutch.
Functions of various components of transmission power, figure (6-1).

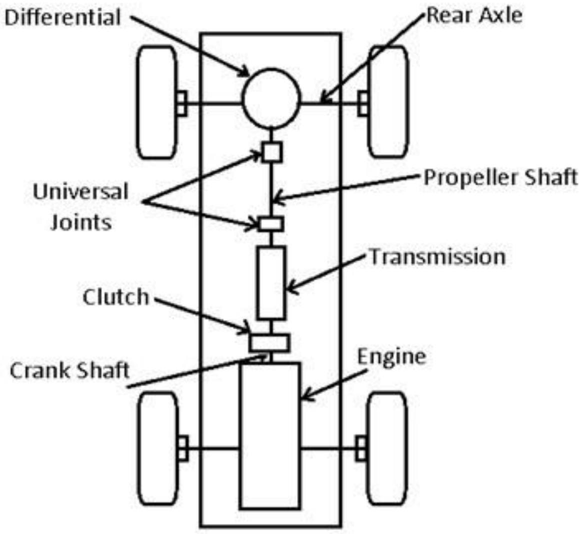


Figure 6-1: Automobile power transmission system.

6-4. The most typical clutches

The following are the various types of clutches:

1. Friction clutch

I. Single plate clutch

II. Multi plate clutch

a. Wet

b. Dry

III. Cone clutch

a. External

b. Internal

2. Centrifugal Clutch

3. Semi-centrifugal clutch

4. Conical spring clutch or Diaphragm clutch

a. Tapered finger type

b. Crown spring type

5. Positive clutch

a. Dog clutch

b. Spline Clutch

6. Hydraulic clutch

7. Electromagnetic clutch

8. Vacuum clutch

9. Overrunning clutch or freewheel unit

6-4-1. Single Plate Clutch

A single-plate clutch is the most common option for completing the transmission of trucks, tractors, buses with a manual transmission. The technologies for the production of auto parts and components are constantly evolving towards simplifying design features and increasing the service life.

6-4-2. Single clutch advantages

1. Simple design and reliable operation,
2. Low price compared to other options,
3. Possibility of converting machines with a double-disc clutch with an additional replacement of only the flywheel and a change in the drive control.

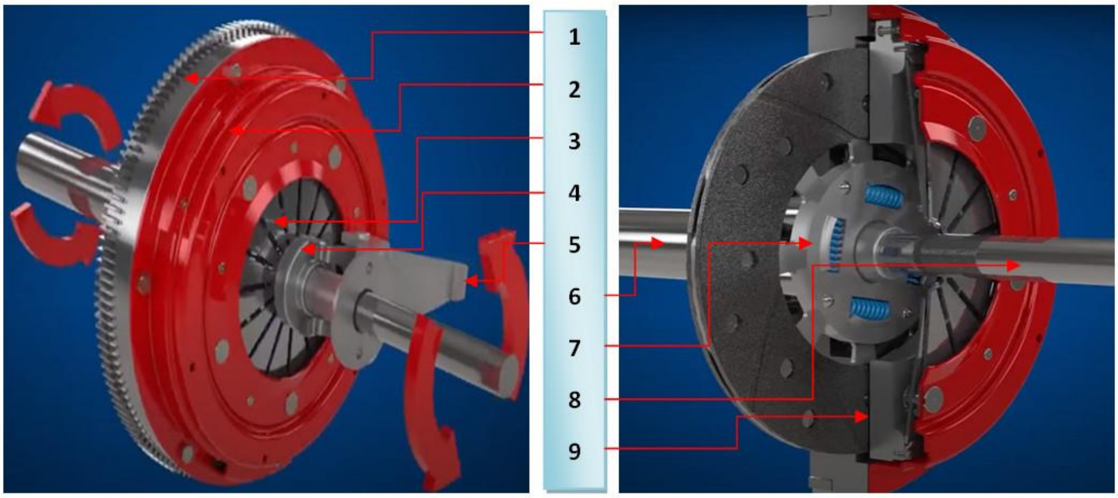
6-4-3. Design and operation of a single-plate clutch

This option consists of three elements that work as an assembly as a clutch in figure (6-2):

1. Pressure disk (basket),
2. Driven disc,
3. Release bearing with clutch.

A disc clutch performs a number of important functions, the main purpose of which is:

1. At the smooth start of the vehicle,
2. Reliable transmission of torque from the gearbox to the internal combustion engine (ICE),
3. Ensuring complete connection / disconnection of the motor and transmission when changing speed modes in motion.



1. Flywheel, 2. Pressure plate assembly, 3. Diagram spring, 4. Release bearing and hub, 5. Clutch Pedal, 6. Engine shaft (Driving member), 7. Clutch disk, 8. Clutch shaft (Driven member), 9. Friction lining.

Figure 6-2: Single Plate Clutch

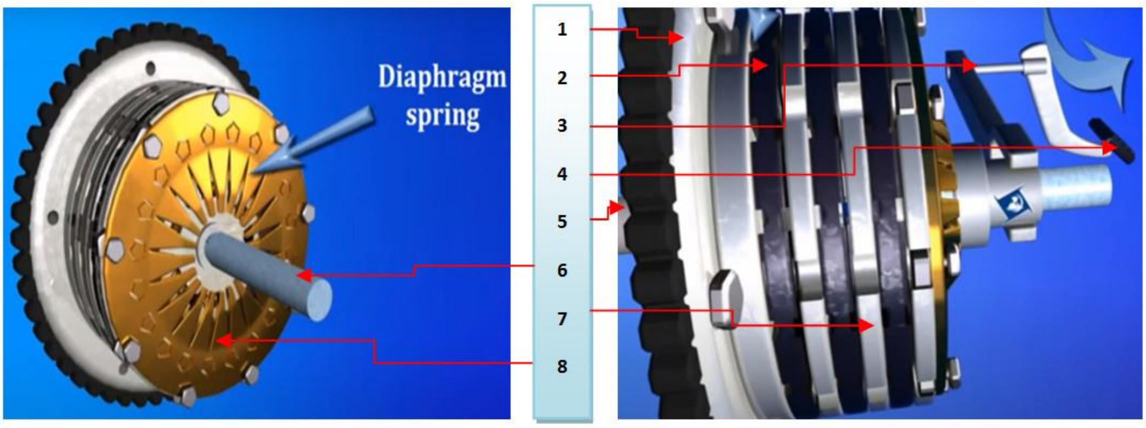
6-4-4. Working single-plate clutch

In a vehicle, use the clutch to disengage the gears by pressing the clutch to peddle. The springs are then compressed, causing the pressure plate to move backwards. The clutch plate is now free between the pressure plate and the flywheel. As a result, the clutch is now disengaged and able to shift gears.

This causes the flywheel to rotate as long as the engine is running, while the clutch shaft speed gradually decreases until it stops rotating. The clutch is said to be disengaged as long as the clutch peddle is pressed; otherwise, the spring forces keep the clutch engaged. When you let go of the clutch pedal, the pressure plate returns to its original position and the clutch engages again.

6-4-5. Multi plate Clutch

The multi-plate clutch is a special type of clutch that can produce high torque. It mainly transmits the power from one shaft to another shaft. One of them is the engine shaft and another one is the transmission shaft. Friction takes place in the engine by the clutch plates. This friction makes high torque, figure (6-3).
Moreover, it can be said that in the automobiles or in pieces of machinery, where high torque is needed like in the gearbox of motorcycles, this multi-plate clutch can be used to assure the precision level of that machine.



1. Flywheel, 2. Friction Disc, 3. Thrust Spring, 4. Clutch Pedal, 5. Engine shaft (Driving member), 6. Clutch shaft (Driven member), 7. Friction lining, 8. Diaphragm Spring

Figure 6-3: Multi Plate Clutch

6-4-6. Cone Clutch

A cone clutch is depicted in this diagram. It is made up of cone-shaped friction surfaces. Figure (6-4) shows how this clutch transmits torque by friction using two conical surfaces. A female cone and a male cone make up the engine shaft. To slide on it, the male cone is mounted on the splined clutch shaft. The conical portion has a friction surface.

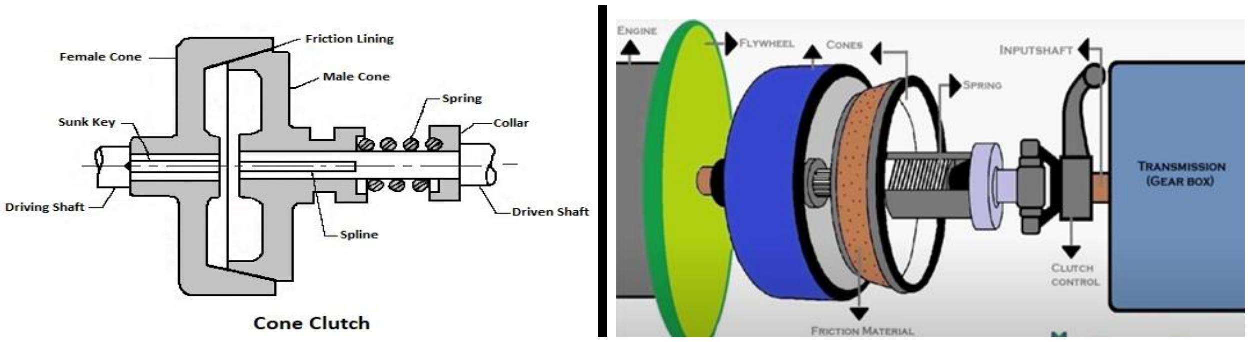


Figure 6-4: Cone Clutch

6-4-6-1. Advantage Cone Clutch

1. Simple construction
2. Less maintenance
3. Affordable
4. Automatic

6-4-6-2. Limitations Cone Clutch

1. Slipping
2. Power less
3. Low capacity
4. Overheating.

6-4-7. Centrifugal Clutch

A centrifugal clutch is depicted in figure (6-5). Centrifugal clutches use centrifugal force rather than spring force to keep the clutches engaged. The clutch in these clutches operates automatically based on the engine speed. As a result, there is no need for a clutch pedal to operate the clutch.

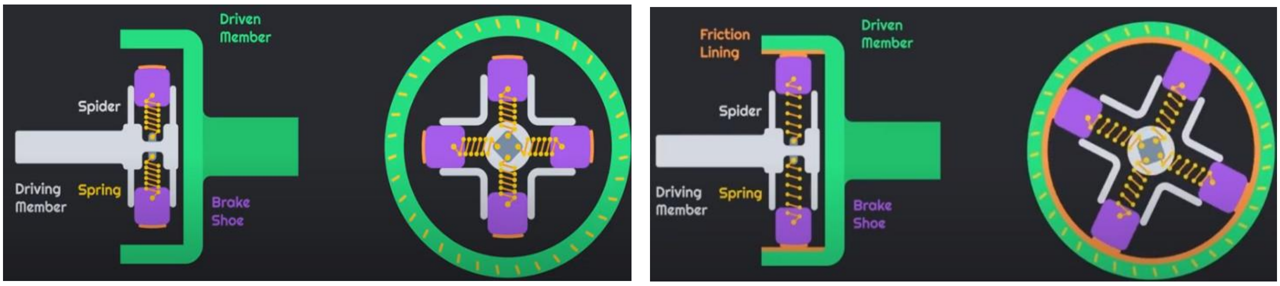


Figure 6-5: Cone Clutch

6-4-7-1. Semi-Centrifugal Clutch

Semi Centrifugal Clutches used in high powered engines and racing car engines where clutch disengagements require appreciable and tiresome driver’s effort. The power transmitted with partly by clutch springs and remaining by the centrifugal action of an extra weight provided in the system figure (6-6).

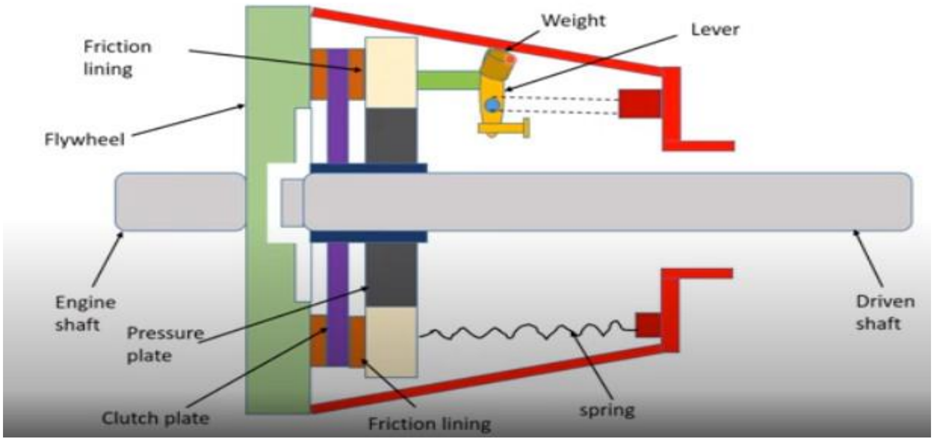


Figure 6-6: Semi-Centrifugal Clutch

6-4-7-2. Working of Semi Centrifugal Clutch

When the engine at low speed the spring keeps the clutch engaged to transmit power, the weighted levers do not have any pressure on the pressure plate.

When engine at high speed the weights fly off and levers exert pressure on the pressure plate which keeps the clutch firmly engaged to transmit high torque.

Thus, instead of having more stiff springs for keeping the clutch engaged firmly at high speeds, they are less stiff because of centrifugal forces of weighted levers, so that the driver may not get any strain in operating the clutch.

when the engine speed decreases, the weights fall and the weighted levers do not exert any pressure on the pressure plate and only spring pressure is exerted on the pressure plate to keep the clutch engaged.

6-4-7-3. Advantages Semi-Centrifugal Clutch

1. Clutch operation is very easy.
2. Less stiff clutch springs are used as they operate only at low engine speeds.

6-4-7-4. Disadvantages Semi-Centrifugal Clutch

1. Springs have transmitted the torque at lower engine speeds only.
2. Centrifugal forces work only at higher engine speed to transmit torque.

6-4-8. Diaphragm Clutch

The diaphragm coupling converts the torque reliably, safely and without wear or maintenance figure (6-7). Equipped with one or two diaphragms, these couplings provide a power range from 100 to 70,000 kW.

Diaphragm coupling consists of half couplings, which are made of aluminum alloy, connected by steel diaphragms and a central ring. They are secured to the shaft by tightening with a screw. They are used in units where compensation of shaft misalignment and angle between shafts is required. They transmit high torque, provide a backlash-free transmission, have a high rotational speed, similar to a cardan joint.

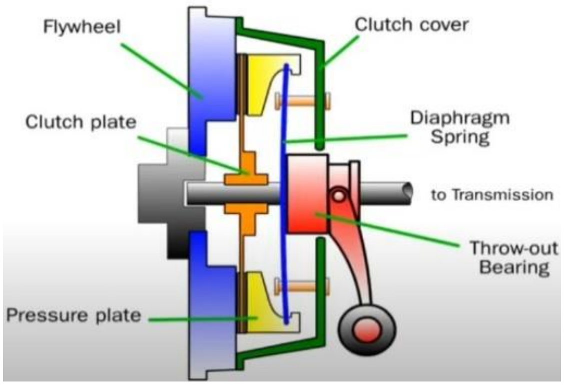


Figure 6-7: Diaphragm Clutch

6-4-8-1. Advantages Diaphragm Clutch

1. It is more compact by means of storing energy. Thus, compact design results in smaller clutch housing.
2. Diaphragm spring is less affected by centrifugal forces.
3. In Diaphragm spring the load deflection curve is not linear, therefore in this case the clutch facing wears the pressure by diaphragm spring gradually increases.
4. The diaphragm spring acts as both, the clamping spring as well as the release lever. So many parts like struts, eye bolts, levers etc.

6-4-8-2. Disadvantages Diaphragm Clutch

1. To get more co-efficient of friction, the size and diameter of Diaphragm is increased.
2. Compare to Diaphragm spring, Coil springs have tendency to distort in the transverse direction at higher speeds.

6-4-9. Dog and spline clutch

A dog clutch is a type of clutch that is used to connect two shafts or to lock two shafts together. The clutch has two parts: a dog clutch with external teeth and a sliding sleeve with internal teeth, as shown in figure (6-8). Both shafts are designed in such a way that one will always rotate at the same speed as the other and will never slip. When the two shafts are connected, the clutch is said to be engaged. To disengage the clutch, move the sliding sleeve back on the splined shaft until it is no longer in contact with the driving shaft.

In manual transmission vehicles, the dog and splined clutch are commonly used to lock different gears.

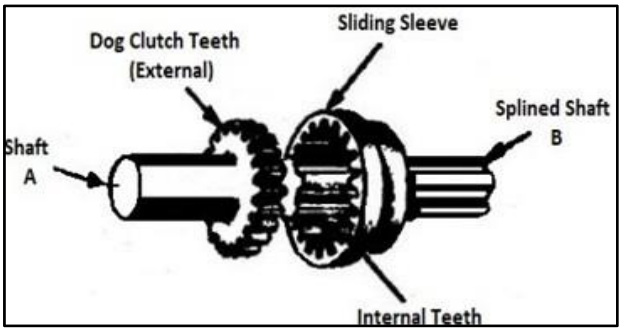


Figure 6-8: Dog and spline Clutch

6-4-10. Electromagnetic Clutch

These clutches are electrically operated, but the torque is transmitted mechanically. This is why these clutches are referred to as electro-mechanical clutches. It evolved into an electromagnetic clutch over the course of a year, as illustrated in figure (6-9).

Because there is no mechanical linkage to control the engagement of these clutches, they operate quickly and smoothly. Electromagnetic clutches are best suited for remote operation, which means you can operate the clutch from a distance.

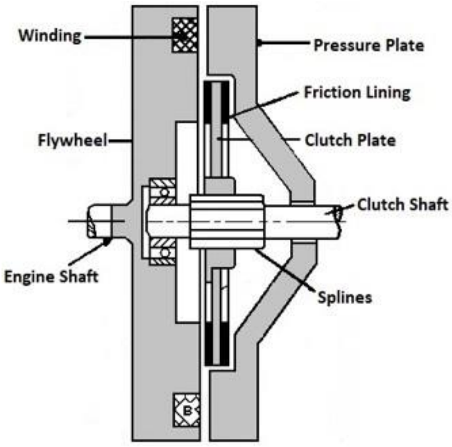


Figure 6-8: Electromagnetic Clutch

The clutch's flywheel is made of winding. The battery provides the electricity. When electricity passes through the winding, it creates an electromagnetic field, which attracts the pressure plate and causes it to engage. When the power is turned off, the clutch disengages.

The gear lever in this clutch system has a clutch release switch, which means that when the driver operates the gear lever to change gears, the switch is activated, cutting off the current supply to the winding, causing the clutch to disengage.

6-4-11. Vacuum clutch

The vacuum clutch mechanism is seen in figure (6-10). This kind of clutch is operated by the engine manifold's built-in vacuum. A reservoir, non-return valve, vacuum cylinder with piston, and solenoid valve make up the vacuum clutch.

6-4-11-1. Construction and working of a Vacuum clutch

The reservoir is connected to the inlet manifold via a non-return valve, as shown in the figure. A solenoid-controlled valve connects a vacuum cylinder to a reservoir. The solenoid is powered by the battery, and the circuit includes a switch located on the gear lever. The switch is activated when the driver changes gear by holding the gear lever, as shown in the figure (6-10).

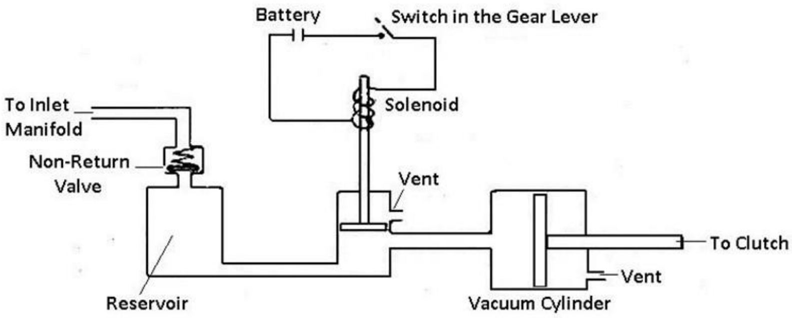


Figure 6-10: Vacuum clutch

6-4-12. Hydraulic clutch

The hydraulic clutch operates in the same way as the vacuum clutch. The main distinction between these two is that the hydraulic clutch is activated by oil pressure, whereas the vacuum clutch is activated by vacuum, as illustrated in figure (6-11).

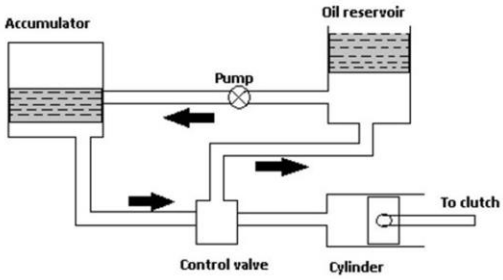


Figure 6-11: Hydraulic clutch

The illustration depicts the mechanism of a hydraulic clutch. It is made up of fewer parts than other clutches. It has an accumulator, a control valve, a cylinder with a piston, a pump, and a reservoir.

6-5 Design disc clutch

Take into account two friction surfaces that are kept in contact by an axial thrust (W), as depicted in figure (6-12).

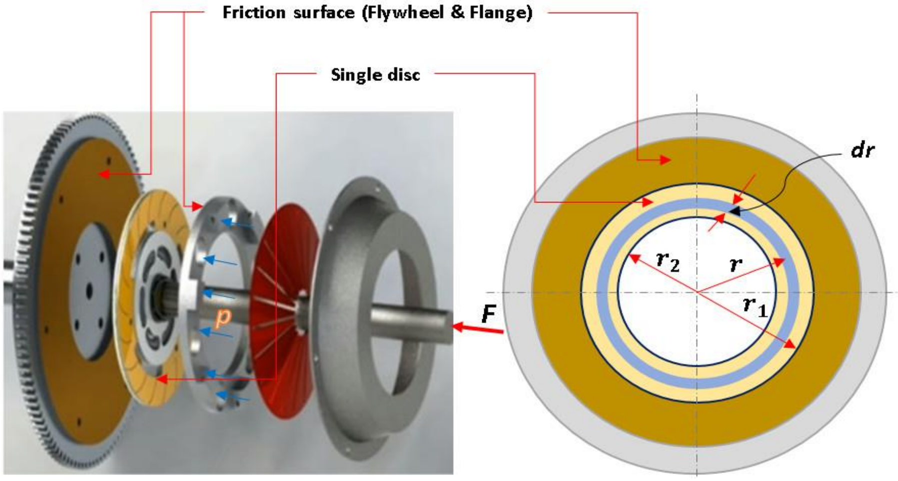


Figure 6-12: Forces on a disc clutch

Let

$$T = Torque\ transmitted\ by\ the\ clutch,$$

$$p = Intensity\ of\ axial\ pressure\ with\ which\ the\ contact\ surfaces\ are\ held\ together,$$

$$r_1\ \&\ r_2 = External\ and\ internal\ radii\ of\ friction\ faces,$$

$$r = Mean\ radius\ of\ the\ friction\ face,$$

$$\mu = Coefficient\ of\ friction.$$

Consider the area of the contact surface or friction surface of an elementary ring with radius (r) and thickness (dr), as illustrated in figure (6-12).

We know that area of the contact surface or friction surface.

$$A = 2\pi\, r.dr \qquad (6-1)$$

∴ Normal or axial force on the ring (F),

$$\delta F = Pressure \times Area = p \times 2\pi\, r.dr \qquad (6-2)$$

and the frictional force on the ring acting tangentially at radius r,

$$Fr = \mu \times \delta F = \mu.p \times 2\pi\, r.dr \qquad (6-3)$$

∴ Frictional torque acting on the ring,

$$Tr = Fr \times r = \mu.p \times 2\pi\, r.dr \times r = 2\,\pi\,\mu\, pr^2.dr \qquad (6-4)$$

Consider the following two cases:

1. When there is a uniform pressure, and
2. When there is a uniform axial wear.

1. Considering uniform pressure

When pressure is uniformly distributed across the entire area of the friction face, as shown in figure (6-12), the pressure intensity:

$$p = \frac{F}{\pi[(r_1)^2 - (r_2)^2]} \qquad (6-5)$$

Where

$$F = Axial\ thrust\ with\ which\ the\ friction\ surfaces\ are\ held\ together.$$

The frictional torque on the elementary ring of radius (r) and thickness (dr), as previously established, is:

$$T_r = 2\pi\,\mu.p.r^2.dr$$

Integrating this equation within the limits from (r_2 to r_1) for the total friction torque.

∴ Total frictional torque acting on the friction surface or on the clutch,

$$T = \int_{r_2}^{r_1} 2\pi\,\mu.p.r^2.dr = 2\pi\,\mu.p\left[\frac{r^3}{3}\right]_{r_2}^{r_1} = 2\pi\,\mu.p\left[\frac{{r_1}^3 - {r_2}^3}{3}\right]$$

$$\because \quad p = \frac{F}{\pi[(r_1)^2 - (r_2)^2]}$$

$$\therefore \quad T = 2\pi\,\mu.\frac{F}{\pi[(r_1)^2 - (r_2)^2]}\left[\frac{{r_1}^3 - {r_2}^3}{3}\right] = \frac{2}{3}\,\mu.F\left[\frac{{r_1}^3 - {r_2}^3}{{r_1}^2 - {r_2}^2}\right]$$

$$Mean\ radius\ of\ the\ friction\ surface\ (R) = \frac{2}{3}\left[\frac{{r_1}^3 - {r_2}^3}{{r_1}^2 - {r_2}^2}\right]$$

$$\therefore \quad T = \mu . F . R \qquad (6-6)$$

2. Considering uniform axial wear

Normal wear is proportional to friction work, which is a key design tenet for machine parts that are subject to sliding friction wear. Normal pressure (p) and sliding velocity (V) are multiplied to get the frictional work. Therefore:

$$Normal\ wear \propto Work\ of\ friction \propto p.V$$

Or

$$p.V = K\ (a\ constant) or\ p = \frac{K}{V} \qquad (6-7)$$

The pressure distribution throughout the entire contact surface is homogeneous when the friction surface is new. Where the sliding velocity is greatest, this pressure will deteriorate most quickly, lowering the pressure between the friction surfaces. Until the product (p.V.) is uniform across the entire surface, this process is repeated. After that, the attire will be uniform, as seen in figure (6-13).

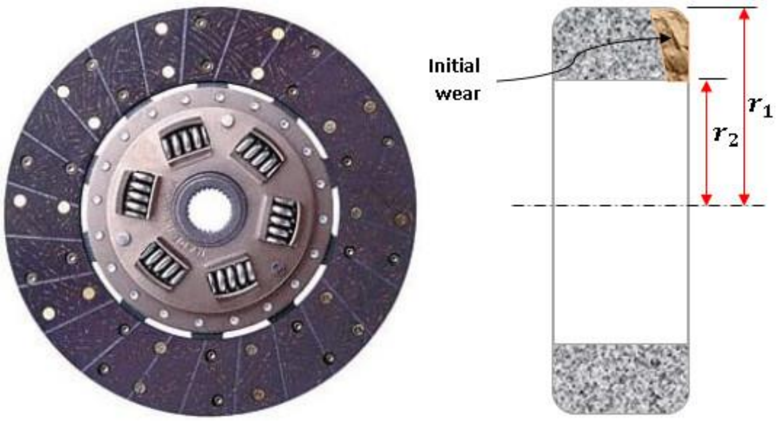


Figure 6-13: Uniform axial wear.

Let

$\boldsymbol{p} = the\ normal\ intensity\ of\ pressure\ at\ a\ distance\ (\boldsymbol{r})\ from\ the\ axis\ of\ the\ clutch.$

Because the intensity of pressure varies inversely with distance, this means:

$$p.r = C\ (a\ constant) or\ p = \frac{C}{r} \qquad (6-8)$$

additionally, the ring's normal force,

$$\delta F = p.2\pi\, r\, dr = \frac{C}{r}.2\pi\, r\, dr = 2\pi\, C\, dr \qquad (6-9)$$

$\therefore$ Total acing force on the friction surface,

$$F = \int_{r_2}^{r_1} 2\pi\, C.dr = 2\pi\, C.[r]_{r_2}^{r_1} = 2\pi\, C.(r_1 - r_2) \qquad (6-10)$$

$$\therefore \quad C = \frac{F}{2\pi\,(r_1 - r_2)} \qquad (6-11)$$

We know that the frictional torque acting on the ring,

$$\because \quad p = \frac{C}{r} \qquad (6-12)$$

$$\therefore\ T_r = 2\pi\,\mu.p.r^2.dr = 2\pi\,\mu.\frac{C}{r}.r^2.dr = 2\pi\,\mu.C.r.dr \qquad (6-13)$$

$\therefore$ Total frictional torque acting on the friction surface (or on the clutch),

$$T = \int_{r_2}^{r_1} 2\pi\,\mu.C.r.dr = 2\pi\,\mu.C\left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi\,\mu.C\left[\frac{{r_1}^2 - {r_2}^2}{2}\right] = \pi\,\mu.C[{r_1}^2 - {r_2}^2]$$

$$\because \quad C = \frac{F}{2\pi[r_1 - r_2]} \qquad (6-14)$$

$$\therefore \quad T = \pi\,\mu.\frac{F}{2\pi[r_1 - r_2]}[{r_1}^2 - {r_2}^2] = \frac{1}{2}\mu.F(r_1 + r_2)$$

$$\because \ Mean\,radius\ of\ the\ friction\ surface\ (R) = \frac{r_1 + r_2}{2}$$

$$\therefore \quad T = \mu.F \qquad (6-15)$$

In general, total frictional torque acting on the friction surfaces (or on the clutch) is given by:

$$T = n.\mu.F.R \qquad (6-16)$$

Where

$$n = Number\ of\ pairs\ of\ friction\ (or\ contact)\ surfaces, and$$

$$R = Mean\,radius\ of\ friction\ surface$$

1- For uniform pressure $R = \frac{2}{3}\left[\frac{{r_1}^3 - {r_2}^3}{{r_1}^2 - {r_2}^2}\right]$

2- For uniform wear $R = \frac{r_1 + r_2}{2}$

6-6 Design Multi Clutch

When a large torque is to be transmitted, a multiple disc clutch, as shown in figure (6-14), may be used. To allow axial motion, the inside discs (usually made of steel) are attached to the driven shaft (except for the last disc). The outside discs (usually made of bronze) are fastened to the housing, which is keyed to the driving shaft. Numerous applications, such as those involving machines and automobiles, utilize multiple disc clutches.

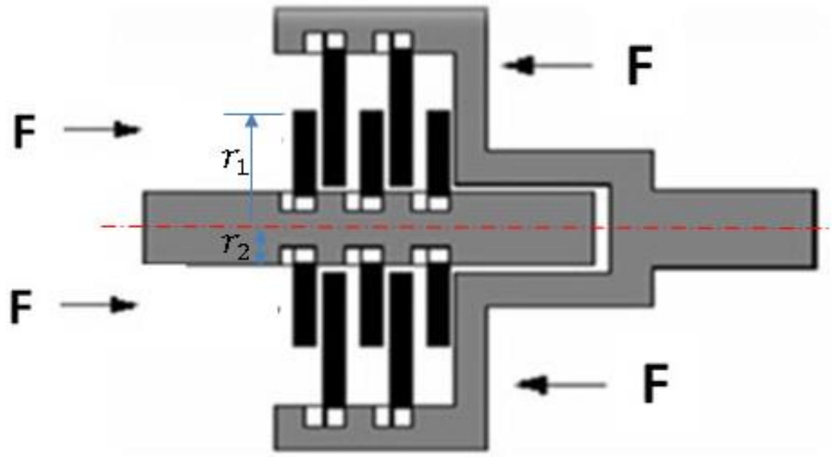


Figure 6-14: Uniform axial wear

Let

$$n_1 = Number\ of\ discs\ on\ the\ driving\ shaft, and$$

$$n_2 = Number\ of\ discs\ on\ the\ driven\ shaft.$$

$$\therefore\ Number\ of\ pairs\ of\ contact\ surfaces,$$

$$n = n_1 + n_2 - 1 \qquad (6-17)$$

and total frictional torque acting on the friction surfaces or on the clutch,

$$T = n.\mu.W.R \qquad (6-18)$$

Where

$$R = Mean\ radius\ of\ friction\ surfaces$$

1- For uniform pressure $R = \frac{2}{3}\left[\frac{{r_1}^3 - {r_2}^3}{{r_1}^2 - {r_2}^2}\right]$

2- For uniform wear $R = \frac{r_1+r_2}{2}$

Where

$$T = Maximum\ power\ transmitted,$$

$$\mu = Coefficient\ of\ friction,$$

$$F = Axial\ force,$$

$$r_1 = Outer\ raduis,$$

$$r_2 = Inner\ raduis,$$

$$n = Number\ of\ friction\ clutch.$$

6-7. A Cone Clutch Design

Consider a cone clutch's friction surfaces in pairs, as shown in figure (6-15). A little thought will reveal that the frustrum of a cone is the region of contact between two friction surfaces.

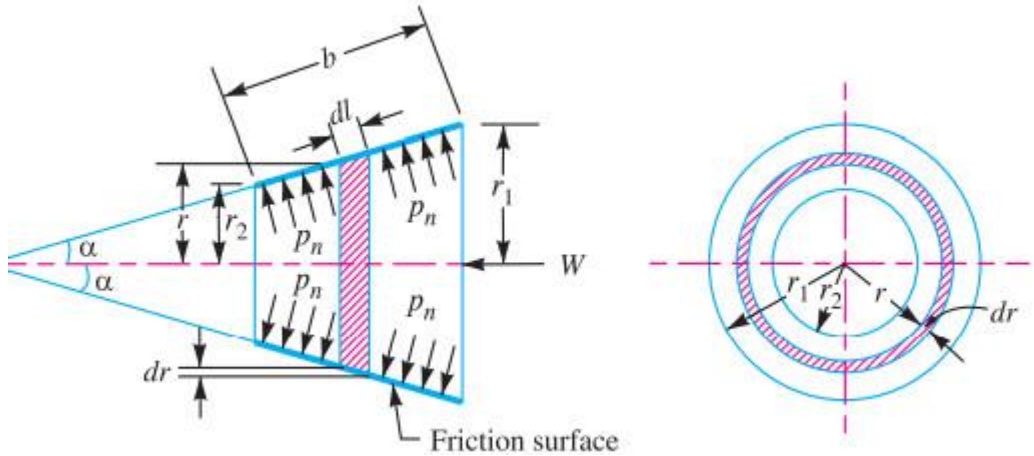


Figure 6-15: Friction surfaces as a frustrum of a cone

Let

$$p_n = Intensity\ of\ pressure\ with\ which\ the\ conical\ friction\ surfaces\ are\ held\ together\ (i.e.\ normal\ pressure\ between\ the\ contact\ surfaces),$$

$$r_1 = Outer\ radius\ of\ friction\ surface,$$

$$r_2 = Inner\ radius\ of\ friction\ surface,$$

$$R = Mean\, radius\, of\ friction\ surface = \frac{r_1 + r_2}{2}$$

$$\alpha = Semi - angle\ of\ the\ cone\ (also\ called\ face\ angle\ of\ the\ cone) or$$

$$angle\ of\ the\ friction\ surface\ with\ the\ axis\ of\ the\ clutch,$$

$$\mu = Coefficient\ of\ friction\ between\ the\ contact\ surfaces, and$$

$$b = Width\ of\ the\ friction\ surfaces\ (also\ known\ as\ face\ width\ or\ cone\ face).$$

Consider a small ring of radius r and thickness dr as shown in figure (6-15).

Let dl is the length of ring of the friction surface, such that,

$$dl = dr\ cosec\ \alpha$$

$$\therefore\ Area\ of\ ring = 2\pi\, r.dl = 2\pi\, r.dr\ cosec\ \alpha \qquad (6-19)$$

We shall now consider the following two cases :

1. When there is a uniform pressure

We know that the normal force acting on the ring,

$$\delta Fn = Normal\, pressure \times Area\ of\ ring = p_n \times 2\pi\, r.dr\ cosec\ \alpha$$

Additionally, the ring's axial force,

$$\delta W = Horizontal\ component\ of\ \delta Fn\ (i.e. in\ the\ direction\ of\ F)$$

$$= \delta Fn \times sin\ \alpha = p_n \times 2\pi\, r.dr\ cosec\ \alpha \times sin\ \alpha = 2\pi \times p_n.r.dr$$

∴ Total axial load applied to the clutch or the necessary axial spring force is:

$$F = \int_{r_2}^{r_1} 2\pi\, .p_n.rdr = 2\pi\, .p_n.\left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi.p_n.\left(\frac{{r_1}^2 - {r_2}^2}{2}\right) = \pi.p_n.({r_1}^2 - {r_2}^2)$$

$$\therefore \qquad p_n = \frac{F}{\pi.({r_1}^2 - {r_2}^2)} \qquad (6-20)$$

We know that frictional force on the ring acting tangentially at radius (r),

$$F_r = \mu.\delta F_n = \mu.p_n \times 2\pi r.dr\ cosec\ \alpha$$

∴ Frictional torque acting on the ring,

$$T_r = F_r \times r = \mu.p_n \times 2\pi r.dr\ cosec\ \alpha \times r = 2\pi\, \mu.p_n\ cosec\ \alpha.r^2\ dr$$

Integrate this expression for the clutch's overall frictional torque while staying within the range of (r_2 to r_1) .

∴ Total frictional torque,

$$T = \int_{r_2}^{r_1} 2\pi\, \mu . p_n . cosec\, \alpha . r^2\, dr = 2\pi\, \mu . p_n . cosec\, \alpha \left[\frac{r^3}{3}\right]_{r_2}^{r_1}$$

$$= 2\pi\, \mu . p_n . cosec\, \alpha \left[\frac{{r_1}^3 - {r_2}^3}{3}\right]$$

Substituting the value of (p_n) from equation,

$$p_n = \frac{F}{\pi . ({r_1}^2 - {r_2}^2)}$$

We get

$$\therefore \quad T = \frac{2}{3}\, \mu . F \times cosec\, \alpha \left[\frac{{r_1}^3 - {r_2}^3}{{r_1}^2 - {r_2}^2}\right] \qquad (6-21)$$

2. When there is a uniform wear

In figur (6-15), let (p_r) be the normal intensity of pressure at a distance r from the axis of the clutch.

Knowing that, in case of uniform wear, the intensity of pressure varies inversely with the distance.

$$\therefore \quad p_r . r = C\ (a\ constant)\ \ or\ \ p_r = C / r$$

Knowing that the ring is being affected by the normal force,

$$\delta Fn = Normal\ pressure \times Area\ of\ ring = p_r \times 2\pi r . dr\ cosec\ \alpha$$

and the axial force acting on the ring,

$$\delta F = \delta Fn \times \sin\alpha = p_r \times 2\pi\, r . dr\ cosec\ \alpha \times \sin\alpha = 2\pi \times p_r . r\ dr$$

$$\because \quad p_r = \frac{C}{r} \qquad (6-22)$$

$$\therefore \quad \delta F = 2\pi . \frac{C}{r} . r\ dr = 2\pi . C . dr$$

Total axial load transmitted to the clutch,

$$F = \int_{r_2}^{r_1} 2\pi . C . dr = 2\pi . C . [r]_{r_2}^{r_1} = 2\pi . c . (r_1 - r_2)$$

$$\therefore \quad C = \frac{F}{2\pi.(r_1 - r_2)} \qquad (6-23)$$

We are aware of the tangentially applied frictional force on the ring at radius (r),

$$Fr = \mu.\delta Fn = \mu.p_r \times 2\pi\, r.dr\, cosec\, \alpha$$

∴ Frictional torque acting on the ring,

$$Tr = Fr \times r = \mu.p_r \times 2\pi\, r.dr\, cosec\, \alpha \times r$$

$$= \mu \times C\, r \times 2\pi\, r.dr\, cosec\, \alpha \times r = 2\pi\, \mu.C\, cosec\, \alpha \times r\, dr$$

Integrating this expression within the limits from $(r_1\ to\ r_2)$ for the overall clutch frictional torque.

∴ Total frictional torque,

$$T = \int_{r_2}^{r_1} 2\pi\, \mu.C.cosec\, \alpha.rdr = 2\pi\, \mu.C.cosec\, \alpha \left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi\, \mu.C.cosec\, \alpha \left[\frac{{r_1}^2 - {r_2}^2}{2}\right]$$

Substituting the value of C from equation,

$$C = \frac{F}{2\pi.(r_1 - r_2)}$$

We have

$$T = 2\pi\, \mu.\frac{F}{2\pi.(r_1 - r_2)}.cosec\, \alpha \left[\frac{{r_1}^2 - {r_2}^2}{2}\right] = \mu.F\, cosec\, \alpha \left[\frac{{r_1}^2 + {r_2}^2}{2}\right] = \mu.F.R\, cosec\, \alpha$$

Where

$$R = \frac{r_1 + r_2}{2} = Mean\, radius\, of\, friction\, surface.$$

Considering the normal force occurring on the friction surface, $(Fn = F\, cosec\, \alpha)$, therefore the equation ,

$$T = \mu.F.R\, cosec\, \alpha$$

May be written as

$$T = \mu\, Fn\, R \qquad (6-24)$$

Figures (6-16) (a) and (b) show the forces acting on a friction surface during steady clutch operation and after the clutch has been engaged.

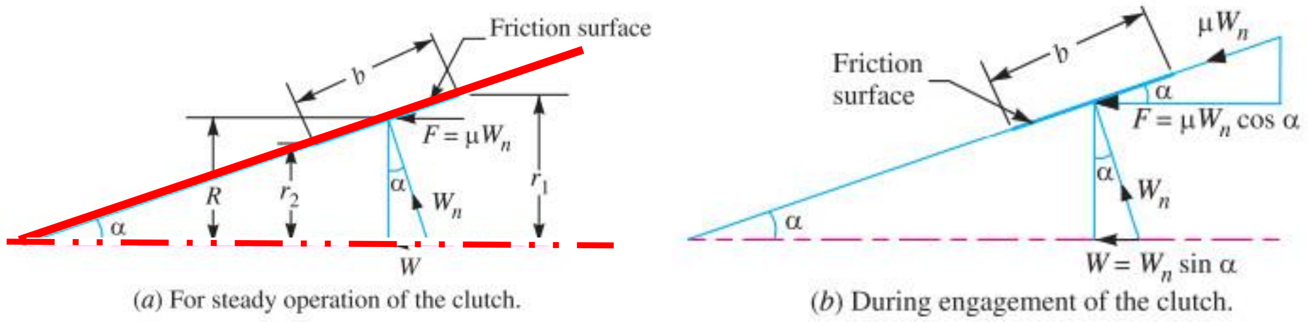


Figure 6-16: Forces on a friction surface

From figure (6-16) (a), we find that

$$r_1 - r_2 = b \sin \alpha \quad \& \quad R = \frac{r_1 + r_2}{2} \quad or \quad r_1 + r_2 = 2R$$

∴ From equation,

$$p_n = \frac{F}{\pi.(r_1{}^2 - r_2{}^2)} \qquad (6-25)$$

Normal pressure acting on the friction surface,

$$p_n = \frac{F}{\pi.(r_1{}^2 - r_2{}^2)} = \frac{F}{\pi.(r_1 - r_2)(r_1 + r_2)} = \frac{F}{2\pi.R.b \ \sin \alpha}$$

$$\therefore \quad F = p_n.2\pi.R.b \ \sin \alpha = F_n \ \sin \alpha$$

Where

$$Fn = Normal\ load\ acting\ on\ the\ friction\ surface = p_n \times 2\pi.R.b$$

$$F = Fn \ \sin \alpha \qquad (6-26)$$

Now the equation ,

$$T = \mu.F.R \ cosec \ \alpha$$

May be written as:

$$T = \mu\,(p_n \times 2\pi\, R.b \sin \alpha)\, R \ cosec \ \alpha = 2\pi\, \mu.p_n R^2.b$$

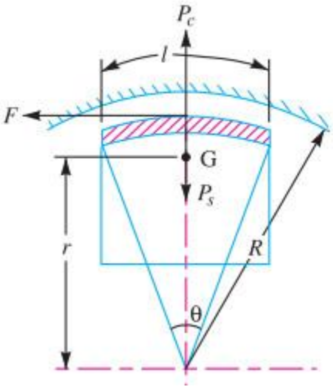
$$T = 2\pi\, \mu.p_n R^2.b \qquad (6-27)$$

6-8 Design of a Centrifugal Clutch

In The weight of the shoe, shoe size, and spring dimensions must all be determined when designing a centrifugal clutch. For the design of a centrifugal clutch, use the following procedure.

1. Mass of the shoes

Think about a centrifugal clutch's individual shoe, as seen in figure (6-17).



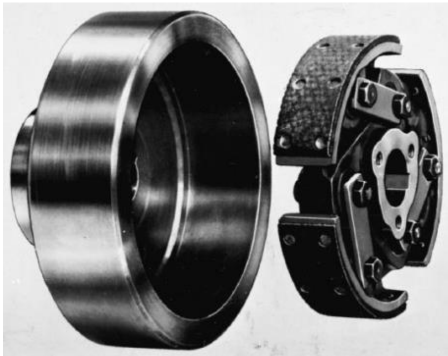


Figure 6-17: Forces on a shoe of a centrifugal cluch

Let

$$m = Mass\ of\ each\ shoe,$$

$$n = Number\ of\ shoes,$$

$$r = Distance\ of\ centre\ of\ gravity\ of\ the\ shoe\ from\ the\ centre\ of\ the\ spider,$$

$$R = Inside\ radius\ of\ the\ pulley\ rim,$$

$$N = Running\ speed\ of\ the\ pulley\ in\ r.p.m.,$$

$$\omega = Angular\ running\ speed\ of\ the\ pulley\ in\ rad\ /\ s = 2\,\pi\,N\ /\ 60\ rad/s,$$

$$\omega_1 = Angular\ speed\ at\ which\ the\ engagement\ begins\ to\ take\ place, and$$

$$\mu = Coefficient\ of\ friction\ between\ the\ shoe\ and\ rim.$$

That at running speed, each shoe is being affected by centrifugal force,

$$P_C = m.\omega^2.r \qquad (6-28)$$

The inward force that the spring exerts on each shoe is determined by the following equation, where the speed at which the engagement begins to occur is typically regarded to be (3/4th) of the running speed:

$$P_s = m.\omega_1{}^2.r = m\left(\frac{3}{4}\omega\right)^2.r = \frac{3}{4}\,m.\omega^2.r$$

At running speed, the net outward radial force (i.e., centrifugal force) with which the shoe presses against the rim is:

$$= P_c - P_s = m.\omega^2.r - \frac{9}{16} m.\omega^2.r = \frac{7}{16} m.\omega^2.r$$

as well as the tangential frictional force acting on each shoe,

$$F = \mu (P_c - P_s)$$

$$\therefore \ Frictional\ torque\ acting\ on\ each\ shoe = F \times R = \mu (P_c - P_s) R$$

and total frictional torque transmitted,

$$T = \mu (P_c - P_s) R \times n = n.F.R \qquad (6-29)$$

The mass of the shoes (m) may be calculated from this expression.

2. Size of the shoes

Let

$$l = Contact\ length\ of\ the\ shoes,$$

$$b = Width\ of\ the\ shoes,$$

$$R = Contact\ radius\ of\ the\ shoes.$$

$$It\ is\ same\ as\ the\ inside\ radius\ of\ the\ rim\ of\ the\ pulley,$$

$$\theta = Angle\ subtended\ by\ the\ shoes\ at\ the\ centre\ of\ the\ spider\ in\ radians, and$$

$$p = Intensity\ of\ pressure\ exerted\ on\ the\ shoe.$$

In order to ensure reasonable life, it may be taken as ($0.1\ N/mm^2$).

We know that

$$\theta = \frac{l}{R} \quad or \quad l = \theta.R = \frac{\pi}{3} R \qquad (6-30)$$

(Assuming:$\theta = 60° = \pi / 3\ rad$)

$$\therefore \ Area\ of\ contact\ of\ the\ shoe = l\ .b \qquad (6-31)$$

and

$$the\ force\ with\ which\ the\ shoe\ presses\ against\ the\ rim = A \times p = l.b.p$$

Because running causes the shoe's rim to be pressed against with such power:

$(Pc - Ps)$,

therefore

$$l.b.p = Pc - Ps \qquad (6-32)$$

From this expression, the width of shoe (b) may be obtained.

3. Dimensions of the spring

As previously stated, the load on the spring is given by the following formula:

$$Ps = \frac{9}{16} m.\omega^2.r \qquad (6-33)$$

The dimensions of the spring can be obtained in the usual manner.

6 - 9. Solve examples

Example 1

Amulti - leaf friction clutch ($n = 7$), has outer diameter ($d_{outer} = 200\ mm$), inner diameter ($d_{inner} = 110\ mm$). The coefficient of the friction ($\mu = 0.3$), axial force ($F = 800\ N$) at speed ($N = 2300\ rpm$). Find maximum power transmitted to work the clutch distributions cases:

1. Uniform Pressure Theory
2. Uniform Wear Theory

Solution:

Given

$$[n = 7, N = 2300\,rpm, r_1 = 0.1\,m, \quad r_2 = 0.055\,m, \mu = 0.3, F = 800\,N].$$

1. **Uniform Pressure Theory**

$$T = n.\mu.F.R$$

$$R = \frac{2}{3}\left[\frac{{r_1}^3 - {r_2}^3}{{r_1}^2 - {r_2}^2}\right] = \frac{2}{3}\left[\frac{0.1^3 - 0.055^3}{0.1^2 - 0.055^2}\right] = 0.079677\,m$$

$$T = n.\mu.F.R = 7 \times 0.3 \times 800 \times 0.079677 = 133.858\,N.m$$

$$Power = \frac{2\pi NT}{60} = \frac{2 \times 3.14 \times 2300 \times 133.858}{60} = 32224\ Watt$$

2. **Uniform Wear Theory**

$$T = n.\mu.F.R$$

$$R = \frac{r_1 + r_2}{2} = \frac{0.1 + 0.055}{2} = \frac{0.155}{2} = 0.05775\,m$$

$$T = n.\mu.F.R = 7 \times 0.3 \times 800 \times 0.05775 = 130.2\,N.m$$

$$Power = \frac{2\pi NT}{60} = \frac{2 \times 3.14 \times 2300 \times 130.2}{60} = 31343\ Watt$$

Example 2

A multiple disc clutch has nine plates having eight pairs of active friction surfaces. If the intensity of pressure is not to exceed $0.233\, N/mm^2$, find the power transmitted at $750\, r.p.m.$ The outer and inner diameter of friction surfaces are $300\, mm$ & $200\, mm$ respectively. Assume uniform wear and take coefficient of friction $\mu = 0.3$.

Solution:

Given

$$n_1 + n_2 = 9 , n = 8 , p = 0.233 \frac{N}{mm^2} , N = 750\, r.p.m.,$$

$$r_1 = 150\, mm , r_2 = 100\, mm , \mu = 0.3$$

We know that for uniform wear, pressure is maximum at the inner radius (r_2), therefore,

$$\because \; p = \frac{C}{r} \qquad \Rightarrow \qquad C = p.r_2 = 0.233 \times 100 = 23.3\, N/mm$$

$$\because \; C = \frac{F}{2\pi[r_1 - r_2]}$$

$$\therefore \; F = 2\pi[r_1 - r_2].C = 2 \times 3.14 \times (150 - 100) \times 23.3 = 7316.2\, N$$

Assum Uniform Wear Theory

$$T = n.\mu.F.R$$

$$R = \frac{r_1 + r_2}{2} = \frac{150 + 100}{2} = \frac{250}{2} = 125\, mm$$

$$\therefore T = n.\mu.F.R = 8 \times 0.3 \times 17316.2 \times 125 = 2194860\, N.mm = 2194.86\, N.m$$

$$Power = \frac{2\pi NT}{60} = \frac{2 \times 3.14 \times 750 \times 2194.86}{60} = 172296.51\, Watt \approx 172.296\, KW$$

Example 3

A cone clutch built inside the flywheel is fitted to an engine producing ($88\, kW$) at ($1300\, rpm$). The cone has a (12.5°) face angle and a maximum mean diameter of ($660\, mm$). ($\mu = 0.3$) is the coefficient of friction. The normal pressure on the clutch face should not be greater than (0.3 N/mm^2).Find the required face width and the axial spring force to engage the clutch.

Solution:

Given $P = 88\,kW = 88\,000\,W\,, N = 1300\,r.p.m.\,, \alpha = 12.5°\,,$

$D = 660\,mm\ or\ R = 330\,mm\,, \quad \mu = 0.2\,, \quad p_n = 0.3\,N/mm^2$

1. **Face width**

   Let

$$b = Face\ width\ of\ the\ clutch\ in\ mm.$$

Torque developed by the clutch , it is calculated as follows:

$$Power = \frac{2\pi NT}{60} \quad \Rightarrow \quad T = \frac{P \times 60}{2\pi N} = \frac{88000 \times 60}{2 \times 3.14 \times 1300} \approx 646.742\,N.m$$

$$= 646742\,N.mm$$

$$\because T = 2\pi\,\mu.p_n R^2.b \qquad \Rightarrow \qquad \therefore b = \frac{T}{2\pi\,\mu.p_n R^2}$$

$$b = \frac{T}{2\pi\,\mu.p_n R^2} = \frac{\mathbf{646742}}{2 \times 3.14 \times 0.3 \times 0.3 \times 330^2} \approx 10.51 \approx 11\,mm$$

2. Axial spring force necessary to engage the clutch

Normal force acting on the contact surfaces , it is calculated as follows:

$$F_n = p_n \times 2\pi R.b = 0.3 \times 2 \times 3.14 \times 330 \times 11 = 6838.92\,N$$

So that, axial spring force necessary to engage the clutch , it is calculated as follows:

$$F_e = F_n\,(sin\,\alpha + 0.25\,\mu\,cos\,\alpha) = 6838.92\,(sin\,12.5° + 0.25 \times 0.3\,cos\,12.5°)$$

$$= 6838.92\,(0.216 + 0.075 \times 0.976) \approx 1977.97\,N$$

Example 4

A centrifugal clutch is to be designed to transmit $(55\,kW)$ at $(1300\ rpm)$. The shoes are six in number. The speed at which the engagement begins is $\left(\frac{3}{5th}\right)$ of the running speed. The inside radius of the pulley rim is $(180\,mm)$. The shoes are lined with Ferrodo for which the coefficient of friction may be taken as $(\mu = 0.3)$. Determine:

1. mass of the shoes, and

2. size of the shoes. Solution.

<u>Solution:</u>

Given : $P = 55\,kW = 55000\,W\,, N = 1000\,rpm\,, n = 6\,,$

$$R = 180\,mm = 0.15\,m\,, \mu = 0.3$$

1. Mass of the shoes

Let

$$m = Mass\ of\ the\ shoes.$$

The angular running speed, it is calculated as follows:

$$\omega = \frac{2\pi N}{60} = \frac{2 \times 3.14 \times 1000}{60} \approx 104.667\ rad/s$$

Since the speed at which the engagement begins is $(\frac{3}{5}th)$ of the running speed, therefore angular speed at which engagement begins , it is calculated as follows:

$$\omega_1 = \frac{3}{5}\omega = \frac{3}{5} \times 104.667 = 63.8\ rad/s$$

Assuming that the centre of gravity of the shoe lies at a distance of $150\,mm\ (30\,mm\ less\ than\ R)$ from the centre of the spider, i.e.

$$r = 150\,mm = 0.15\,m$$

The centrifugal force acting on each shoe, it is calculated as follows:

$$Pc = m.\omega^2.r = m \times 104.667^2 \times 0.15 \quad \Rightarrow \quad \therefore\ Pc = 1643.277\,m \ldots (1)$$

and the inward force on each shoe exerted by the spring i.e. the centrifugal force at the engagement speed, ω_1,

$$Ps = m.(\omega_1)^2.r = m \times 63.8^2 \times 0.15 \quad \Rightarrow \therefore\ Ps = 610.566\,m \ldots (2)$$

The torque transmitted at the running speed , it is calculated as follows:

$$T = \frac{P \times 60}{2\pi N} = \frac{55000 \times 60}{2 \times 3.14 \times 1000} \approx 525.478\,N.m$$

The torque transmitted (T), it is calculated as follows:

$$T = \mu\,(P_c - P_s\,)\,R \times n$$

$$525.478 = 0.3\,(1643.277\,m - 610.566\,m) \times 0.18 \times 6$$

$$525.478 = 0.3\,(1032.711\,m) \times 0.18 \times 6$$

$$525.478 = 334.598\,m$$

$$m = \frac{525.478}{395.91} \approx 1.57\,Kg$$

2. Size of the shoes

Let

$$l = Contact\ length\ of\ shoes\ in\ mm, and$$

$b = Width\ of\ the\ shoes\ in\ mm.$

Assuming that the arc of contact of the shoes subtend an angle of

($\theta = 60°\ or\ \frac{\pi}{3}\ radians$), at the centre of the spider, therefore

$$l = \theta . R = \frac{\pi}{3} \times 180 = 188.4\,mm$$

Area of contact of the shoes,

$$A = l.b = 188.4\,b$$

Assuming that the pressure (p) applied to the shoes is (0.1 N/mm^2), the force with which the shoe presses against the rim is:

$$F = A.p = 188.4\,b \times 0.1 = 18.84\ b\,N \qquad (1)$$

The force with which the shoe presses against the rim, it is calculated as follows:

$$F = Pc - Ps = 1643.277\,m - 610.566\,m = 1032.711\ m$$

$$= 1032.711 \times 1.57 = 1621.356\,N \qquad (2)$$

From equations (1) and (2), we find that

$$b = \frac{Pc - Ps}{A.p} = \frac{1621.356}{18.84} = 86.06\,mm$$

6-10 Chapter Questions

1. The type of clutch used in trucks is ------- clitch.

- **a. multi-plate clutch**
- b. centrifugal clutch
- c. cone clutch
- d. single plate clutch

2. The ———— operates automatically based on engine speed.

- a. cone clutch
- b. multi-plate clutch
- c. single plate clutch
- **d. centrifugal clutch**

3. The clutch's friction material should have

- a. low coefficient friction
- b. high endurance limit strength
- **c. high coefficient friction**
- d. surface hardness that is high

4. Cone clutches have become obsolete as a result of

- a. difficult to disengage
- b. easy to disengage
- c. difficult construction
- **d. strict coaxiality requirement for two shafts**

5. At the start of a centrifugal clutch engagement,

- a. the spring force are greater or less than the centrifugal force on the shoe
- b. the spring force is greater than the centrifugal force on the shoe.
- **c. the centrifugal force exerted by the shoe is slightly greater than the spring force.**
- d. the spring force is equal the centrifugal force on the shoe.
- e. .

6. The net force acting on the drum when the centrifugal clutch is operating is equal to:

- **a. less the centrifugal force acting on the shoe from the spring.**
- b. the combined spring and centrifugal forces on the shoe.
- c. the force of a spring.
- d. a shoe's reaction to centrifugal force.

7. Oil is used in the case of multi-disk clutches.

- a. To remove the heat.
- **b. To remove the heat, to lessen friction, and lubricate the surfaces in contact.**
- c. To lessen friction.
- d. To lubricate the surfaces in contact.

8. The clutch's ability to transmit torque depends on:

- **a. Friction lining dimensions, axial force required to engage the clutch, and coefficient of friction.**
- b. friction lining dimensions.
- c. friction coefficient.
- d. The axial force used to engage the clutch.

9. As opposed to friction moment under uniform pressure, the friction moment in a clutch with uniform wear is:

- a. more
- b. more or less depending on speed
- **c. Less**
- d. equal

10. When a new clutch is compared to an old clutch, the friction radius will be:

- **a. more**
- b. depending on clutch size, more or less
- c. Less
- d. equal

11. In the case of a cone clutch, a relatively tiny axial force can transmit a specific torque if the semi-cone angle is:

- a. more
- b. depending on clutch size, more or less
- **c. Less**
- d. equal

12. Are used in units where shaft misalignment and angle between shafts must be compensated for

- a. Dog and spline clutch
- **b. Diaphragm Clutch**
- c. Electromagnetic Clutch
- d. Vacuum clutch

13. This type of clutches uses the existing vacuum in the engine manifold to operate the clutch.

- a. Dog and spline Clutch
- b. Diaphragm clutch
- c. Electromagnetic Clutch
- **d. Vacuum clutch**

14. It is made up of fewer parts than other clutches. It has an accumulator, a control valve, a cylinder with a piston, a pump, and a reservoir.

- a. Electromagnetic clutch
- b. Dog and spline Clutch
- c. Diaphragm Clutch
- **d. Vacuum Hydraulic clutch**

15. A multiple disc clutch has five plates having four pairs of active friction surfaces. If the intensity of pressure is not to exceed ($p = 0.127\ N/mm^2$), find the power transmitted at ($N = 500\ r.p.m.$) The outer and inner dimeter of friction surfaces are ($d_o = 250\ mm$ & $d_i = 150\ mm$) respectively. Assume uniform wear and consider the coefficient of friction. ($\mu =\ 0.3$).

$\boldsymbol{Ans}$: $\boldsymbol{P\ =\ 18.8\ Kw}$

16. To transmit, a single plate clutch that is effective on both sides is required.($P = 25\ kW$) at ($N = 3000\ r.p.m$). Determine the outer and inner diameters of frictional surface if the coefficient of friction is ($\mu = 0.255$), ratio of diameters is($\frac{d_O}{d_I} = 1.25$) and the maximum pressure is not to exceed ($P = 0.1 \frac{N}{mm^2}$) . Also, determine the axial thrust to be provided by springs. Assume the theory of uniform wear.

$\boldsymbol{Ans}$: $\boldsymbol{r_i\ =\ 96\ mm}$ & $\boldsymbol{r_o\ =\ 120\ mm}$ & $\boldsymbol{F\ =\ 1447\ N}$

17. Three discs on the driving shaft and two on the driven shaft comprise a multi-disc clutch. The contact surface's inner diameter is ($d_i = 120\ mm$). The maximum pressure between the surface is limited to ($p = 0.1\ N/mm^2$). Find the outer diameter for transmitting ($P = 25\ kW$) at ($N = 1575\ r.p.m$). Assume that the wear condition is uniform and that the coefficient of friction is ($\mu =\ 0.3$).

$\boldsymbol{Ans}$: $\boldsymbol{r_o\ =\ 158\ mm}$

18. An engine developing $P = 45\ kW\ at\ N = 1000\ r.p.m.$ is fitted with a cone clutch built inside the flywheel. The cone has an angle of($\theta = 25°$ and an outside diameter of ($d_o = 400\ mm$). The coefficient of friction is ($\mu = 0.2$). The normal pressure on the clutch face is not to exceed ($p = 0.1 \frac{N}{mm^2}$) . Establish the clutch engagement face width and axial spring force requirements.

$\boldsymbol{Ans}$: $\boldsymbol{b\ =\ 90\ mm}$ & $\boldsymbol{F\ =\ 2515\ N}$

19. A centrifugal clutch is to transmit ($P = 15\ kW\ at\ N = 900\ r.p.m$). The shoes are four in number. The speed at which the engagement begins is ($\omega_1 = 3/4th$) of the running speed. The inside dimeter of the pulley rim is ($d_i = 300\ mm$) and the center of gravity of the shoe lies at ($R = 120\ mm$) starting from the spider's middle. Ferro do is used to lining the shoes, and its coefficient of friction is ($\mu = 0.25$). Determine:

1. Mass of the shoes, and
2. Size of the shoes, if angle subtended by the shoes at the center of the spider is (60º) and the pressure exerted on the shoes is ($p = 0.1\ N/mm^2$).

[$\boldsymbol{Ans}$: $\boldsymbol{m = 2.27\ kg}$, $\boldsymbol{l\ =\ 157.1\ mm}$, $\boldsymbol{b\ =\ 67.1\ mm}$]

20. The interior cylindrical surface of a rim keyed to the driven shaft is in touch with the four shoes of a centrifugal clutch, which glide radially in a spider keyed to the driving shaft. Each shoe in the clutch is pulled against a stop when the clutch is at rest by a spring, leaving a radial space of ($c = 5\ mm$) between the shoe and the rim. The spring's pull is then equal to ($S = 500\ N$). The distance between the clutch's axis and the shoe's mass center is ($r = 160\ mm$). Find the power transmitted by the clutch at a given speed if the internal diameter of the rim is ($400\ mm$), the mass of each shoe is ($m = 8\ kg$), each spring is stiff at ($s = 50\ N/mm$), and the coefficient of friction between the shoe and the rim is ($\mu = 0.3$) ; find the power transmitted by the clutch at ($N = 1440\ r.p.m$). [$\boldsymbol{Ans}$: $\boldsymbol{P = 36.1\ Kw}$]

Chapter 7

Types of springs

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7. Types of springs

7-1. Introduction

A spring is characterized as an elastic body that can store mechanical energy, deforms under load, and straightens out after the load is lifted. When a spring is loaded, it deforms, then when the load is removed, it takes on its original shape.

7-2. The various applications of springs

Spring applications include the following:

1. To cushion, absorb, or control energy caused by shock and vibration, as in bicycle or automobile springs, railway buffers, shock absorbers, aircraft landing gear, and vibration dampers,
2. To exert force, such as in brakes, clutches, and spring-loaded valves,
3. To control motion by keeping two elements in contact, as in cams and followers,
4. Force measurement, as in spring balances and engine indicators,
5. Energy storage, as in toys and watches.

7-3. Types of springs

Figure (7-1) shows types of springs.

Figure 7-1: Types of springs

7-3-1. On the basis of shape

Following are the spring types according to shape:

7-3-1-1. Helical Springs (Coil Springs)

It is a spring made of coiled wire in the shape of a helix. It is designed to withstand tensile and compressive loads, as illustrated in the figure (7-2).

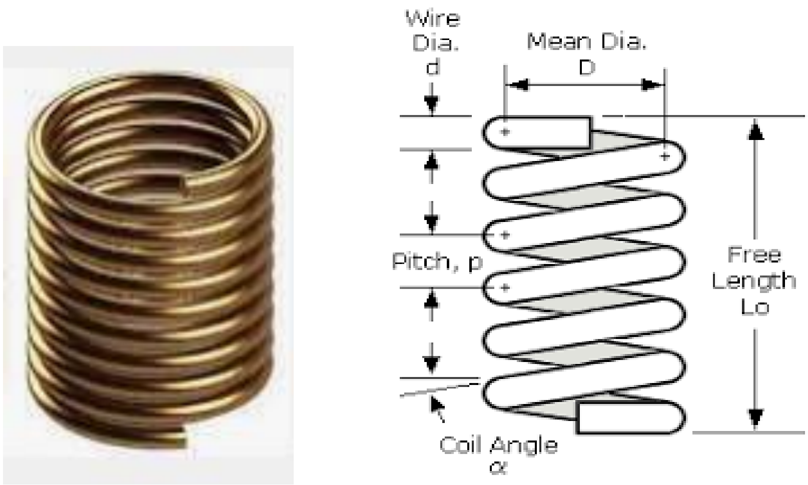


Figure 7-2: Helical springs or Coil Springs

7-3-1-2. Conical and Volute Springs

The compression springs have conical shapes. Conical springs have a uniform pitch, whereas volute springs have a paraboloid shape with constant pitch and lead angles. When compressed, the coils of these springs slide past each other, causing the spring to compress to a very short length, as illustrated in the figure (7-3).

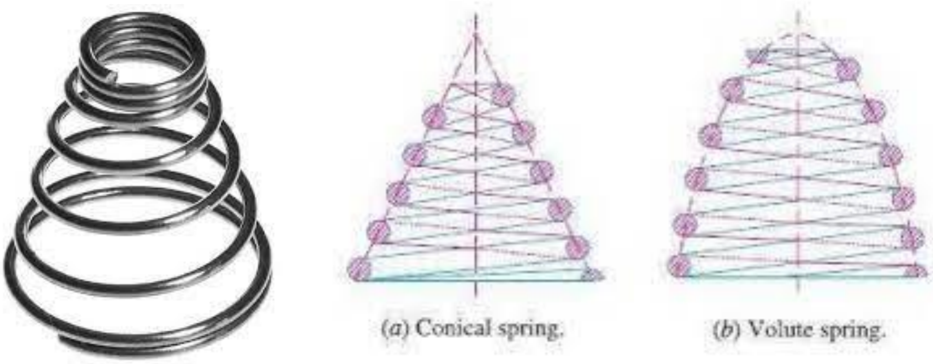


Figure 7-3: Conical and Volute Springs

7-3-1-3. Torsion Springs

It is a torsion or twisting spring. When twisted, it stores mechanical energy, as illustrated in the figure (7-4).

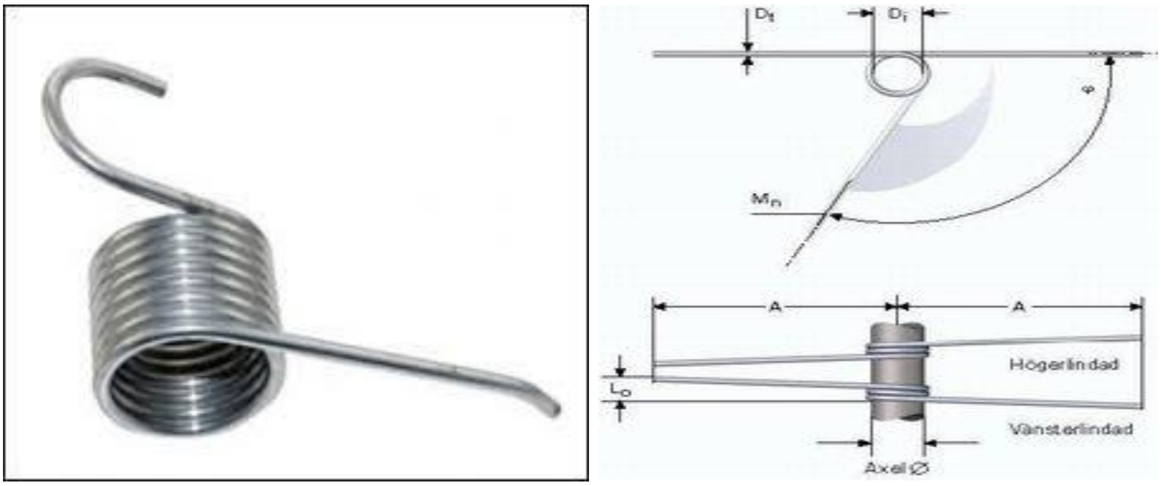


Figure 7-4: Torsion Springs

7-3-1-4. Laminated spring (Leaf Spring)

It is a type of spring that is commonly used in vehicle suspension, electrical switches, and bows. It is made up of a series of flat plates (known as leaves) of varying lengths that are held together with clamps and bolts, as illustrated in the figure (7-5).

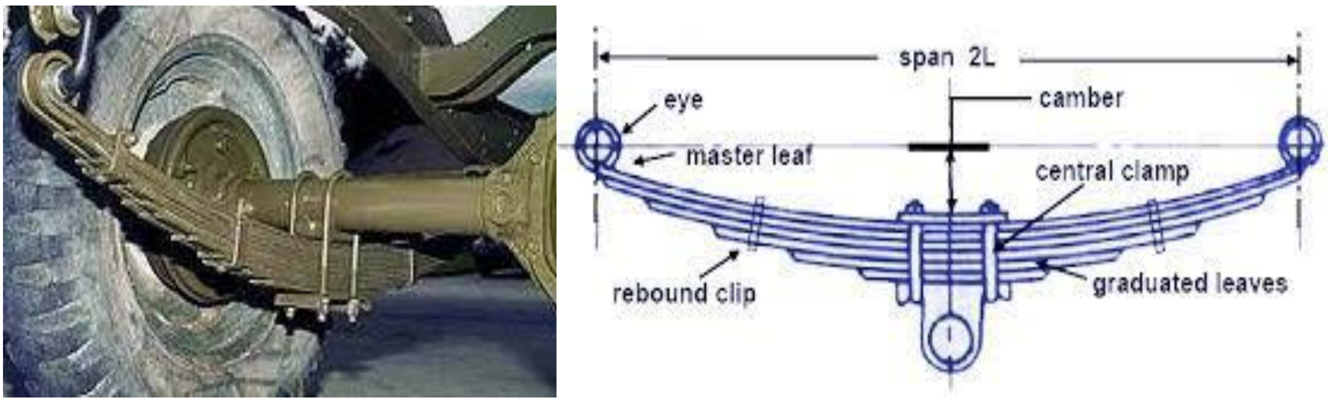


Figure 7-5: Laminated or Leaf Springs

7-3-1-5. Disc or Belleville Springs

It is a spring in the shape of a disc. It is commonly used to tighten a bolt. Belleville washers and conical compression washers are other names for it. as illustrated in the figure (7-6).

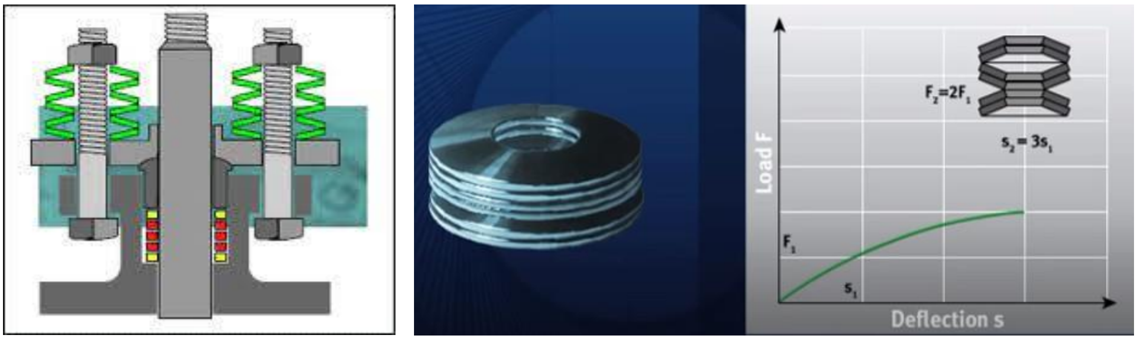


Figure 7-6: Disc or Belleville Springs

7-3-2. Spring tension varies depending on how the load force is applied

Springs are divided into the following categories based on how the load force is applied:

7-3-2-1. Tension spring (Extension spring)

Tension or extension springs work by applying tension loads. When a tensile load is applied to this spring, it stretches to a certain length, as illustrated in the figure (7-7).

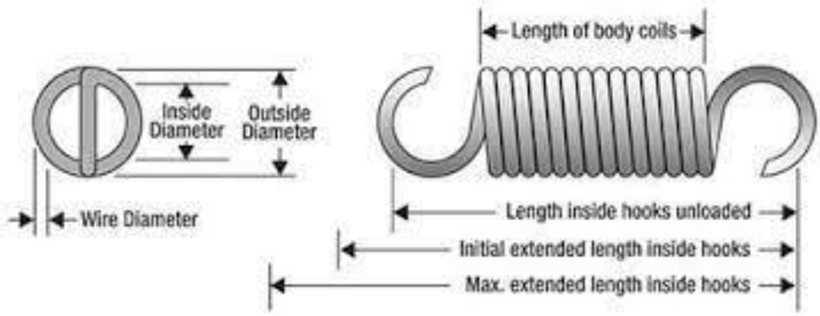


Figure 7-7: Tension or Extension Spring

7-3-2-2. Compression Spring

Compression springs are intended to function when a compressive load is applied to them. It shrinks when compressed, as illustrated in the (7-8).

Figure 7-8: Compression Spring

7-3-2-3. Torsion Spring

It is design to operate while being twisted. It can be twisted to store mechanical energy, as illustrated in the (7-9).

Figure 7-9: Torsion Spring

7-3-2-4. Constant Spring

It is a particular kind of spring where the supported load stays constant throughout the deflection cycle, as illustrated in the (7-10).

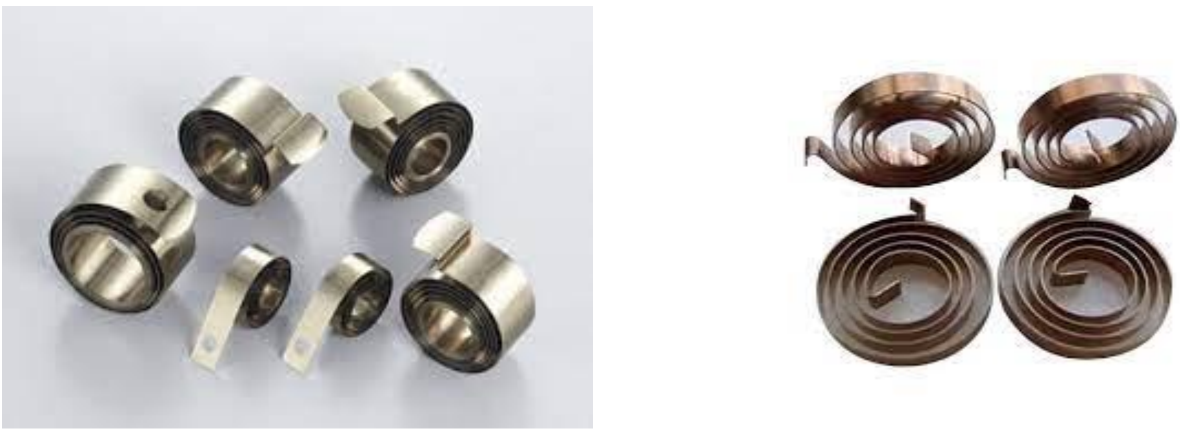


Figure 7-10: Constant Spring

7-3-2-5. Variable Spring

A variable spring is one that adjusts its coil's resistance to load during compression, as illustrated in the (7-11).

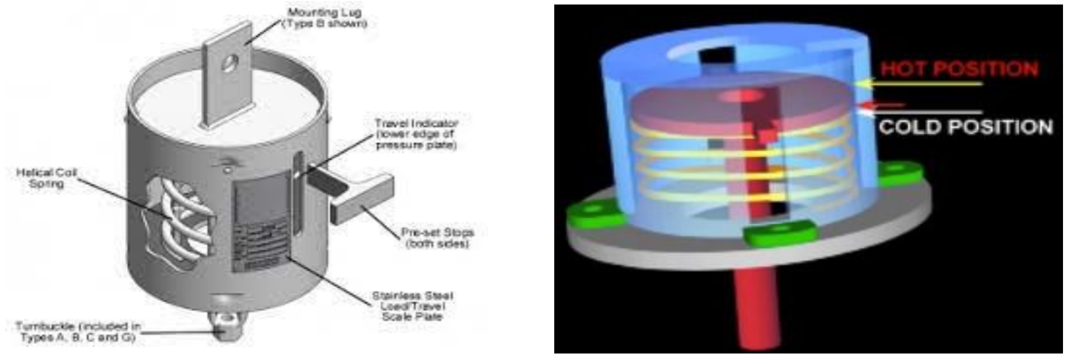


Figure 11: Variable Spring

7-4. Design of the springs

7-4-1. Design of Helical spring

The helical spring's design has three goals in mind. They are listed as follows:

It should have the following qualities:

1. It should be strong enough to support the external load,
2. It should have the following qualities,
3. It should be strong enough to support the external load.

The main dimension of helical spring subjected to compressive force as shown in figure (7-12). They are as follows:

$$d = Wire\ diameter\ of\ spring\ (mm)$$

$$D_i = Inside\ diameter\ of\ spring\ coil\ (mm)$$

$$D_o = Outside\ diameter\ of\ spring\ coil\ (mm)$$

$$D = Mean\ coil\ diameter\ (mm).$$

Therefore

$$D = \frac{D_i + D_o}{2} \qquad (7-1)$$

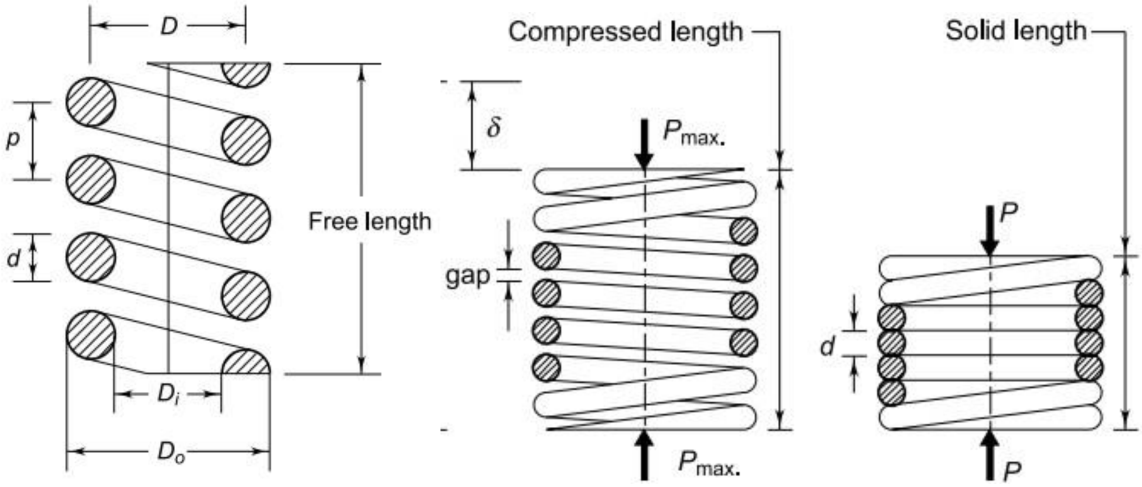


Figure 7-12: Dimension of helical spring

In the spring design, the following primary dimensions must be calculated:

1. Wire diameter (*d*),
2. Mean coil diameter (*D*),
3. Number of active coils (*N*),
4. The load stress equation is used to determine the first two, while the load deflection equation is used to calculate the third.

The force necessary to cause unit delfection is what determines the spring's stiffness (k). therfore,

$$k = \frac{P}{\delta} \qquad (7-2)$$

Where

$$\delta = Axial\ deflection\ of\ the\ spring\ (mm),$$

$$P = Axial\ spring\ force\ (N),$$

$$k = Stiffness\ of\ spring\ (\frac{N}{mm})$$

The loaddeflection equation can be applied easily. The following shear stress ($\boldsymbol{\tau}$) equation yields a useful load stress equation that incorporates the spring index as a component.

$$\tau = K.\left\{\frac{8\ P_{Max.}D}{\pi d^3}\right\} \qquad (7-3)$$

Where :

$$K = Stress\ factor\ or\ wahl\ factor, \qquad P = Maximum\ axial\ force$$

$$K = \frac{4\ C-1}{4\ C-4} + \frac{0.615}{C} \qquad (7-4)$$

$C = The\ spring\ index$

1. To calculate wire diameter(d), we use the followng equation:

$$\therefore \tau = K.\left\{\frac{8\ P_{Max.}C}{\pi d^2}\right\} \rightarrow\ d = \sqrt{\frac{8\ P\ C\ K}{\pi\ \tau}} \qquad (7-5)$$

2. To calculate mean wire diameter(D), we use the followng equation:

$$D = C\ .d \qquad (7-6)$$

3. To calculate the number of active coils (N), we use the followng equation:

$$\delta = \frac{8\ PD^3N}{G\ d^4} \quad \rightarrow \quad N = \frac{\delta\ G\ d^4}{8\ PD^3} \qquad (7-7)$$

$G = Modulus\ of\ rigidity$ (N/mm^2).

4. To calculate the total number of coils, we use the following relatioship:

$$N_{Total} = N + 2 \qquad (7-8)$$

5. To calculate the solid length of the spring, we use the equation:

$$L_S = N_{Total}\ .d \qquad (7-9)$$

6. Using the connection below, one may determine the spring's free length:

$$Free\ length\ spring\ (L_F) = Soild\ length\ spring\ (L_S) + Total\ axial\ gap + \delta$$

$$Total\ axial\ gap = (N_T - 1) \times gap\ between\ two\ adjacent\ coils$$

7. To calculate the pitch of coil (p), by using the equation:

$$Pitch\ of\ coils\ (p) = \frac{L_F}{(N_T - 1)} \qquad (7-10)$$

8. To calculate the required spring rate (R), by using the equation:

$$R = \frac{P_{Max.} - P_{Min.}}{\delta} \qquad (7-11)$$

9. To calculate the actual spring rate (R_a), by using the following relationship:

$$R_a = \frac{G d^4}{8\ N\ D^3} \qquad (7-12)$$

Example 1:

A compression spring constructed of circular wire and an oil-hardened and tempered steel helical spring are exposed to axial forces that range from (4 KN) to (6 KN). The deflection of the spring should be about during this range of force (33.204 mm). The spring index is interpreted as (4). If the spring's ultimate tensile strength is (850.9 N/mm^2) and its rigidity modulus is (81330 N/mm^2), it has square and ground ends. The spring wire's allowable shear stress should be calculated as (33%) of its maximum tensile strength.

Design the spring and perform the following calculations.

1. Wire diameter (d),
2. Mean coil diameter (D),
3. Number of active coils (N),
4. Total number of coils (N_{Total})
5. Solid length of the spring (L_S),
6. Free length of the spring (L_F),
7. Pitch of coils (p),

8. Required spring rate (R_r),
9. Actual spring rate (R_a), and
10. Draw a neat sketch of the spring showing various dimensions.

Solution

Given

$$[P_{Min.} = 4000\ \mathrm{N}, P_{Max.} = 6000\ \mathrm{N}, \delta = 8\ \mathrm{mm}, C = 4, S_{ut} = 850.9 \frac{\mathrm{N}}{\mathrm{mm}^2},$$

$$G = 81330 \frac{\mathrm{N}}{\mathrm{mm}^2}, \tau = 0.33\ S_{ut}, P = P_{Max.} - P_{Min.} = 6000 - 4000 = 2000\ \mathrm{N}]$$

1. Wire diameter (d)

$$\tau = 0.33\ S_{ut} = 0.33 \times 850.9 = 280.8\ \mathrm{N/mm^2}$$

$$\because K = \frac{4\ C - 1}{4\ C - 4} + \frac{0.615}{C} = \frac{(4 \times 4) - 1}{(4 \times 4) - 4} + \frac{0.615}{7} = \frac{15}{12} + 0.088 = 1.338$$

Also, from equation

$$\tau = K. \left\{\frac{8\ P_{Max.} C}{\pi d^2}\right\} \rightarrow \ 445.5 = 1.338 \left\{\frac{8 \times 6000 \times 4}{3.14 \times d^2}\right\}$$

$$d^2 = \frac{1.338 \times 8 \times 6000 \times 4}{3.14 \times 280.8} = \frac{256896}{881.712} = 291.36$$

$$\therefore d = \sqrt{291.355} = 17.069 \ \approx 18\ \ \mathrm{mm}$$

2. Mean coil diameter (D)

$$D = C \times d = 4 \times 18 = 72\ \ \mathrm{mm}$$

3. Number of active coils (N)

$$\delta = \frac{8\ PD^3 N}{G\ d^4}$$

$$\therefore N = \frac{\delta\, G\, d^4}{8\, P\, D^3} = \frac{8 \times 81330 \times 18^4}{8 \times 2000 \times 72^3} = \frac{68301584640}{5971968000} = 11.44 \;\approx 12 \text{ Coils}$$

4.Total number of coils (N_{Total})

The square and ground ends, the number of inactive Coils is two, therefore,

$$N_{Total} = N + 2 = 12 + 2 = 14 \text{ Coils}$$

5. Solid length of the spring (L_S)

$$L_S = N_{Total} \,.\, d = 14 \times 18 = 252 \text{ mm}$$

6. Free length of the spring (L_F) and pitch of coils (p)

The actual deflection of the spring under the maximum force is given by:

$$\delta = \frac{8\, P_{Max.} D^3 N}{G\, d^4} = \frac{8 \times 6000 \times 72^3 \times 12}{81330 \times 18^4} = \frac{214990848000}{8537698080} = 25.18 \text{ mm}$$

$$\text{Total axial gap} = (N_{Total} - 1) \times 0.5 = (12 - 1) \times 0.5 = 5.5 \text{ mm}$$

$$\text{Free length spring } (L_F) = \text{Soild length spring } (L_S) + \text{Total axial gap} + \delta$$

$$\therefore L_F = 252 + 5.5 + 25.18 = 282.68 \;\approx 283 \text{ mm}$$

7. pitch of coils (p)

$$\text{Pitch of coils (p)} = \frac{L_F}{(N_T - 1)} = \frac{283}{(14 - 1)} = 21.77 \text{ mm}$$

8. Required spring rate (R_r)

$$R = \frac{P_{Max.} - P_{Min.}}{\delta} = \frac{6000 - 4000}{8} = 250 \text{ N/mm}$$

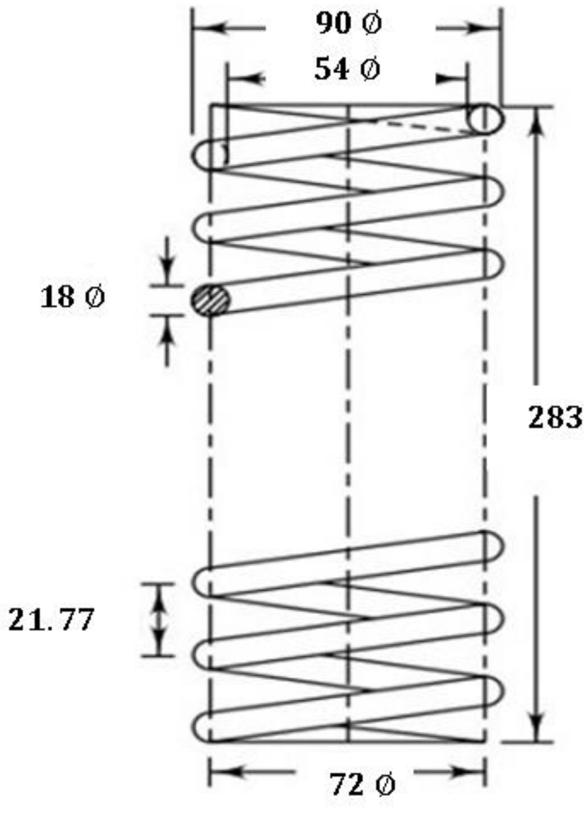
9. Actual spring rate (R_a)

$$R_a = \frac{G d^4}{8\, N\, D^3} = \frac{81330 \times 18^4}{8 \times 12 \times 72^3} = \frac{8537698080}{35831808} = 238.27 \text{ N/mm}$$

10. Draw a neat sketch of the spring showing various dimensions

$$\because \mathrm{D} = \frac{\mathrm{D_i} + \mathrm{D_o}}{2} \ , \mathrm{D_o} = \mathrm{D} + \mathrm{d} \ , \mathrm{D_i} = \mathrm{D} - \mathrm{d}$$

$$\mathrm{D_o} = \mathrm{D} + \mathrm{d} = 72 + 18 = 90 \text{ mm} \ , \mathrm{D_i} = \mathrm{D} - \mathrm{d} = 72 - 18 = 54 \text{ mm}$$



All dimension in mm

7-4-2. Design Leaf spring

Leaf springs are subdivided into longitudinal and transverse leaf springs. Longitudinal leaf springs are used only on rigid axles, more commonly on commercial vehicles and trailers. Figure (7-13).

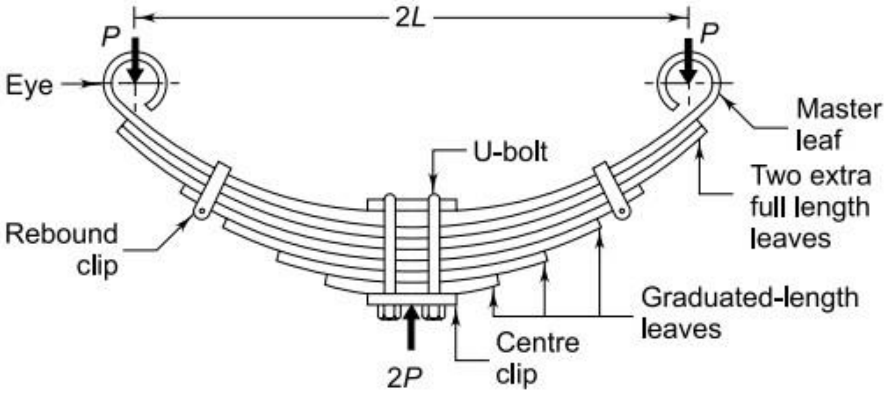


Figure 7-13: Semi elliptic Leaf Spring

Were,

$$n_f = Number\ of\ extra\ full\ length\ leaves,$$

$$n_g = Number\ of\ graduated\ length\ leaves\ including\ master\ leaf,$$

$$n = Total\ number\ of\ leaves,$$

$$b = Width\ of\ each\ leaf\ (mm),$$

$$t = Thickness\ of\ each\ leaf\ (mm),$$

$$L = Length\ of\ the\ cantilever\ or\ the\ half\ the\ length\ of\ semi$$
$$-\ ellitic\ spring\ (mm),$$

$$P = Force\ applied\ at\ the\ end\ of\ the\ spring\ (N),$$

$$P_f = Portion\ of\ (P) taken\ by\ the\ extra\ full - length\ leaves\ (\ N),$$

$$P_g = Portion\ of\ (P) taken\ by\ the\ graduated\ length\ leaves\ (N),$$

$$P_i = Initial\ pre - load,$$

$$\sigma_b = Bending\ stress\ in\ the\ plate\ ,$$

$$\delta_g = Delection\ of\ graduated\ lenth\ leaves,$$

$$\delta_f = Delection\ of\ full\ length\ leaves.$$

1- To calculate the width and thickness of the leaves (b):

The stress is equal in all leaves

$$\because \sigma_b = \frac{6PL}{nbt^2}\ , b = 9t$$

$$\therefore t^3 = \frac{6PL}{9n\sigma_b}$$

$$b = 9t \qquad (6-13)$$

2- To calculate initial nip (C)

$$\because \delta_g = \frac{6P_gL^3}{En_gbt^3}\ , \qquad \delta_f = \frac{4P_fL^3}{En_fbt^3}$$

$$C = \delta_g + \delta_f = \frac{6P_gL^3}{En_gbt^3} - \frac{4P_fL^3}{En_fbt^3}$$

Also, we know

$$P_g = \frac{n_g\ P}{n} \& P_f = \frac{n_f\ P}{n} \& \quad n = n_g + n_f$$

$$\therefore C = \frac{2PL^3}{Enbt^3} \qquad (6-14)$$

3- To calculate initial pre- load (P_i)

Under the action of pre-load, we know

$$C = (\delta_g)_i + (\delta_f)_i$$

$$\frac{2PL^3}{Enbt^3} = \frac{6P_gL^3}{En_gbt^3} + \frac{4P_fL^3}{En_fbt^3}$$

$$\therefore P_i = \frac{2n_gn_fP}{n(3n_f + 2n_g)} \qquad (6-15)$$

Example 2:

A semi-elliptic leaf spring used for automobile suspension consists of (4) extra full-length leaves and (18) graduated length leaves, including the master leaf. The center-to-center distance between two eyes of the spring (1.3 m). The spring can withstand a maximum force of (66000 N). The proportion of each leaf's breadth to thickness is (7:1). The leaf material has an elastic modulus of (330000 N/mm^2). The leaves are pre-stressed so that, when the force is greatest, all of the leaves experience stresses equal to and up to (330 N/mm^2). Find out the following:

1- The thickness and width of the leaves,
2- Initial nip, and
3- The first pre-load needed to bridge the gap (C) between extra full-length leaves and graduated length leaves.

<u>Solution</u>

Given: $[2P\ =\ 66000\,N \rightarrow P = 33000\,N, 2L =\ \ 1300\,mm \rightarrow L = 650\,mm\,,$

$$b = 9t\ \ \rightarrow, n_f = 4, n_g = 18, E = 330000\ \frac{N}{mm^2}, \sigma_b = 330\ N/mm^2$$

1. *The width and thickness of the leaves (b)*

$$n = n_f + n_9 = 4 + 18 = 22$$

$$\because \sigma_b = \frac{6PL}{nbt^2}$$

$$\therefore t^2 = \frac{6PL}{nb\sigma_b} = \frac{6 \times 33000 \times 650}{22 \times 9t \times 330000}$$

$$\therefore t^3 = \frac{6 \times 33000 \times 650}{22 \times 9 \times 330} = \frac{128700000}{65340} = 1969.70$$

$$\therefore t = \sqrt[3]{1969.70} = 12.535 \approx 13\,mm$$

$$\therefore b = 9t = 9 \times 13 = 117\,mm$$

2. *The initial nip(C)*

$$C = \frac{2PL^3}{Enbt^3} = \frac{2 \times 33000 \times 650^3}{330000 \times 22 \times 117 \times 13^3} = \frac{18125000000000}{1866200000000} = 9.712\,mm$$

3. *The first pre-load necessary (Pi) to cover the gap (C) between additional full-length leaves and graduated length leaves.*

$$P_i = \frac{2n_g n_f P}{n(3n_f + 2n_g)} = \frac{2 \times 4 \times 18 \times 33000}{22(3 \times 18 - 2 \times 4)} = \frac{4752000}{1012} = 4695.65\,N$$

7-5. Chapter Questions

1. Springs are employed to:

- **a. force measurement, shock and vibration absorption, energy storage, and energy release.**
- b. energy storage and release.
- c. force measurement.
- d. absorb vibrations and shocks.

2. In automobiles, the following sort of spring is used to absorb shocks and vibrations:

- a. A spiral spring
- b. A helical extension springs.
- c. A Bellville springs.
- **d. A multiple-leaf spring.**

3. The kind of spring used in mechanical watches to store and release energy is:

- **a. A spiral spring.**
- b. A multi-leaf springs.
- c. A helical extension springs.
- d. A helical torsion springs.

4. Door hinge springs are of the following kind:

- a. A spiral spring.
- b. A multi-leaf springs.
- c. A helical extension springs.
- **d. A helical torsion springs.**

5. The kind of spring used in spring balance to measure weights is:

- a. A spiral spring.
- b. A multi-leaf springs.
- **c. A helical extension springs.**
- d. A helical torsion springs.

6. The valve mechanism uses the following sort kind of spring:

- a. A spiral spring.
- b. A multi-leaf springs.
- **c. A helical compression springs.**
- d. A helical torsion springs.

7. The kind of springs utilized in automotive clutches are:

- a. Spiral springs.
- b. Multi-leaf springs.
- **c. Belleville springs and helical compression springs.**
- d. Helical torsion springs.

8. The sort of stress that is created in the spring wire when the helical compression spring is subjected to an axial compressive force is:

a. **Torsional shear stress.**
b. tensile stress.
c. bending stress.
d. compressive stress.

9. The kind of stress created in the spring wire when the helical extension spring is subjected to axial tensile force is:

a. **Torsional shear stress.**
b. Tensile stress.
c. Bending stress.
d. Compressive stress.

10. In spring wire, the maximum shear stress is produced at:

a. Central surface of the coil.
b. Outer surface of the coil.
c. **Inner surface of the coil.**
d. End coils.

11. The kind of stress that is created in the spring wire when the helical torsion spring is torqued is:

a. Torsional shear stress.
b. Tensile stress.
c. **Bending stress.**
d. Compressive stress.

12. The multi-leaf spring leaves are subjected to:

a. Torsional shear stress.
b. Tensile stress.
c. **Bending stress.**
d. Compressive stress.

13. The spring works.

a. Within the viscoelastic range
b. Within the range of plastic
c. Beyond the limit of elastic
d. **Within an elastic region**

14. When a helical spring is cut in half, the stiffness of each half spring is as follows:

a. Half of the original spring.
b. Same as the original spring
c. **Double of the original spring.**
d. Quarter of the original spring.

15. The load shared by each spring when two concentric springs of the same material, similar free length, and equal axial compression are used is proportional to:

- **a. Each spring's square of wire diameter.**
- b. Each spring's index is given.
- c. Average coil diameter for each spring.
- d. The diameter of each spring's wire.

16. The purpose of a multi-leaf spring in an automobile is to:

- a. activate the mechanism
- **b. absorb shocks and vibrations**
- c. measure the force
- d. store and release energy

17. The spring's stiffness is:

- **a. force necessary to cause a unit deflection.**
- b. deflection per axial force unit.
- c. average coil diameter to wire diameter ratio.
- d. force per unit of a spring's cross-sectional area.

18. What is the spring index?

- **a. It is the force necessary to cause a unit deflection.**
- b. It is a ratio between the wire diameter and the mean coil diameter.
- c. It measures the proportion of wire diameter to mean coil diameter.
- d. It measures the spring's force per unit of cross-sectional area.

19. The spring's ends that come into contact with the seat are as follows:

- a. Transmit the most force possible.
- **b. Coils that are active.**
- c. Do not exert any force.
- d. Coils that are not in use.

20. A helical compression spring made of unique cold-drawn steel wire that can withstand a maximum force of (1250 N). The spring's deflection should be nearly equal to the maximum force (30 mm). The spring index is interpreted as (6). If the stiffness modulus is (81370 N/mm^2) and the ultimate tensile strength is (1090 N/mm^2). The spring wire's allowable shear stress should be calculated as (50 %) of its maximum tensile strength.

Design the spring and perform the following calculations.

1. Wire diameter (d),
2. Mean coil diameter (D),
3. Number of active coils (N),
4. Total number of coils (N_{Total})
5. Solid length of the spring (L_S),
6. Free length of the spring (L_F),
7. Pitch of coils (p),
8. Required spring rate (R_r),

9. Actual spring rate (R_a), and
10. Draw a neat sketch of the spring showing various dimensions.

21. A semi-elliptic leaf spring used for the suspension of the rear axle of a truck. It consists of (*2*) extra full-length leaves and (*10*) graduated length leaves, including the master leaf. The center-to-center distance between two eyes of the spring (*1.2 m*). The leaves are constructed of steel 55Si2Mo90 with a modulus of elasticity of (207 GPa), the ultimate strength equal to (S_{yt} = *1500 N/mm*^{2}), and a safety factor of (2.5). The spring must be constructed to withstand a maximum force of (30000 N). To equalize pressures across all leaves, the leaves are pre-stressed. Find out the following:

1. The cross section of leaves, and
2. The deflection at the end of the spring.

22. The dimensions of a compression coil spring constructed of alloy steel are as follows: mean coil diameter ($D = 50\ mm$), wire diameter ($d = 5\ mm$), and number of active coils ($N = 20$). Calculate the maximum shear stress that the spring material will withstand if it is subjected to an axial load of ($P_{Max.} = 500\ N$), neglect the curvature effect.

$[\boldsymbol{Ans}: \quad \boldsymbol{\tau = 534.7\ MPa}]$

23. Design a spring for a balance to measure $(0\ to\ 1000\ N)$ over a scale of length $(\delta = 80\ mm)$. The spring is to be enclosed in a casing of $(d = 25\ mm)$ diameter. The approximate number of turns is $(N = 30)$. The modulus of rigidity is $(G = 85\frac{kN}{mm^2})$. Also calculate the maximum shear stress induced.

$[\boldsymbol{Ans}: \quad \boldsymbol{D = C.d = 4.84 \times 4 = 19.36\ mm\ , Do = D + d = 19.36 + 4 = 23.36\ mm\ , \tau = 1018.2\ N/mm^2 = 1018.2\ MPa}]$

24. A truck spring has $(N = 12)$ number of leaves, two of which are full length leaves. The spring supports are $1.05\ m$ apart and the central band is $85\ mm$ wide. The central load is to be $5.4\ kN$ with a permissible stress of $280\ MPa$. Determine the thickness and width of the steel spring leaves. The ratio of the total depth to the width of the spring is 3. Also determine the deflection of the spring.

$[\boldsymbol{Ans}: \quad \boldsymbol{b = 40\ mm\ , \delta\ 16.7\ mm}]$

Chapter 8

Types of BEITS

8. Types of Belts

8-1. Introduction

A belt is a looped strip of flexible material, used to mechanically link two or more rotating shafts. They may be used to move objects, to efficiently transmit mechanical power, or to track relative movement. Belts are looped over pulleys. In a two-pulley system, the belt may either drive the pulleys in the same direction, or the belt may be crossed so that the shafts move in opposite directions. A conveyor belt is built to continually carry a load between two points.

8-2. Types of Belts

Figure (8-1) shows types of the belts.

1. Flat belts
2. V-belts
    a. Standard V-belts
    b. Narrow V-belts
    c. Wide V-belts (variable speed belts)
    d. Double V-belts (hex-belts)
    e. Kraft bands
    f. Poly V-belts (serpentine belts)
3. Round belts (Rope belt)
4. Timing belts (synchronous belts)

Flat belt V-belt Round belt Timing belt

Figure 8-1: Types of Belts

8-3. Material used for Belts

1. Leather belts
2. Cotton or fabric belts
3. Rubber belts
4. Balata belts

8-4. Flat belts

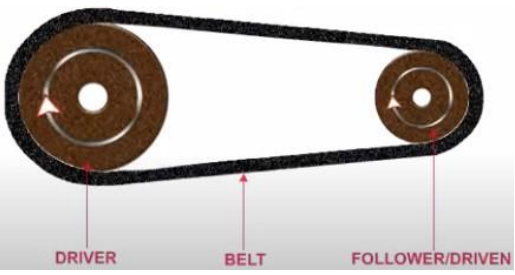
The simplest type of belt is the flat belt. It has a rectangular cross-section and was often made of leather in the early days. Today, however, steel or high-strength synthetic materials such as polyamide or aramids are used for tension cords. These force-transmitting cords are embedded in a rubber core between a top cover and a bottom cover. The bottom layer where the belt has contact with the pulley, can be coated with special rubber to increase friction and wear resistance. The top layer on the opposite side only has a protective function.

8-4-1. Types of flat Belts

The power transmission flat belt can be used in many forms of power transmission. It is known as a two-pulley drive, consisting of a driving pulley, a driven pulley, and the belt. Below are examples of pulley design variations.

1. Open belt driver

Figure (8-2) show open belt driver.



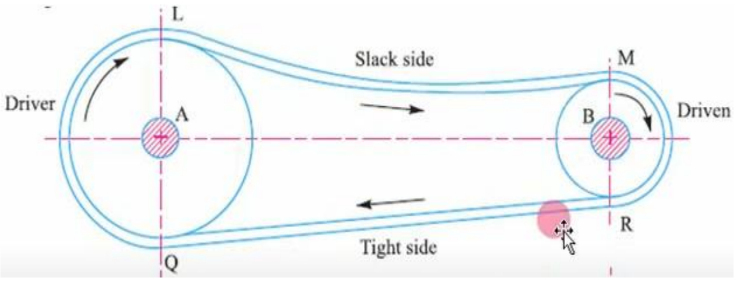
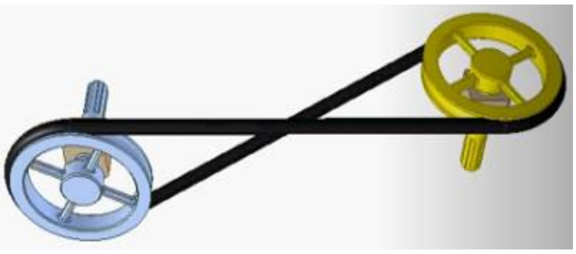


Figure 2; Open belt driver

2. Crossed or twist belts

Figure (8-3) show Crossed or twist belt.



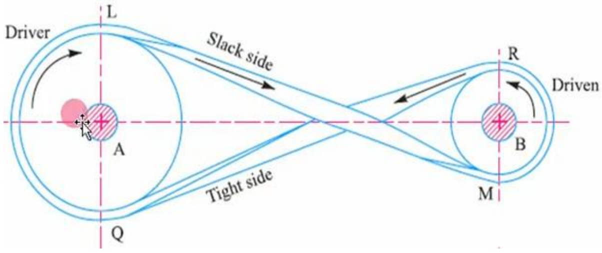
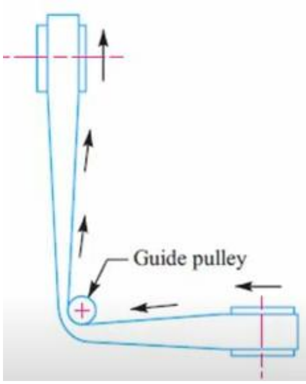
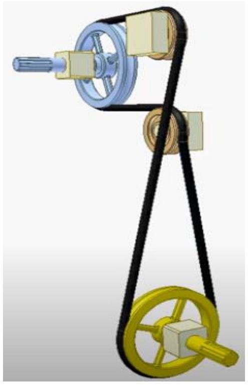


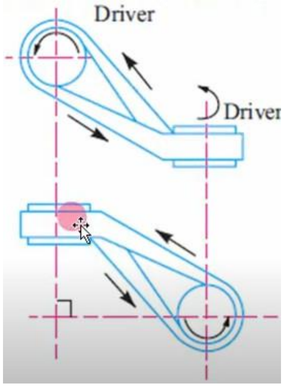
Figure 8-3; Crossed or twist belt

3. Quarter turn belt drive

Figure (8-4) shows Quarter turn belt drive.







Angular drive   Half cross drive

Figure 8-4: Quarter turn belt drive

4. Belt drive with idler pulleys

Figure (8-5) shows belt drive with idler pulleys.

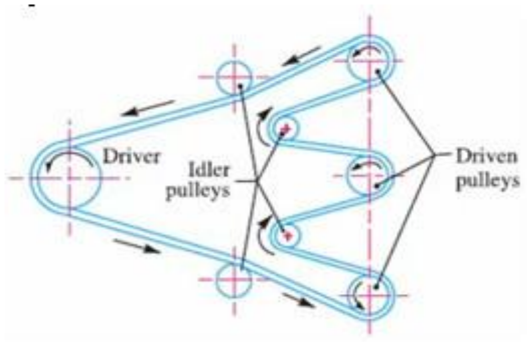


Figure 8-5: Belt drive with idler pulleys

5. Compound belt drive

Figure (8-5) show Compound belt drive.

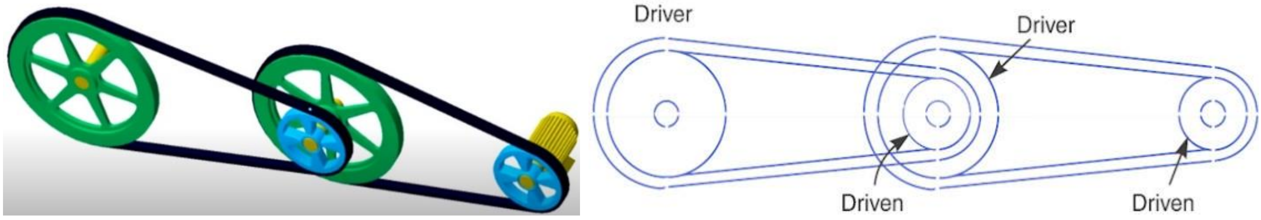


Figure 8-6: Compound belt drive

6. Stepped or cone pulley drive

Figure (8-7) show Stepped or cone pulley drive.

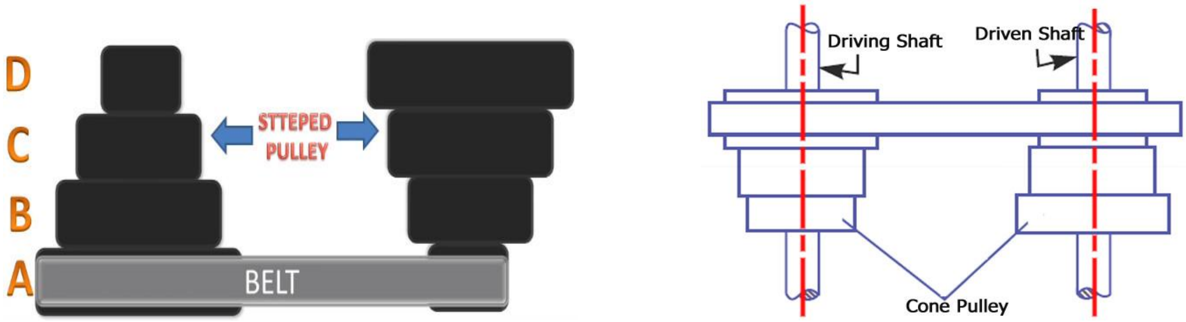


Figure 8-7: Stepped or cone pulley drive

7. Fast and loose pulley drive

Figure (8-8) show fast and loose pulley drive.

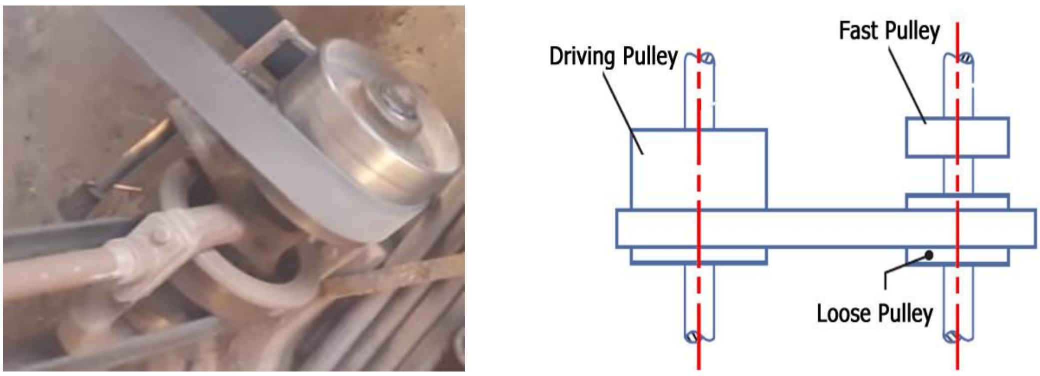


Figure 8-8: Fast and loose pulley drive

8-5. Power transmitted by belts

Belt power transmission depends on the following factors:

1. The force exerted on the pulleys by the belts when they are under tension,
2. The speed of the belt,
3. Contact arc between a belt and a pulley.

8-6 Calculation of the belt dimension and tensions load

8-6-1 Velocity ratio of a belts drive

1. The following equation can be used to determine the length of the belt that passes over the driver once every minute:

$$L_d = \pi\, d_1 N_1 \qquad (8-1)$$

2. The following equation can be used to determine the length of the belt that passes over the follower once every minute:

$$L_f = \pi\, d_2 N_2 \qquad (8-2)$$

3. Given that the length of the belt that goes over the driver in a minute is the same as the length of the belt that passes over the follower in a minute, the following conclusions follow:

$$\therefore \quad L_d = L_f$$

$$\pi\, d_1 N_1 = \pi\, d_2 N_2$$

$$\frac{N_1}{N_2} = \frac{d_2}{d_1} \qquad (8-3)$$

4. When the belt's thickness (t) is taken into account, the velocity ratio is:

$$\frac{N_1}{N_2} = \frac{d_2 + t}{d_1 + t} \qquad (8-4)$$

5. The belt's peripheral velocity at the driving pulley is:

$$V_1 = \frac{\pi\, d_1\, N_1}{60} \quad \left(\frac{m}{s}\right) \qquad (8-5)$$

6. The peripheral velocity of the belt on the driven pulley is:

$$V_2 = \frac{\pi\, d_2\, N_2}{60} \quad \left(\frac{m}{s}\right) \qquad (8-6)$$

6. When the is no slip, then

$$V_1 = V_2 \qquad (8-7)$$

$$\frac{\pi\, d_1\, N_1}{60} = \frac{\pi\, d_2 N_2}{60}$$

$$\frac{\pi\, d_1\, N_1}{60} = \frac{\pi\, d_2 N_2}{60} \qquad or \qquad \frac{N_2}{N_1} = \frac{d_1}{d_2}$$

7. In case of a compound drive, example eight velocities. The speed ratio is given by the following equation:

$$\frac{N_8}{N_1} = \frac{d_1 \times d_3 \times d_5 \times d_7}{d_2 \times d_4 \times d_6 \times d_8} \qquad (8-8)$$

8. The general equation for calculating speed ratio can be written as follows:

$$\therefore \quad \frac{Speed\ of\ last\ driven}{Speed\ of\ first\ driver} = \frac{Product\ of\ diameter\ of\ drivers}{Product\ of\ diameter\ of\ drivens} \qquad (8-9)$$

9. If The length of the belt is determined by: If (D1 & D2) are the diameters of the smaller and larger pulleys, respectively, and (C) is the center distance between the axes of the pulleys. The length of the belt is determined by:

$$L = 2C + \frac{\pi(D_1 + D_2)}{2} + \frac{(D_1 - D_2)^2}{4C}\ , \qquad (fore\ open\ belt\ length) \qquad (8-10)$$

$$L = 2C + \frac{\pi(D_1 + D_2)}{2} + \frac{(D_1 + D_2)^2}{4C}, \qquad (fore\ cross\ belt\ length) \qquad (8-11)$$

10. Angle of wrap on smaller (θ_1) and larger pulleys (θ_2) are given by,

$$Angle\ of\ wrap\ (\theta_1) = 180 - 2\sin^{-1}\frac{(R_2 - R_1)}{C}, (fore\ open\ belt\ drive) \quad (8-12)$$

$$Angle\ of\ wrap\ (\theta_2) = 180 + 2\sin^{-1}\frac{(R_2 - R_1)}{C}, (fore\ open\ belt\ driven) \quad (8-13)$$

$$Angle\ of\ wrap\ (\theta_1 = \theta_2)$$

$$= 180 + 2\sin^{-1}\frac{(R_2 - R_1)}{C}, (fore\ cross\ belt\ drive) \qquad (8-14)$$

11. The following equation gives the ratio of tension in the tight and slack sides:

$$\frac{T_2}{T_1} = e^{\mu\theta}\ , \qquad (fore\ open\ belt\ driven) \qquad (8-15)$$

$$\frac{T_2}{T_1} = e^{\mu\theta}\ \csc\beta, \qquad (fore\ cross\ belt\ driven) \qquad (8-16)$$

Were,

$$\mu\ =\ coefficient\ of\ friction\ between\ belt\ \&\ pulley$$

$$\theta\ =\ angle\ of\ contact$$

$$\beta\ =\ half\ of\ the\ groove\ angle\ of\ v-belt$$

8-6-2. Slip of the Belts

The motion of belts and pulleys assuming a firm frictional grip between the belts and the pulleys. But sometimes, the frictional grip becomes in sufficient. This may cause some forward motion of the driver without carrying the belt with it. This is called slip of the belt and is generally expressed as a percentage.

$$S_1\,\% = Slip\ between\ the\ driver\ and\ the\ belt,$$

$$S_2\,\% =\ Slip\ between\ the\ belt\ and\ the\ follower.$$

∴ The belt's velocity as it passes the driver per second is:

$$V_1\ = \frac{\pi\ d_1\ N_1}{60} - \frac{\pi\ d_1\ N_1}{60} \times \frac{S_1}{100}$$

$$V_1\ = \frac{\pi\ d_1\ N_1}{60}\left(1 - \frac{S_1}{100}\right) \qquad (8-17)$$

And the belt's velocity per second as it passes the follower is:

$$V_2 = \frac{\pi\ d_2\ N_2}{60} = V_1 - V_1.\left(\frac{S_2}{100}\right) = V_1.\left(1 - \frac{S_2}{100}\right) \qquad (8-18)$$

Substituting equation 10 into equation 11 we get:

$$\frac{\pi\ d_2\ N_2}{60} = \frac{\pi\ d_1\ N_1}{60}\left(1 - \frac{S_1}{100}\right)\left(1 - \frac{S_2}{100}\right)$$

$$Neglecting \quad \rightarrow \quad \left(\frac{S_1 \times S_2}{100 \times 100}\right)$$

$$\therefore \quad \frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \frac{S_1}{100} - \frac{S_2}{100}\right)$$

$$\frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \left(\frac{S_1 + S_2}{100}\right)\right)$$

$$\frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \left(\frac{S}{100}\right)\right) \qquad (8-19)$$

$Where \quad S = S_1 + S_2 \;\; Total\, percentage\, of\, slip$

If the belt's thickness (t) is taken into account, then

$$\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}\left(1 - \left(\frac{S}{100}\right)\right) \qquad (8-20)$$

8-6-3. Cases where slip occurs in belt

1. In some situation during power or rotary transmission the pulley may not carry the belt with it which means the belt slips over the pulley.
2. This is mainly due to frictional grip between the pulley and belt.
3. This is also due to high power from the driver pulley that cannot be transmitted by the belt.

8-6-4. Creep in the Belt

1. A piece of the belt stretches when the belt moves from the slack side to the tight side, and it contracts again when the belt moves from the tight side to the slack side. The surfaces of the belt and pulley move relative to one another as a result of these variations in length. Creep is the name given to this relative motion,
2. Overall, creep slows down the driven pulley or follower's speed a little,
3. σ_1 & σ_2 = Both the tight and slack sides of the belt are under stress.

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \times \frac{E + \sqrt{\sigma_2}}{E + \sqrt{\sigma_1}} \qquad (8-21)$$

Example 1:

A flat belt runs on driver pulley diameter ($3\ m$) and power transmits ($10\ KN$) at speed ($500\ rpm$) and diameter of driven ($1.3\ m$). Distance between pulleys ($C = 12\ m$). Assuming angle of lap as ($\theta_1 = 130°$) and cofficent of friction as (0.25). Find the necessary width of belt if the maximum pull is not to exceed ($180\ N/cm$) width of the belt and find length of the belt. Neglect centrifugal tension.

<u>Solution</u>

Given

$$[D_1 = 3\ m, D_2 = 1.3\ m, P = 10 \times 1000 = 10000\ N, N_1 = 500\ rpm,$$

$$C = 12\ m, \qquad \theta_1 = 130^{\circ} = 130° \times \frac{\pi}{180} = 2.27\ Radians.]$$

Power transmitted by belt is:

$$P = (T_1 - T_2)\, V_1 \qquad (1)$$

Belt tension ratio is:

$$\frac{T_1}{T_2} = e^{\mu.\theta_1}$$

$$\frac{T_1}{T_2} = e^{0.25 \times 2.27} = 1.764$$

$$\therefore \quad T_1 = 1.764\ T_2 \qquad (2)$$

Velocity of belts is:

$$V_1 = \frac{\pi\ D_1\ N_2}{60} = \frac{3.14 \times 3 \times 500}{60} = 78.5\ m/s$$

Put all values in equation (1)

$$P = (T_1 - T_2)\, V_1 \qquad (1)$$

$$10000 = (1.764\, T_2 - T_2) \times 78.5$$

$$10000 = (0.764\, T_2) \times 78.5$$

$$T_2 = \frac{10000}{0.764 \times 78.5} = \frac{10000}{59.974} = 166.739\ N$$

$$\therefore \quad T_1 = 1.764\ T_2 = 1.764 \times 166.739 = 294.127\ N \rightarrow T_{Maximum}$$

But the maximum pull is not to exceed(*180 N/cm*) width of the belt.

$$1 \ cm = 180 \ \ N \qquad \& \qquad b = 294.124$$

$$\therefore \quad b = \frac{1 \times 294.124}{180} = 1.64 \ cm \ \ \approx \ \ 17 \ mm$$

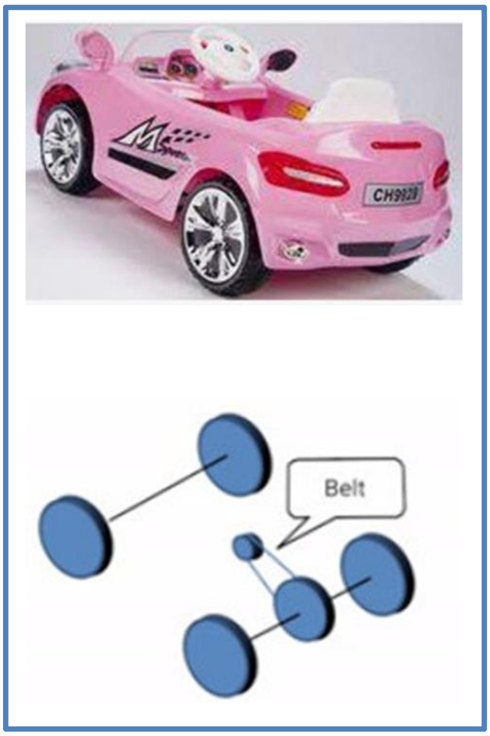
To find length of the belt, we applied the following equation:

$$L = 2C + \frac{\pi(D_1 + D_2)}{2} + \frac{(D_1 - D_2)^2}{4C}$$

$$L = 2 \times 12 + \frac{\pi(3 + 1.3)}{2} + \frac{(3 - 1.3)^2}{4 \times 12} = 24 + 6.751 + 0.060 = 30.811 \, m$$

8-7. Designing a flat belt drive

Example 1: Car in the game city weighing ($233 \ kg$), as shown in figure. Wheel diameter ($300 \, mm$), Large pulley diameter ($150 \, mm$) and small pulley diameter ($75 \, mm$). Distance between two pulleys ($C \ = \ 300 \, mm$) and $Cofficent \ of \ frictio$ (μ)=0.35. Force (F_T) is ($500 \ N$). Find Total extra force on bearing (F_B), and what is load percentage from the belt drive?



Required torque on the drive axle.

Draw an (FBD) of the wheel that will show the following:

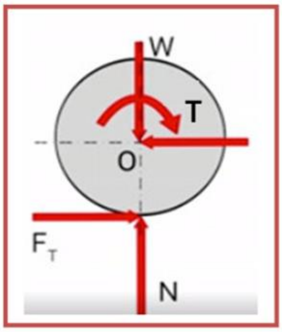
1. The traction force at the drive wheels,
2. The gravity load from the axle,
3. The horizontal force from the axle,
4. The normal force at the road,
5. The torque that the belt drive will apply to the axle.

Solution

Analysis for required torque

The equilibrium equation to find the value for the torque in terms of force (F_T).

$Wheel\ diameter(D_W) = 300\ mm\ = 0.3\ m\ ,\ F_T = 500\ N$



Find the numerical value of torque (T)?

$$\Sigma M_O = 0$$

$$T - \frac{F_T\ \times\ D_W}{2} = 0$$

$$\therefore T = \frac{F_T\ \times\ D_W}{2} = \frac{500 \times\ 0.3}{2} = 75\ N.m$$

Produce to find the following:

1. Use the rope around a bollard analysis to find the maximum ratio of the tensions around each pulley?
2. Use moment equilibrium to find the different tensions on either side of each pulley?
3. Solve to get the two tension?
4. Find the additional forcess on the bearing?

Rope around a bollard

$$\frac{T_1}{T_2} = e^{\mu.\theta_1}$$

The small pulley will slip first.

So, the angle wrap is smaller, so ($e^{\mu.\theta_1}$) is also smaller.

$$Angle\ of\ wrap\ (\theta_1) = 180° - 2\sin^{-1}\frac{(R_2 - R_1)}{C}$$

$$\theta_1\ = 180° - 2\sin^{-1}\frac{(0.075 - 0.0375)}{0.3} = 180° - 2\sin^{-1}(\,0.125 \times \frac{180°}{\pi})$$

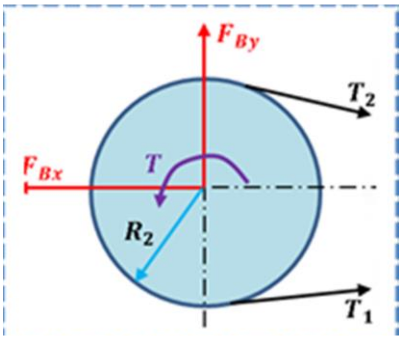
$$= 180° - 2sin(7.166°) = 180° - 0.25 = 179.75\ °$$

FBD Showing the tension difference

Tension ratio

Draw a (FBD) of the large that will show

1. The two-belt tension,
2. The reaction from the shaft on the pulley that balances the two tensions,



3. The torque from the shaft on the large pulley.

$$We\acute{}ll\ specify\ that\ T_2 > T_1$$

Now we´ll apply equilibrium to find the difference in the tensions.

$\Sigma M_O = 0$

$(T_2 - T_1) \times R_2 - T = 0$

$$(T_2 - T_1) = \frac{T}{R_2} = \frac{75}{0.0\,75} = 100\ N \qquad (1)$$

$$\because\ \frac{T_2}{T_1} = e^{\mu\theta} \quad \Longrightarrow \quad \frac{T_2}{T_1} = e^{0.35\times 179.75\times\frac{\pi}{180}} = e^{1.097} = 2.995$$

$$\therefore \quad T_2 = 2.995\ \ T_1 \qquad (2)$$

Substituting the second equation into the first equation is produced.

$$(T_2 - T_1) = 1000 \quad \Longrightarrow \quad T_2 = 1000 + T_1 \quad \Rightarrow \quad 2.995\,T_1 = 1000 + T_1$$

$$\therefore \quad 2.995\,T_1 - T_1 = 1000$$

$$T_1 = \frac{1000}{1.995} = 501.25\ \ N \qquad (3)$$

By substituting the third equation into the second equation to get a value of T_2 .

$$T_2 = 2.995\,T_1 = 2.995 \times 501.25 = 1501.25\ \ N$$

Total extra force on bearing (F_B)

$$\Sigma F_x = 0$$

$$F_{Bx} = (T_1 + T_2) \times \cos\left[\sin^{-1}\left(\frac{R_2 - R_1}{C}\right)\right]$$

$$F_{Bx} = (501.25 + 1501.25) \times \cos\left[\sin^{-1}\left(\frac{0.075 - 0.0375}{0.3}\right)\right]$$

$$F_{Bx} = (2002.5) \times \cos\left[\sin^{-1}\left(0.125 \times \frac{180}{\pi}\right)\right]$$

$$F_{Bx} = (2002.5) \times \cos(7.166) = (2002.5) \times 0.992 = 1986.86\ \ N$$

$$\Sigma F_y = 0$$

$$F_{By} = (T_2 - T_1) \times \sin\left[\sin^{-1}\left(\frac{R_2 - R_1}{C}\right)\right]$$

$$F_{By} = (1501.25 - 501.25) \times \sin\left[\sin^{-1}\left(\frac{0.075 - 0.0375}{0.3}\right)\right]$$

$$F_{By} = (1000) \times \sin\left[\sin^{-1}\left(0.125 \times \frac{180}{\pi}\right)\right]$$

$$F_{By} = (1000) \times \sin(7.166) = (1000) \times 0.125 = 120\ N$$

$$F_B = \sqrt{(F_{Bx})^2 + \left(F_{By}\right)^2} = \sqrt{(1986.86\ )^2 + (120)^2} = \sqrt{3962.10^6} = 1990.48\ N$$

Total mass of vehicle is (*233kg*), what is load percentage from the belt drive.

$$Bearing\ load\ from\ vehicle\ mass = m\ .\ g = 233\ \times 9.81 = 2285.73\ N$$

$$Bearing\ load\ from\ belt\ drive = 1990.48\ N$$

$$Extra\ load\ from\ the\ belt\ drive = \frac{Bearing\ load\ from\ belt\ drive}{Bearing\ load\ from\ vehicle\ mass}\ .100\ \%$$

$$Extra\ load\ from\ the\ belt\ drive\ \% = \frac{1990.48}{2285.73}\ .100\ \% = 87.09\ \%$$

8-8. V-Belt belts

The V-belts are transmission belts used in auto-industry. These belts are used to transmit the power from the engine to the ancillary components. They are considered as low-cost and efficient means of transmitting power and V-Belt are called V-Belt belt because it has a V shape cross section area. From giant rock crushers to tiny sewing machines, V-belts have found their way into countless industrial applications. Today's V-belts are marvels of modern technology, reflecting the latest advances in mechanical and chemical engineering. Unlike flat belts, which rely solely on friction and can track and slip off pulleys, V-belts have sidewalls that fit into corresponding sheave grooves, providing additional surface area and greater stability.

Following are some environmental and application design criteria that will influence belt selection:

1. Ambient temperature

2. Oil resistance
3. Ozone resistance
4. Static conductivity
5. Power capacity
6. Pulsation or shock loading
7. Small sheave diameters
8. Backside idlers
9. Misalignment tolerance
10. Serpentine or quarter turn layout
11. Minimal take-up
12. Clutching
13. High speeds
14. Energy efficiency
15. Dust and abrasives

$$Fc = Centrifugal\ Force\ (N)$$

$$R = Pulley\ reaction\ Force\ (N)$$

$$P = Max\ power\ transferred\ kW$$

$$T = Belt\ tension$$

$$Tc = Belt\ tension\ due\ to\ centrifugal\ force$$

$$\mu = Coefficient\ of\ friction.$$

$$f = Effective\ coefficient\ of\ friction = \mu/\sin\beta$$

$$b = Belt\ width\ (m)$$

$$\omega = Angular\ velocity\ of\ pulley\ (rad/s)$$

$$n = Rotational\ Speed\ (RPM)$$

$$\theta = Angle\ of\ belt\ lap$$

$$2.\beta = Internal\ Angle\ of\ Vee$$

$$v = Linear\ velocity\ of\ belt\ (m/s)$$

8-8-1. Types of V-Belts

There are three types of V-Belts.

1. **Classical V-belt (standard V-belt)**

Classical V-belts are standardized in Germany according to DIN 2215 and have a height to width ratio of (1:1.6). Tension cords made of steel, aramid, polyester or glass are embedded in an elastomer core covered by a top layer. The tension cords run at the level of the nominal width, figure (8-9).

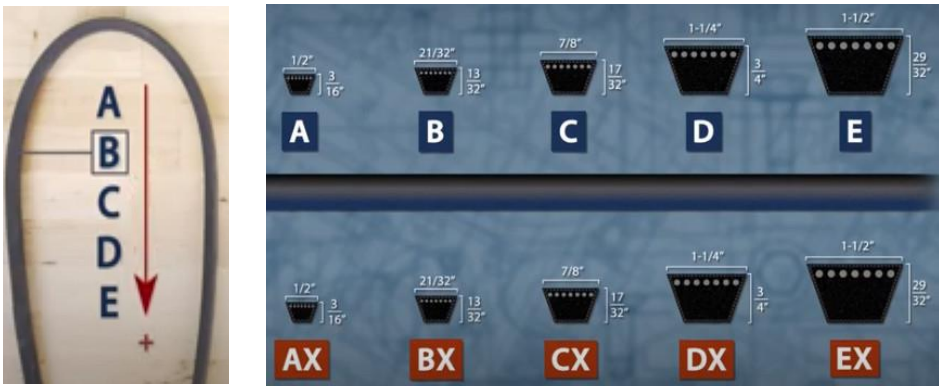


Figure 8-9: Classical V-belt (standard V-belt)

2. **Narrow belt**

Narrow belts are optimum for load transfer and force distribution because of their greater depth to width ratio. That's their advantage over classical V belts. Narrow belts are also suitable for drives with high belt speeds, again, for their powerfully compact size. Narrow belts have the ability to transmit up to three times the horsepower of classical V-belt in the same drive space. They can handle drives from 1 to 1000 horsepower, figure (8-10).

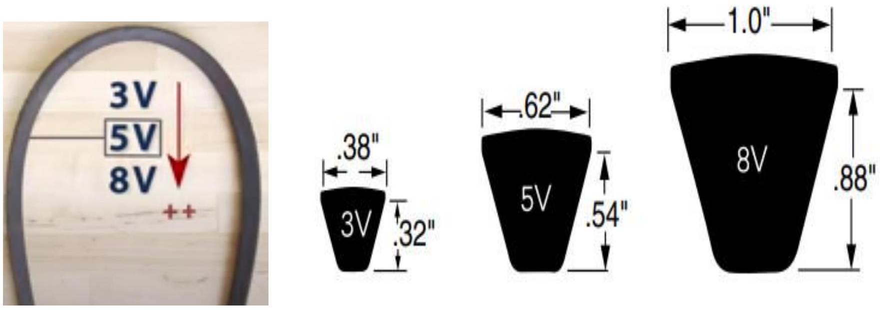


Figure 8-10: Narrow V-Belt

3. **Fractional Horsepower Belt**

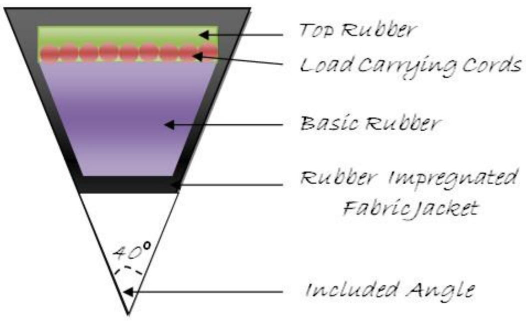
Fractional Horsepower Belt light duty V-belts are used most often as single belt on drives of 1 horsepower or less. Its design is for relatively light loads. The common applications for this V-belt type are domestic washing machines, small fans, refrigerators, and garage equipment. They are identified with a [3L, 4L or 5L] prefix. The numerical prefix indicates the belt top width in one eighth of an inch followed by nominal outside length in inches. For example, 3L300 part number indicates 3/8” top width with 30.0” outside length, figure (8-11).

Figure 8-11: Fractional Horsepower V-Belt

8-8-2. Cross section of V-Belt

Figure (8-12) show the cross section of V-Belt.

1. Double layer rubberized belt wrap,
2. Polyester tensile carrier – cord,
3. A layer of neoprene that connects and protects the cord,
4. Special transverse reinforcing cord,
5. The "body" of the belt is made of neoprene rubber, which takes the compressive load.



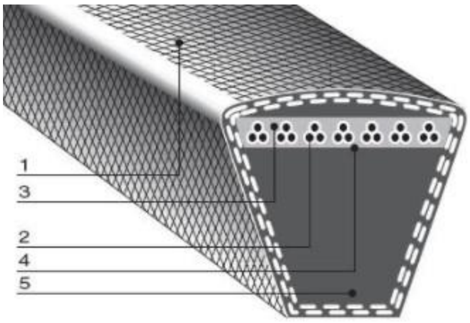


Figure 8-12: Cross section of V-Belt

8-8-3. Standard V-belt sections

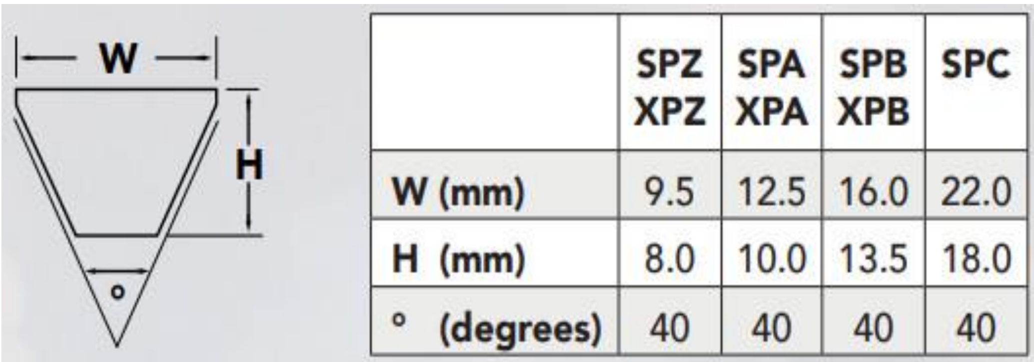
The standard V-belt sections are [A, B, C, D and E]. The table below contains design parameters for all the sections of V-belt. The kW rating given for a particular section indicates that, belt section selection depends solely on the power transmission required, irrespective of number of belts. If the required power transmission falls in the overlapping zone, then one has to justify the selection from the economic view point also, table (8-1).

Table 8-1: Standard V-belt sections

Section	Hours power range (KW)	Minimum pulley pitch (mm)	Width (mm)	Thickness (mm)
A	0.4 - 4	125	13	8
B	1.5 - 15	200	17	11
C	10 - 70	300	22	14
D	35 - 150	500	32	19
E	70 - 260	630	38	23

8-8-4. SP Belts - European Standard DIN 7753

European standards DIN 7753 and ISO 4184 are based on the metric system of measure and have different cross section designations, figure (8-13).



	SPZ XPZ	SPA XPA	SPB XPB	SPC
W (mm)	9.5	12.5	16.0	22.0
H (mm)	8.0	10.0	13.5	18.0
° (degrees)	40	40	40	40

Figure 8-13: SP Belts - European Standard DIN 7753

8-9. V-belt Pulleys

Groove angles and dimensions for pulleys shall confirm to figure (8-14).

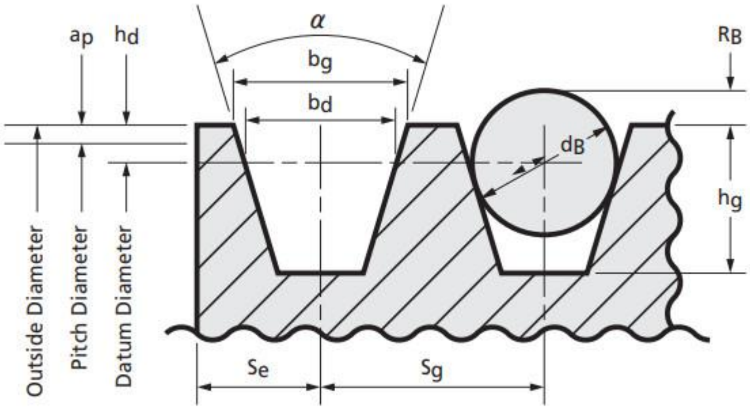


Figure 8-14: Standard Groove Dimensions

8-10. Compare between Flat belt and V-Belt

Table (8-1) Show compare between Flat belt and V-Belt.

Table 8-2: Compare between Flat belt and V-Belt

NO.	Flat Belt Drive	V - Belt Drive
1.	Flat belt has rectangular cross section,	V- Belts are characterized by their trapezium shaped cross section,
2.	Simple design, inexpensive cross section,	Compared to flat belt pulleys, V-Belt and V-Groove pulley construction is difficult and expensive,
3.	The slip may occur,	Slip is negligible due to wedging action between the belt and V-groove pulley
4.	Significantly greater pretension required to transmit a particular torque,	Require little pretension,
5.	Up to 15 meters, flat belts can be utilized for extended lengths,	V-belt cannot use for long distance because weight per unit length of the belt is greater than that of the flat belt,
6.	Power transmission capacity is low,	V-belt can transmit more power for the same coefficient of friction,
7.	Efficiency is higher than V-belt drive,	Efficiency is lower than flat belt,
8.	Flat belt not used in the vertical direction.	V- belt can run even the belt is vertical.

8-11. Design V-belts

The following equations to design and calculation of the V-belt, figure (8-15).

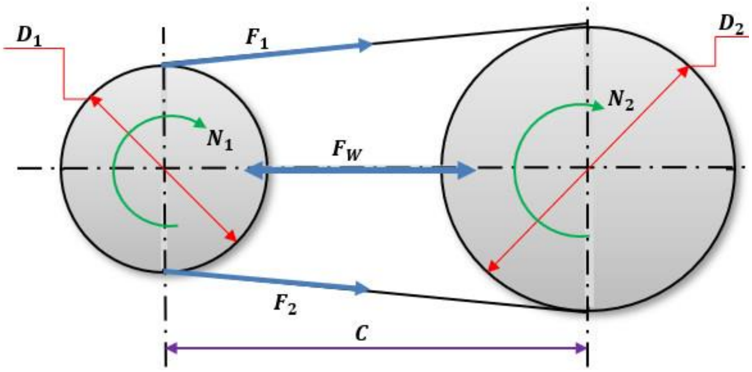


Figure 8-15: V- belt transmit

1. Belt cross section

Select standard V-belt cross section from European standards DIN 7753 and ISO 4184, figure (8-16) based on motor power (kW)

2. Pulley diameters

Calculate the diameters of the smaller and larger pulley using the relation:

$$i = \frac{D_2}{D_1} = \frac{N_1}{N_2} \qquad (8-22)$$

$D_2 -$ Pitch diameter of larage pulley (mm)

$D_1 -$ Pitch diameter of small pulley (mm)

$N_2 -$ Speed of larage pulley (rpm)

$N_1 - Speed\ of\ small\ pulley\ (rpm)$

$i - velocity\ ratio$

3. Designation of V - belt

Pitch diameter is allegedly used as the basis for the calculations for V-belt drives. However, V-belts are identified with a nominal inside length (this is easily measurable compared to pitch length). Consequently, the interior length can be calculated using the relationship shown below.

$$Inside\ length + X = Pitch\ Length \qquad (8-23)$$

Table 8-3: Show value of (X) in millimeter

Section	A	B	C	D	E
X (mm)	36	43	56	79	93

4. V- belt Equation

Because of the presence of a wedge, V-belts have increased friction grip. As a result, the equation for belt tension must be altered. The equation is altered as follows:

$$\frac{T_1 - mv^2}{T_2 - mv^2} = e^{\mu\alpha \sin\frac{\theta}{2}} \qquad (8-24)$$

Where (θ) is the belt wedge angle

5. **Center distance, (C) should be such that**,

$$B = 4\mathrm{L} - 6.28\,(D_2 + D_1) \qquad (8-25)$$

$$\mathrm{C} = \frac{\mathrm{B} + \sqrt{B^2 - 32(D_2 - D_1)^2}}{16} \qquad (8-26)$$

$$D_2 < C < 3(D_2 + D_1)$$

6. **Belt length open $(\mathbf{L_o})$**

$$L_o = 2\mathrm{C} + 1.57(D_2 + D_1) + \frac{(D_2 - D_1)^2}{4C} \qquad (8-29)$$

$$\because Inside\ length + X = Pitch\ Length$$

7. **V - belt design factors**

a. Calculate design power

$$Design\ power,\ P_{design} = service\ factor\ (C_{sev.}) \times required\ power\ (P)$$

$$C_{sev.} = (1.1\ to\ 1.8)\ for\ light\ to\ heavy\ shock.$$

b. Modification of $(\boldsymbol{kW})$ rating

Power rating of a typical V-belt section requires modification, since, the ratings are given for the conditions other than operating conditions. The factors are as follows,

c. Equivalent smaller pulley diameter

In a belt drive, both the pulleys are not identical, hence to consider severity of flexing, equivalent smaller pulley diameter is calculated based on speed ratio. The power rating of V-belt is then estimated based on the equivalent smaller pulley diameter (D_{ES}).

$$D_{ES} = C_{SR}.D_1 = 355 \times 1.12 = 398\, mm$$

Where, (C_{SR}) is a factor dependent on the speed ratio.

d. Angle of wrap correction factor

The power rating of V-belts is based on angle of wrap, ($\alpha = 180°$) . Hence, Angle of wrap correction factor (C_{VW}) is incorporated when (α) is not equal to ($180°$) .

e. Belt length correction factor

There is an optimum belt length for which the power rating of a V-belt is given. Let, the belt length is small then, in a given time it is stressed more than that for the optimum belt length. Depending upon the amount of flexing in the belt in a given time a belt length correction factor (C_{VL}) is used in modifying power rating. Therefore, incorporating the correction factors,

$$Modified\ power\ rating\ of\ a\ belt\ (kW)$$
$$= Power\ rating\ of\ a\ belt\ (kW)\ x\ C_{VW}\ x\ C_{VL}$$

f. Selection of V- belt

Depending on the amount of power to be transmitted, a suitable V-belt section and a transmission ratio for the V-belt drive are selected a range of (1:15). A V-belt drive's belt speed should be around ($20\ \frac{m}{s}\ to\ 25\ \frac{m}{s}$), but should not exceed($30\ \frac{m}{s}$. From the speed ratio, and chosen belt speed, pulley diameters are to be selected from the standard sizes available. Depending on available space the center distance is selected, however, as a guideline,

The belt pitch length can be calculated if (C):

$$D_2 < C < 3(D_2 + D_1) \qquad (8-30)$$

g. Choice for belt section is (C).

$$P = \frac{\pi.D.N}{60} = \frac{\pi.D_1.N_1}{60}$$

$$D_1 = \frac{60\ P}{\pi.N_1} \qquad \& \qquad D_2 = \frac{D_1 \times N_1}{N_2}$$

Compare $(D_1\ \&\ D_2)$ with standard sizes and choice highest and near record standard $sizes$ $(D_{1,standar}\ \&\ D_{2,standar})$.

$$speed\ ratio\ (i)_{actual} = \frac{D_2}{D_1} = \frac{N_1}{N_2}$$

$$speed\ ratio\ (i)_{standard} = \frac{D_2}{D_1} = \frac{N_1}{N_2}$$

As a result, the second combination is preferable because it is very close to the given speed ratio.

h. Number of belts

$$Number\ of\ belts = \frac{Design\ \ Power}{Modified\ power\ rating\ of\ a\ belt} \tag{31}$$

Example 3:

For the following information, design a V-belt drive: Drive: AC motor with a 1440 rpm working speed and greater than (10 hours). A compressor that requires 20 kW of power transmission and spins at (900 rpm) is the apparatus being driven.

Solution:

$$Design\ power,\ P_{design} = service\ factor\ (C_{sev.}) \times required\ power\ (P)$$

$$= \ 1.3 \times \ 20 = \ 26\ kW = 26000\ W$$

For the given service condition, the value (1.3) is chosen from the design data book.

As a result, C is the obvious choice for the belt section.

$$P = \frac{\pi.D.N}{60} = \frac{\pi.D_1.N_1}{60}$$

$$26000 = \frac{3.14 \times D_1 \times 1440}{60}$$

$$\therefore \quad D_1 = \frac{26000 \times 60}{3.14 \times 1440} \approx 345\, mm \approx 355\, mm\, Standard\ size$$

$$D_2 = \frac{D_1 \times N_1}{N_2} = \frac{345 \times 1440}{900} = 552\, mm \approx 560\, mm\, Standard\ size$$

$$speed\ ratio\ (i)_{actual} \ = \frac{D_2}{D_1} = \frac{552}{345} = 1.6$$

$$speed\ ratio\ (i)_{standard} \ = \frac{D_2}{D_1} = \frac{560}{355} = 1.58$$

Therefore, since the second combination is so close to the specified speed ratio, it is preferable to employ it.

As a result, the second combination is preferable because it is very close to the given speed ratio.

As a result, the chosen pulley diameters are ($D_1 = 355\, mm$) and (($D_2 = \ 560\, mm$).

The center distance, C, should be chosen so that:

$$\boldsymbol{D_2 \ < \ C \ < \ 3(D_2 \ + \ D_1\ )}$$

$$C > \ D_2 \quad \rightarrow \quad C > 560 \ \ mm$$

$$C < 3(D_2 \ + \ D_1\ ) \quad \rightarrow \quad C < 3 \times (560 + 350) < 2730\, mm$$

Let us consider, C = 1500 mm, this value satisfies the above condition.

$Belt\ length\ open\ (L_o)$

$$L_o = 2\mathrm{C} + 1.57(D_2 + D_1) + \frac{(D_2 - D_1)^2}{4C}$$

$$L_o = 2 \times 1500 + 1.57(560 + 355) + \frac{(560 - 355)^2}{4 \times 1500}$$

$$L_o = 3000 + 1436.55 + 7 \approx 4444\, mm$$

$$\because Inside\ length\ + \ X = Pitch\ Length$$

$$\therefore Inside\ length\ of\ belt \ = \ 4444 \ - \ 56 \ = \ 4388\, mm$$

The nearest value of belt length for C-section is $4394\, mm$ (from design data book)

Therefore, the belt designation is $\boldsymbol{C}$: $\mathbf{4394/173}$

Power rating (kW) of one C-section belt

Equivalent small pulley diameter is,

$$D_{\ ES} = C_{SR}.D_1 = 355 \times 1.12 = 398\,mm$$

$$C_{SR} = 1.12\ \ is\ obtained\ from\ the\ hand\ book$$

For the belt speed of$(26\,\frac{m}{\sec})$, the given power rating $\{(kW) = 12.1\,kW\}$ For the obtained belt length, the length correction factor$(C_{vl} = 1.04)$.

Determination of angle of wrap:

$$\beta = \sin^{-1}(\frac{D_2 - D_1}{2C}) = \frac{560 - 355}{2 \times 1500} = 3.92^{\circ}$$

$$\alpha_1 = 180 - 2\beta = 1800 - 2(3.92) = 172.16° = 3\,rad$$

$$\alpha_2 = 180 + 2\beta = 1800 + 2(3.92) = 187.84° = 3.28\,rad$$

For the angle of wrap of $(3.00\,radian)$ (smaller pulley) the angle of wrap factor, (C_{VW}) is found to (0.98).for a C section belt.

Therefore, incorporating the correction factors,

Modified power rating of a belt:

For a C section belt with an angle of wrap of (3.00 radian) (smaller pulley), the angle of wrap factor, (C VW), is found to be (0.98). As a result of incorporating the correction factors، Belt power rating modification:

$$(kW\ ) =\ Power\,rating\ of\ a\ belt\ (kW) \times\ C_{VW}\ \times C_{VL}$$

$$= 12.1 \times 0.98 \times 1.04 = 12.33\,KW$$

$$Number\ of\ belts = \frac{Design\ \ Power}{Modified\,power\,rating\ of\ a\ belt} = \frac{26}{12.33} = 2.1 \approx 2$$

Two numbers of $(C\,\frac{4394}{173})$ belts are required for the transmission of $(20\,kW)$.

Dr. Emad T. Karash Dr. Jamal N. Sultan Ahmed M. Mosa Iman Z. Alshih Yahya

8-12 Chapter Questions

1. **The power transmitted by belt drive is determined by the following factors:**
   a. **Belt velocity, arc of contact, and initial belt tension.** b. Initial belt tension.
   b. Belt velocity. d. Arc of contact.
2. **Belts in agricultural machinery can be made of the following materials:**
   a. leather. b. cotton duck.
   c. balata gum. d. rubber.
3. **Belts in flour mills can be made of the following materials:**
   a. leather. b. rubber.
   c. canvas or cotton duck. d. rubber balata gum.
4. **When does the belt's speed increase?**
   a. When a maximum power is reached before the transmitted power starts to decline.
   b. When the transmission's power rises.
   c. When the amount of power delivered stays constant.
   d. When the transmission's power drops.
5. **The reasons behind the belt's creep are:**
   a. Effect of temperature on belt.
   b. Stresses beyond elastic limit of belt material.
   c. Unequal extensions in the belt due to tight and slack side tensions.
   d. Material of belt.
6. **In belt drive, the coefficient of friction is determined by:**
   a. Belt material
   b. Belt and pulley materials
   c. Belt and pulley materials
   d. Velocity of a belt
7. **Which of the following has a positive drive?**
   a. Drive belt, flat
   b. Drive belts crossed
   c. Belt timing
   d. V-belt transmission
8. **When using a V-belt drive, the belt comes into contact at:**
   a. **The groove sides of the pully.**
   b. The bottom of the pulley's groove.
   c. The bottom and sides of the pulley's groove.
   d. The top of the pulley's groove.
9. **In belts, the centrifugal tension:**
   a. Reduces the amount of power sent.
   b. **Tension is increased of the belt without affecting power transmission.**
   c. Expand the wrap angle.
   d. Increases the amount of power sent.
10. **The belt slips as a result of:**
    a. **Loose belt, heavy load, and too small driving pulley.**
    b. A heavy load.
    c. Driving pulley is too small.
    d. Belt should be loose.

11. **When using the same pulley diameters, center distance, belt speed, and belt and pulley materials,**
    a. **Power is transmitted more efficiently by crossed belt drive than by open belt drive.**
    b. Power is transmitted more efficiently by open belt drive than by crossed belt drive.
    c. Power transmission is not dependent on open or crossed constructions.
    d. The power transmitted by open and crossed belt drives is the same.
12. **The belt drive's transmission power can be increased by:**
    a. Dressing the belt to increase the coefficient of friction, increasing the belt's initial tension, and increasing wrap angle with an idler pulley.
    b. Increasing the belt's initial tension.
    c. Increasing the coefficient of friction by dressing the belt.
    d. Increasing the wrap angle with an idler pulley.
13. **When replacing V belts, a complete set of new belts is used rather than replacing a single damaged belt because,**
    a. Only one belt can be used in conjunction with other used belts.
    b. Belts, both new and old, will cause vibration.
    c. Belts are sold in sets.
    d. The new belt will carry more than its fair share, resulting in a short lifespan.
14. **The goal of 'crowning' belt drive flat pulleys is to:**
    a. Increase the belt's speed.
    b. improve the transfer of power capacity.
    c. prevent against the belt surface being harmed by the joint.
    d. prevent the belt fall off the pulley.
15. **The pulleys' arms for flat belt drive have the following:**
    a. Main axis twice the minor axis, elliptical in cross-section, and major axis in plane of rotation.
    b. Cross-section that is elliptical.
    c. Twice as much major as minor axis.
    d. Major axis in the rotational plane.
16. **Idler pulleys in belt drives have the following goals:**
    a. Increase the capacity of power transmission.
    b. The belt's propensity to slip should be reduced.
    c. The belt's wrap angle and tension should be increased, as should the capacity for power transmission and the likelihood of belt slippage.
    d. Belt tension and wrap angle should be increased.
17. **When using a V- belt drive?**
    a. The belt should only make contact with the pulley's bottom groove; it should not touch the sides.
    b. The pulley's bottom groove should be touched by the belt.
    c. The pulley's bottom groove should be touched by the belt.
    d. The pulley's groove's sides shouldn't be touched by the belt.

Chapter 9

Design of shafts

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9. Design of shafts

9-1. Introduction

A shaft is defined as a rotating machine element, usually circular in cross-section, which is used to transmit power from one part to another, or from a machine that produces power to a machine that absorbs power. Shafts are important elements of the machines. They are the elements that support rotating parts like gears and pulleys and in turn are themselves supported by bearings resting in the rigid machine housings. The shafts perform the function of transmitting power from one rotating member to another supported by it or connected to it. Thus, they are subjected to torque due to power transmission and bending moment due to reactions on the members that are supported by them. Shafts are to be distinguished from axles which also support rotating members but do not transmit power. Axles are thus subjected to only bending loads and not to the torque. Most the times, shafts have circular cross-section and could be either solid or hollow.

9-2. Difference between shafts and axles

1. **Shaft:** are designed to carry links which transmit torque and experience both bending and torsion.
2. **Axles**: are intended to support rotating parts that do not transmit torques and are subjected to bending only, figure (9-1). The shaft is a live member while the axle is a dead one. The shaft used to transmit power at a short distance while axle transmits power at long distance. The shaft can be meant for balancing or transferring torque while the axle is meant for balancing or transferring bending moment.

a. Shaft

b. Axel

Figure 9-1: Shaft and axel

9-3. Classification of Shaft

Shafts can be classified in terms of different loads on the shaft, the shape of the shaft and the application of the shaft.

1. **According to the purpose of the shaft**:
   a. Shafts of various drives (gear drives, belt drives, chain drives and so on),
   b. Main shafts of mechanisms and machines whose function is to carry not only drive elements but other elements that do not unmet torques such as rotors, flywheels, turbine disks, etc.
2. **According to the structure shape**: figure (9-2).
   1. Plain transmission shaft,
   2. Stepped,
   3. Axel,
   4. Machine shafts,
      a. Crank shaft,
      b. Cam shaft,
      c. Cross shaft,
      d. Eccentric shaft.
   5. Flexible.

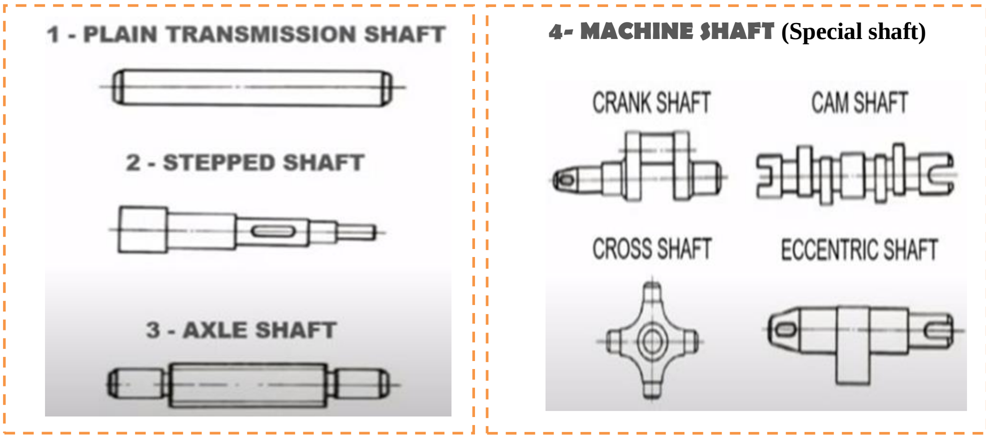

Figure 9-2: Shafts according to the structure

3. **According to the construction**: figure (9-3).
   a. Shaft of constant cross section (without steps),
   b. Shaft of variable cross section of stepped configuration,
   c. Shaft made solid with gears or worms (Spline shaft).

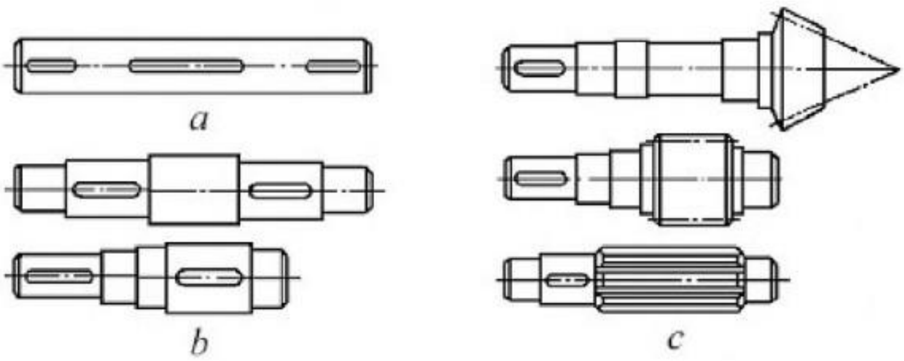

Figure 9-3: Classification shafts according to construction

4. **According to the construction**: figure (9-4).

   A. Shafts with solid circular cross section,

   B. Shafts with hollow circular cross section,

   C. Shafts with keyways,

   D. Shafts with splines,

   E. Shafts with rectangular cross section.

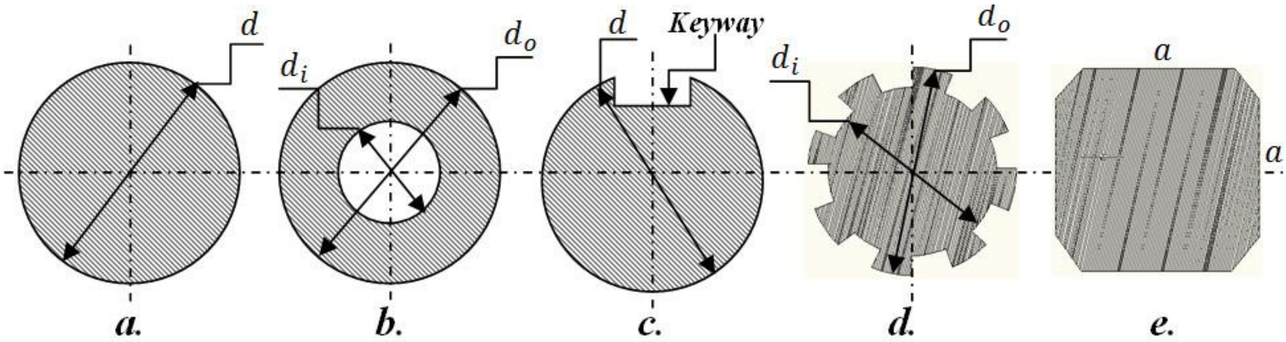

Figure 9-4: Classification shafts according to cross section

9-4. various shaft types

Shafts can be broadly divided into four sorts:

1. **Transmission shafts:** are employed to transfer power between the machine consuming power and the source. Countershafts, line shafts, and all factory shafts are a few examples.

2. **Machine shafts:** are inextricably linked to the machine. example: crankshaft.
3. **Axle shafts:** are utilized in automobiles.
4. **A spindle shafts:** are rotating shafts that have a fixture on them to hold a tool or a piece of work.

9-5. Materials for shafts

Mild steel is the material used to make standard shafts. Alloy steels like nickel, nickel-chromium, or chromium-vanadium steel are utilized when great strength is required. Shafts are typically hot rolled into shape, then cold drawn, turned, or ground to size.

The following characteristics must be present in the material used for the shafts:

1. It should be well-mechanized,
2. It should be really strong,
3. It ought to have a low sensitivity factor,
4. It needs to have favorable heat treatment characteristics.
5. It must possess high wear-resistant qualities.

For normal shafts, carbon steel in the grades [40 C8, 45 C8, 50 C4, and 50 C12] is utilized.

9-6. Standard size of a shafts

The following shaft sizes are typical:

1. Transmission shaft standard sizes are:
   a. 25 mm to 60 mm, and 5 mm steps
   b. 60 mm to 110 mm, and 10 mm steps
   c. 110 mm to 140 mm, and 15 mm steps and
   d. 140 mm to 500 mm, and 20 mm steps
   e. The shafts' standard lengths are (5, 6,and 7 meter).

2. Machine shafts are available in a variety of standard sizes:

    a. Up to 25 mm steps of 0.5 mm

9-7. Manufacturing of the shafts

Shafts are usually produced by hot rolling and are prepared for shape by cold drawing or turning and grinding. Cold rolled shafts are stronger than hot-rolled shafts, but with higher residual stresses.

Residual stress can cause deformation of the shafts when it is mechanized, especially when slots or keys are cut.

Shafts of larger diameter are usually forged and are shaped into a lathe.

9-8. Advantages of Shafts

The advantage of shafts is:

1. It is less probable for the shaft system to jam,
2. When a tube is connected to the drive shaft, a chain system requires less maintenance,
3. A hollow shaft is lighter than a solid shaft while transmitting the same amount of torque,
4. The internal shape of the hollow shaft is hollow, requiring less material.
5. The shaft has a lower failure probability and is stronger,
6. High polar moments of inertia,
7. Significant tensional strength.

9-9. Disadvantages of Shafts

The disadvantage of shafts is:

1. Shafts can vibrate when rotating, which can lead to continual noise production,
2. Power loss from coupling failure,

3. The manufacturing process is challenging, 5. Maintenance and manufacturing costs were high,
4. The mechanical issues caused the downtime to be lengthier,
5. A loss of velocity between shafts may result through the use of flexible couplings, such as a leaf spring coupling.
6. It was difficult to change the speed, and
7. The overhead shafting was dripping with oil.

9-10. Stress in the Shafts

The following stresses are induced in the shafts:

1. Shear stresses due to the transmission of torque (i.e., due to torsional load),
2. Bending stresses (tensile or compressive) due to the forces acting upon machine elements as gears, pulleys etc. as well as due to the weight of the shaft itself,
3. stresses due to combined tensional and bending loads.

9-11. Design Shaft

The shafts may be designed on the basis of

1. Strength,
2. Rigidity and stiffness.

In designing shafts on the basis of strength, the following cases may be considered:

1. Shafts subjected to twisting moment or torque only,
2. Shafts subjected to bending moment only,
3. Shafts subjected to combined twisting and bending moments,
4. Shafts subjected to axial loads in addition to combined tensional and bending loads.

9-11-1. Shafts subjected to twisting moment only

1. **Solid shaft**

When shafts are subjected to twisting moment or torque only, the diameter of the solid shaft be obtained by using the torsion equation.

$$\frac{\tau}{r} = \frac{T}{J} = \frac{G\varphi}{L} \qquad (9-1)$$

Were,

$T \ = \ twisting\ moment\ or\ torque,$

$r \ = \ distance\ from\ neutral\ axis\ to\ the\ outer\ ,$

$G \ = \ modulus\ of\ rigidity,$

$\tau \ = \ torsinal\ \ shear\ stress,$

$L \ = \ length\ of\ shaft,$

$\varphi \ = \ torsion\ angle.$

$J \ = \ polar\ moment\ of\ inertia\ of\ the\ cross\ section.$

$J = \dfrac{\pi d^4}{32} \qquad , \quad For\ solid\ circular\ section$

$d = diameter\ of\ the\ shaft = 2r\ \ , where\ (r)\ raduis\ of\ the\ shaft$

Then we get,

$$\frac{\tau}{r} = \frac{T}{J} \ \Longrightarrow \ \frac{\tau}{\frac{d}{2}} = \frac{T}{\frac{\pi d^4}{32}} \ \Longrightarrow \ T = \frac{\pi.\tau.d^3}{16} \ \Longrightarrow \ \tau = \frac{T}{\frac{\pi d^3}{16}} \ \Longrightarrow \ \tau = \frac{16T}{\pi d^3} \Longrightarrow d^3 = \frac{16T}{\pi.\tau}$$

$$d \ = \sqrt[3]{\frac{16T}{\pi.\tau}} \qquad , \qquad Equation\ to\ find\ diameter\ of\ the\ solid\ shaft. \quad (9-2)$$

2. **Hollow shaft**

The polar moment inertia of the hollow shaft is:

$$J = \frac{\pi(d_o^4 - d_i^4)}{32} \qquad , \quad For\ hollow\ circular\ section$$

$$d_o = outer\ diameter\ of\ the\ shaft, \quad r = \frac{d_o}{2}$$

$$d_i = inner\ diameter\ of\ the\ shaft.$$

$$\frac{\tau}{r} = \frac{T}{J} \quad \Longrightarrow \quad \frac{\tau}{\frac{d_o}{2}} = \frac{T}{\frac{\pi(d_o^4 - d_i^4)}{32}} \quad \Longrightarrow \quad \tau = \frac{T}{\frac{\pi.(d_o^4 - d_i^4)}{16\ d_o}}$$

$$\tau = \frac{16\ d_o.T}{\pi\ (d_o^4 - d_i^4)} \quad \Longrightarrow \quad T = \frac{\pi.\tau.(d_o^4 - d_i^4)}{16.d_o}$$

$Where,$

$$k = \frac{d_i}{d_o} \qquad (9-3)$$

$$\therefore\ T = \frac{\pi.\tau.(d_o^4 - d_i^4)}{16\ d_o} \times \frac{d_o^4}{d_o^4} \quad \Rrightarrow \quad T = \frac{\pi.\tau.d_o^4\ (\frac{d_o^4 - d_i^4}{d_o^4})}{16\ d_o}$$

$$T = \frac{\pi.\tau.d_o^3\ (1-k^4)}{32} \qquad or \qquad \tau = \frac{32\ T}{\pi.d_o^3\ (1-k^4)}$$

$$\therefore\ d_o = \sqrt[3]{\frac{16\ T}{\pi.\tau.(1-k^4)}} \qquad (9-4)$$

$$Equation\ to\ find\ outerdiameter\ of\ the\ hollow\ shaft\ (\ d_o).$$

<u>Note:</u>

1. The twisting moment (T) also be obtained by using the following relation:

$$P = \frac{2\pi\ .N\ .T}{60} \quad \Longrightarrow \quad T = \frac{60\ P}{2\ \pi\ .N}$$

$Where,$

$$T = Twisiting\ moment\ in\ (N.m)\ \&\ N = Speed\ of\ theshaft\ in\ (rpm).$$

2. In case of belt drives, can find twisting moment (T) by the following equation:

$$T = (T_1 - T_2).R \qquad (9-5)$$

$Where,$

$$T_1 \ \& \ T_2 = Tension\ in\ the\ tight\ and\ slack\ sides\ of\ the\ belt\ \ in\ (N),$$

$$R = Radius\ of\ pulley\ in\ (m).$$

9-11-2. Shafts subjected to bending moment only

1. To find diameter solid shaft

When shafts are subjected to bending moment only, the diameter of the solid shaft be obtained by using the bending equation.

$$\frac{M}{I} = \frac{\sigma_b}{y} \qquad (9-6)$$

Where:

$M \ = \ Bensing\ moment,$

$I \ = \ Moment\ of\ inertia\ , \qquad I = \dfrac{\pi d^4}{64} \quad For\ solid\ circular\ section,$

$\sigma_b = Bending\ stress,$

$y = Dsitance\ from\ neutralaxis\ to\ the\ outer\ most\ fiber, \quad y = \dfrac{d}{2}.$

$$\because \ \frac{M}{I} = \frac{\sigma_b}{y} \quad \Rrightarrow \quad \therefore \ \frac{M}{\frac{\pi d^4}{64}} = \frac{\sigma_b}{\frac{d}{2}} \quad \Rrightarrow \quad \frac{M}{\frac{\pi d^3}{32}} = \frac{\sigma_b}{1}$$

$$\therefore \ M = \frac{\pi\ .d^3.\sigma_b}{32} \qquad or \qquad \sigma_b = \frac{32\ M}{\pi\ .d^3}$$

$$\therefore d = \sqrt[3]{\frac{32\ M}{\pi\ .\sigma_b}}\ ,equaion\ to\ find\ diameter\ solid\ shaft\ (d). \qquad (9-7)$$

2. To find outer diameter hollow shaft

Also to find diameter hollow shaft, we use this equation:

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

Where:

$I = Moment\ of\ inertia\ ,\qquad I = \frac{\pi\ (d_o^4 - d_i^4)}{64}\quad For\ hollow\ circular\ section,$

$y = Dsitance\ from\ neutralaxis\ to\ the\ outer\ most\ fiber,\quad y = \frac{d_o}{2}.$

$$\because\ \frac{M}{I} = \frac{\sigma_b}{y}\quad \Rightarrow\ \therefore\ \frac{M}{\frac{\pi\ (d_o^4 - d_i^4)}{64}} = \frac{\sigma_b}{\frac{d_o}{2}}\quad \Rightarrow\quad \frac{M}{\frac{\pi\ (d_o^4 - d_i^4)}{32}} = \frac{\sigma_b}{d_o}$$

$$\therefore\ M = \frac{\pi.\sigma_b.(d_o^4 - d_i^4)}{32\ d_o}\qquad or\qquad \sigma_b = \frac{32\ d_o\ .M}{\pi.(d_o^4 - d_i^4)}$$

$Where,\qquad k = \frac{d_i}{d_o}$

$$\therefore\ M = \frac{\pi.\sigma_b.(d_o^4 - d_i^4)}{32\ d_o} \times \frac{d_o^4}{d_o^4}\qquad \Rightarrow\qquad M = \frac{\pi.\sigma_b.d_o^4\ (\frac{d_o^4 - d_i^4}{d_o^4})}{32\ d_o}$$

$$M = \frac{\pi.\sigma_b.d_o^3\ (1 - k^4)}{32}\qquad or\qquad \sigma_b = \frac{32\ M}{\pi.d_o^3.(1 - k^4)}$$

$$d_o = \sqrt[3]{\frac{32\ M}{\pi.\sigma_b.(1 - k^4)}}\qquad (9-8)$$

$Equation\ to\ find\ outerdiameter\ of\ the\ hollow\ shaft\ (\ d_o).$

9-11-3. Shafts subjected to combined twisting and bending moment

1. To find diameter solid shaft

a. When shafts are subjected according to maximum shear stress theory, the diameter of the solid shaft be obtained by using the maximum shear stress equation.

$$\tau_{max.} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\ \tau^2}\qquad (9-9)$$

Where:

$$\sigma_b = \frac{32\ M}{\pi\ .d^3}\qquad (9-10)\qquad \&\qquad \tau = \frac{16T}{\pi d^3}\qquad (9-11)$$

$$\therefore \tau_{max.} = \frac{1}{2}\sqrt{\left(\frac{32\,M}{\pi\,.\,d^3}\right)^2 + 4\,(\frac{16T}{\pi d^3})^2} = \frac{1}{2}\sqrt{\frac{1024\,\mathrm{M}^2}{\pi^2\,.\,d^6} + 4\,\frac{256\,\mathrm{T}^2}{\pi^2\,.\,d^6}}$$

$$\therefore \boldsymbol{\tau_{max.}} = \frac{\mathbf{1}}{\mathbf{2}}\sqrt{\frac{1024\,\mathrm{M}^2}{\boldsymbol{\pi^2}\,.d^6} + \frac{1024\,\mathrm{T}^2}{\boldsymbol{\pi^2}\,.d^6}} = \frac{\mathbf{32}}{\mathbf{2\pi}.d^3}\sqrt{\mathrm{M}^2 + \mathrm{T}^2} = \frac{\mathbf{16}}{\boldsymbol{\pi d^3}}\sqrt{\mathbf{M^2 + T^2}}$$

$$T_{equ.} = \frac{\pi d^3}{16}\,.\tau_{max.} = \sqrt{\mathrm{M}^2 + \mathrm{T}^2} \qquad (9-12)$$

Where, $T_{equ.}$ = Equivelent twisting moment.

b. When shafts are subjected according to normal stress theory, the diameter of the solid shaft be obtained by using the maximum normal stress equation.

$$\sigma_{(b)max.} = \frac{1}{2}\sigma_b + \frac{1}{2}\sqrt{(\sigma_\mathrm{b})^2 + 4\,\tau^2}$$

Where:

$$\sigma_b = \frac{32\,M}{\pi\,.\,d^3} \qquad \& \qquad \tau = \frac{16T}{\pi d^3}$$

$$\therefore \sigma_{(b)max.} = \frac{1}{2}\times\frac{32\,M}{\pi\,.\,d^3} + \frac{1}{2}\sqrt{\left(\frac{32\,M}{\pi\,.\,d^3}\right)^2 + 4\,(\frac{16T}{\pi d^3})^2} = \frac{32\,M}{\pi\,.\,d^3}\left[\frac{1}{2}\ \ (\mathrm{M} = \sqrt{\mathrm{M}^2 + \mathrm{T}^2})\right]$$

$$\tau_{max.} = \frac{16}{\pi\,.\,d^3}\left[\,(\mathrm{M} + \sqrt{\mathrm{M}^2 + \mathrm{T}^2})\right]$$

$$\therefore M_e = \frac{\pi\,.\,d^3}{16}\,\tau_{max.} = \left[\,(\mathrm{M} + \sqrt{\mathrm{M}^2 + \mathrm{T}^2})\right] \qquad (9-13)$$

Where, $M_{equ.}$ = Equivelent bending moment.

1. To find diameter outer hollow diameter shaft

a. When shafts are subjected according to maximum shear stress theory, the outer diameter of the hollow shaft be obtained by using the maximum shear stress equation.

$$\tau_{max.} = \frac{1}{2}\sqrt{(\sigma_\mathrm{b})^2 + 4\,\tau^2}$$

Where:

$$\sigma_b = \frac{32\,M}{\pi.d_o^3.(1-k^4)} \qquad \& \qquad \tau = \frac{16\,T}{\pi.d_o^3\,(1-k^4)}$$

$$\therefore \tau_{max.} = \frac{1}{2}\sqrt{\left(\frac{32\,M}{\pi.d_o^3.(1-k^4)}\right)^2 + 4\,[\frac{16\,T}{\pi.d_o^3\,(1-k^4)}]^2}$$

$$= \frac{1}{2}\sqrt{\frac{1024\,\mathrm{M}^2}{\pi^2\,.d_o^6\,(1-k^4)^2} + 4\,\frac{256\,\mathrm{T}^2}{\pi^2\,.d_o^6\,(1-k^4)^2}}$$

$$\therefore \tau_{max.} = \frac{1}{2}\sqrt{\frac{1024\,\mathrm{M}^2}{\pi^2\,.d_o^6\,(1-k^4)^2} + \frac{1024\,\mathrm{T}^2}{\pi^2\,.d_o^6\,(1-k^4)^2}} =$$

$$\frac{32}{2\pi.d_o^6(1-k^4)}\sqrt{\mathrm{M}^2+\ \mathrm{T}^2} = \frac{16}{\pi\,d_o^6(1-k^4)}\sqrt{\mathrm{M}^2+\ \mathrm{T}^2}$$

$$T_{equ.} = \frac{\pi\,.\,d_o^6.\,(1-k^4)}{16}\,.\tau_{max.} = \sqrt{\mathrm{M}^2+\ \mathrm{T}^2} \qquad (9-14)$$

Where, $T_{equ.}$ = Equivelent twisting moment.

b. When shafts are subjected according to normal stress theory, the outer diameter of the hollow shaft be obtained by using the maximum normal stress equation.

$$\sigma_{(b)max.} = \frac{1}{2}\sigma_b + \frac{1}{2}\sqrt{(\sigma_\mathrm{b})^2 + 4\,\tau^2}$$

Where:

$$\sigma_b = \frac{32\,M}{\pi.d_o^3.(1-k^4)} \qquad \& \qquad \tau = \frac{16\,T}{\pi.d_o^3\,(1-k^4)}$$

$$\therefore \sigma_{(b)max.} = \frac{1}{2}\times\frac{32\,M}{\pi.d_o^3.(1-k^4)} + \frac{1}{2}\sqrt{\left(\frac{32\,M}{\pi.d_o^3.(1-k^4)}\right)^2 + 4\left(\frac{16\,T}{\pi.d_o^3\,(1-k^4)}\right)^2}$$

$$= \frac{16\,M}{\pi.d_o^3.(1-k^4)} + \left[\frac{1}{2}\sqrt{\left(\frac{32\,M}{\pi.d_o^3.(1-k^4)}\right)^2 + \left(\frac{32\,T}{\pi.d_o^3\,(1-k^4)}\right)^2}\right]$$

$$\sigma_{(b)max.} = \frac{16}{\pi.d_o^3.(1-k^4)}\left[\,(\mathrm{M} + \sqrt{\mathrm{M}^2+\ \mathrm{T}^2})\right]$$

$$\therefore M_e = \frac{\pi.d_o^3.(1-k^4).\sigma_{(b)max.}}{16} = \left[\,(\mathrm{M} + \sqrt{\mathrm{M}^2+\ \mathrm{T}^2})\right] \qquad (9-15)$$

Where, $M_{equ.}$ = Equivelent bending moment.

It is suggested that diameters of the hollow shaft may be obtained by using both the theories and larger of the two values is adopted.

9-11-4. Shafts subjected to axial load in addition to combine torsion and bending loads:

When the shaft is subjected to an axial load *(*F*)* in addition to torsion and bending loads as in propeller shafts of ships and shafts tor driving worm gears, then the stress due to axial load must be added to the bending stress *(*σ_b*)*. We know that bending equation is:

$$\frac{M}{I} = \frac{\sigma_b}{y} \quad \Rrightarrow \quad \sigma_b = \frac{M \cdot y}{I} = \frac{\mathrm{M} \cdot \frac{\mathrm{d}}{2}}{\frac{\pi \cdot \mathrm{d}^4}{64}} = \frac{32\ \mathrm{M}}{\pi\ \mathrm{d}^3}$$

Stress due to axial load is:

$$\sigma_b = \frac{F}{A} = \frac{\mathrm{F}}{\frac{\pi \cdot \mathrm{d}^2}{4}} = \frac{4\ \mathrm{F}}{\pi \cdot \mathrm{d}^2}\ , \qquad For\ round\ solid\ shaft.$$

$But\ for\ round\ hollow\ shaft\ is$:

$$\sigma_b = \frac{F}{A} = \frac{\mathrm{F}}{\frac{\pi \cdot (\mathrm{d}_\mathrm{o}^2 - \mathrm{d}_\mathrm{i}^2)}{4}} = \frac{4\ \mathrm{F}}{\pi \cdot (\mathrm{d}_\mathrm{o}^2 - \mathrm{d}_\mathrm{i}^2)} = \frac{4\ \mathrm{F}}{\pi \cdot \mathrm{d}_\mathrm{o}^2 (1 - \mathrm{k}^2)}\ , where, k = \frac{d_o}{d_i}$$

Result stress (tensile or compressive) for round solid shaft is:

$$\sigma_R = \frac{32\ M}{\pi\ d^3} + \frac{4\ F}{\pi \cdot d^2} = \frac{32\ M}{\pi} (M + \frac{F \times d}{8})$$

And result stress (tensile or compressive) for round hollow shaft is:

$$\sigma_R = \frac{32\ M}{\pi\ d^3} + \frac{4\ F}{\pi \cdot d^2} = \frac{32\ M}{\pi} (M + \frac{F \times d}{8})$$

When a shaft is hollow, the resulting stress is:

$$\sigma_R = \frac{32\ M}{\pi\ d^3\ (1 - k^4)} + \frac{4\ F}{\pi \cdot d^2\ (1 - k^2)} = \frac{32\ M}{\pi d^3\ (1 - k^4)} \left[M + \frac{F . d\ (1 - k^2)}{8} \right] \quad (9 - 16)$$

Example 1:

Solid shaft is transmitting ($750\ KW$) at ($360\ rpm$). Determine the diameter of the shaft if the maximum torque transmitted exceeds the mean torque by ($33\ \%$). Take the maximum allowable shear stress as ($83\ MPa$).

Solution

$Given,$

$$P = 750\ KW = 750000\ W, N = 360\ rpm, T_{max.} = 1.33\ T_{mean}, \tau = 83\ MPa$$

$$T = \frac{60\ P}{2\ \pi . N}$$

$$T_{mean} = \frac{60\ P}{2\ \pi . N} = \frac{60 \times 750000}{2 \times 3.14 \times 360} \approx 39808.917\ N.m = 39808917\ N.mm$$

$$\therefore T_{max.} = 1.33\ T_{mean} = 1.33 \times 39808917 = 52945859.61\ N.mm$$

$$d = \sqrt[3]{\frac{16\ T}{\pi . \tau}} \quad , \quad Equation\ to\ find\ diameter\ of\ the\ solid\ shaft.$$

$$d = \sqrt[3]{\frac{16\ T}{\pi . \tau}} = \sqrt[3]{\frac{16 \times 52945859.61}{3.14 \times 83}} = \sqrt[3]{3250455.65866} = 148.13\ mm$$

$$\approx 150\ mm$$

Example 2:

Find the outside and inside diameter of a hollow steel shaft to transmit ($45\ kW$) at ($1500\ rpm$), when the inside-to-outside diameter ratio is ($k = 0.7$). The steel's ultimate shear stress can be taken as ($424\ MPa$), and a factor of safety as (4).

$Given,$

$$P = 45\ KW = 45000\ W, N = 1500\ rpm, k = o.7, \tau = 424\ MPa, F.\boldsymbol{S} = \boldsymbol{4},$$

$$d_i = inside\ diameter\ \&\ d_o = outside\ diameter$$

$$\tau = \frac{\tau_{ult.}}{F.S} = \frac{424}{4} = 61\ MPa$$

$$T = \frac{60\ P}{2\ \pi . N}$$

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$$T = \frac{60\ P}{2\ \pi . N} = \frac{60 \times 45000}{2 \times 3.14 \times 1500} \approx 286.624\ N.m = 286624\ N.mm$$

$Equation\ to\ find\ outerdiameter\ of\ the\ hollow\ shaft\ (\ d_o)\ is:$

$$d_o = \sqrt[3]{\frac{16\ T}{\pi . \tau . (1 - k^4)}}$$

$$d_o = \sqrt[3]{\frac{16\ \times 286624}{3.14 \times 424 \times\ (1 - 0.7^4)}} = \sqrt[3]{\frac{4585984}{1331.36 \times (1 - 0.2401)}}$$

$$d_o = \sqrt[3]{\frac{4585984}{1331.36 \times 0.7599}} = \sqrt[3]{4532.9464} = 16.55\ mm \approx 20\ mm$$

$$k = \frac{d_i}{d_o} \quad \Rightarrow \quad d_i = k\ .\ d_o = 0.7 \times 20 = 14\ mm$$

Example 3:

A pair of wheels of a railway wagon carries a load of ($133\ kN$) on each axle box, acting at a distance of (120 mm) outside the wheel base. The gauge of the rails is ($1.3\ m$). Find the diameter of the axle between the wheels, if the ultimate bending stress is ($122\ MPa$) and a factor of safety as (3), If a hollow shaft is to be used instead of a solid shaft, calculate the inside and outside diameters when the inside to outside diameter ratio is ($k = 0.4$).

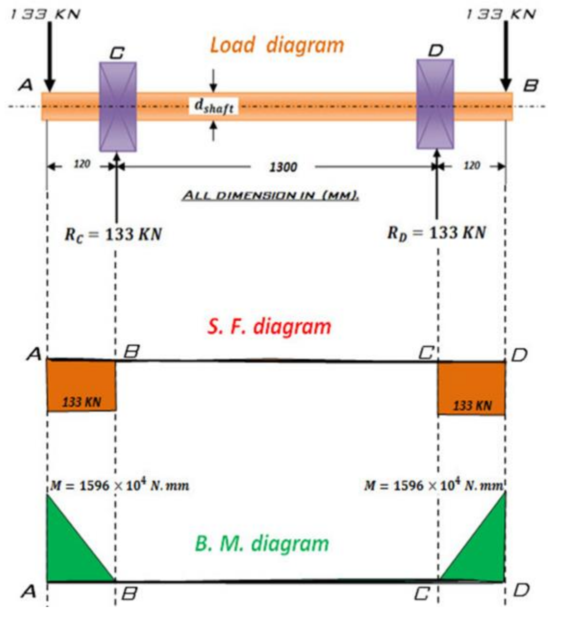

Figure (9-3): *The axle with wheels' diagram.*

Solution

$\boldsymbol{Given},$

$P = 133\ KW = 133000\ W, L = 120\ MM,$

$x = 1.3\ m, k = o.4, \sigma_b = 120\ MPa, F.\boldsymbol{S} = \mathbf{3},$

$d_i = inside\ diameter\ \&\ d_o$

$= outside\ diameter$

$$M = W\ .L = 133000 \times 120 = 1596 \times 10^4\ N.mm$$

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$$\sigma_b = \frac{\sigma_{b(ult.)}}{F.S} = \frac{120}{3} = 40\ MPa$$

1. If solid shaft use

$$d = \sqrt[3]{\frac{32\ M}{\pi\ .\sigma_b}}\ , equaion\ to\ find\ diameter\ solid\ shaft\ (d).$$

$$d = \sqrt[3]{\frac{32\ \times 1596 \times 10^4}{3.14 \times 40}} = \sqrt[3]{4066242} \approx 159.6\ mm = 160\ mm$$

1. If hollow shaft use

$Equation\ to\ find\ outerdiameter\ of\ the\ hollow\ shaft\ (\ d_o).$

$$d_o = \sqrt[3]{\frac{32\ M}{\pi .\sigma_b .(1-k^4)}}$$

$$d_o = \sqrt[3]{\frac{32\ \times 1596 \times 10^4}{3.14 \times 40 \times (1-0.4^4)}} = \sqrt[3]{\frac{32\ \times 1596 \times 10^4}{3.14 \times 40 \times (1-0.0256)}} = \sqrt[3]{4173072.699}$$

$$\approx 161\ mm = 165\ mm$$

$$k = \frac{d_i}{d_o} \quad \Rightarrow \quad d_i = k\ .\ d_o = 0.4 \times 165 = 66\ mm$$

Example 4:

A steel solid shaft transmitting (30 kW) at (330 rpm) is supported on two bearings ($AB = 588\ mm$) apart and has two gears keyed to it, as in figure. The pinion having (45) teeth of (7 mm) module is located ($AC = 588\ mm$) to the left of the right hand bearing and delivers power horizontally to the right. The gear having (80) teeth of (7 mm) module is located ($BD = 88\ mm$) to the right of the left hand bearing and receives power in a vertical direction from below. Using an allowable stress of (66 MPa) in shear, determine the diameter of the shaft. Find the inside and outside diameters of a hollow shaft when the ratio of inside to outside diameters is ($k = 0.5$) and maximum bending stress is (90 MPa).

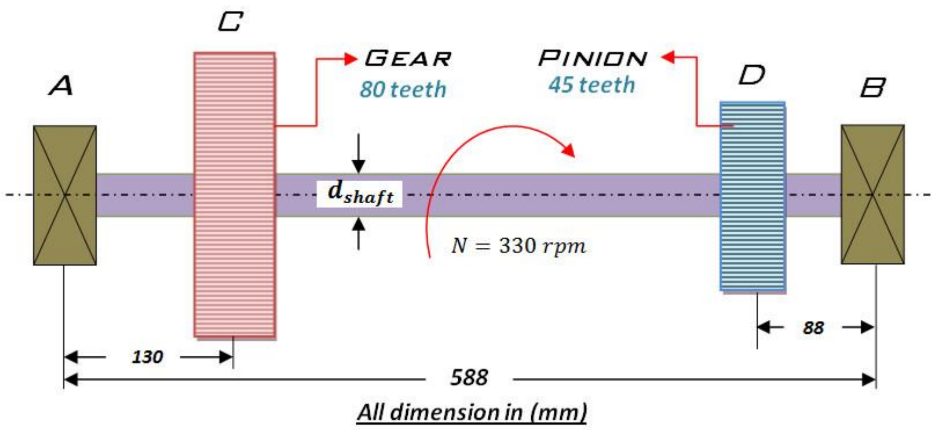

Figure (9-4): *The steel solid shaft diagram*

Given,

$$P = 30\ KW = 30000\ W, N = 330\ rpm, AB = 588\ mm, n_D = 45\ , M_D = 7\ mm, BD = 88\ mm, \quad n_C = 80\ \ , M_C = 8\ mm, AC = 130\ mm, \tau = 66\ MPa$$

$$Torque\ \ (T) = \frac{60\ P}{2\ \pi\ N} = \frac{60 \times 30000}{2 \times 3.14 \times 330} \approx 868.558\ N.m = 868558\ N.mm$$

$$Diameter\ gear\ \ \ (d_G) = Number\ of\ teeth\ (n_C)\ \times Module\ (M)$$

$$\therefore\ d_G = 80\ \ \times 7 = 560\ mm$$

$$Diameter\ pinion\ \ \ (d_P) = Number\ of\ teeth\ (n_D)\ \times Module\ (M)$$

$$\therefore\ d_P = 45\ \ \times 7 = 315\ mm$$

Assuming torque ($T = T_C = T_D$), therefore tangential force (F_{TC}) on the Gear (C) and tangential force (F_{TD}) on the Pinion, acting down words,

$$\therefore F_{TC} = \frac{T_D}{R_C} = \frac{T_D}{\frac{d_C}{2}} = \frac{2\ T_C}{d_C} = \frac{2 \times 868558}{560} \approx 3102\ N$$

$$F_{TD} = \frac{T_D}{R_D} = \frac{T_D}{\frac{d_D}{2}} = \frac{2\ T_D}{d_D} = \frac{2 \times 868558}{315} \approx 5515\ N$$

Find the maximum bending moment for vertical and horizontal loading, from the figure first find the reactions in ($R_{AV}, R_{BV}, R_{AH}, R_{BH}$), figure (9-5).

$$R_{AV} +\ R_{BV} = 3102\ \ \ \ldots \ldots \ldots (1)$$

Taking moment about point (B), we get

$$R_{AV} \times 588 -\ 3102 \times 458 = 0$$

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$$\therefore\ R_{AV} = \frac{3102 \times 458}{588} \approx 2416\ N \ \ldots\ldots\ldots (2)$$

$$R_{BV} = 3102 - R_{AV} = 3102 - 2416 = 682\ N$$

Moment at (C) point is:

$$M_{CV} = R_{AV} \times 130 = 2416 \times 130 = 314080\ N.mm$$

$$M_{DV} = R_{BV} \times 88 = 682 \times 88 = 60016\ N.mm$$

To find (R_{AH}, R_{BH}), we use

$$R_{AH} + R_{BH} = 5515 \ \ldots\ldots\ldots (3)$$

Taking moment about point (B), we get

$$R_{AH} \times 588 - 5015 \times 88 = 0$$

$$\therefore\ R_{AV} = \frac{5015 \times 88}{588} \approx 751\ N \ \ldots\ldots\ldots (4)$$

$$R_{BH} = 5515 - R_{AH} = 5515 - 751 = 4764\ N$$

Moment at (C) point is:

$$M_{CH} = R_{AH} \times 130 = 751 \times 130 = 97630\ N.mm$$

$$M_{DH} = R_{BH} \times 88 = 4764 \times 88 = 419232\ N.mm$$

The resultant bending moment at (C) is:

$$M_C = \sqrt[2]{(M_{CV})^2 + (M_{CH})^2} = \sqrt[2]{(314080)^2 + (97630)^2} \approx 328900\ N.mm$$

Also, resultant bending moment at (D) is:

$$M_D = \sqrt[2]{(M_{DV})^2 + (M_{DH})^2} = \sqrt[2]{(60016)^2 + (419232)^2} \approx 423510\ N.mm$$

$$\therefore\ Maximum\ B.M\ (M_{Max.}) = M_D = 423510\ N.mm$$

Equivalent twisting moment combine torque and bending is:

$$\therefore\ T_{equ.} = \frac{\pi d^3}{16} . \tau_{max.} = \sqrt{\mathrm{M}^2 + \mathrm{T}^2}$$

$$\therefore\ T_{equ.} = \sqrt{\mathrm{M}^2 + \mathrm{T}^2} = \sqrt{(423510)^2 + (868558)^2} = 966310\ N.mm = 966.31\ \mathrm{N.m}$$

$$\because\ T_{equ.} = \frac{\pi d^3}{16} . \tau_{max.}$$

$$\therefore d = \sqrt[3]{\frac{16\, T_{equ.}}{\pi \,.\, \tau_{max.}}} = \sqrt[3]{\frac{16 \times 966310}{3.14 \times 66}} \approx 42.1\, mm = 45\, mm$$

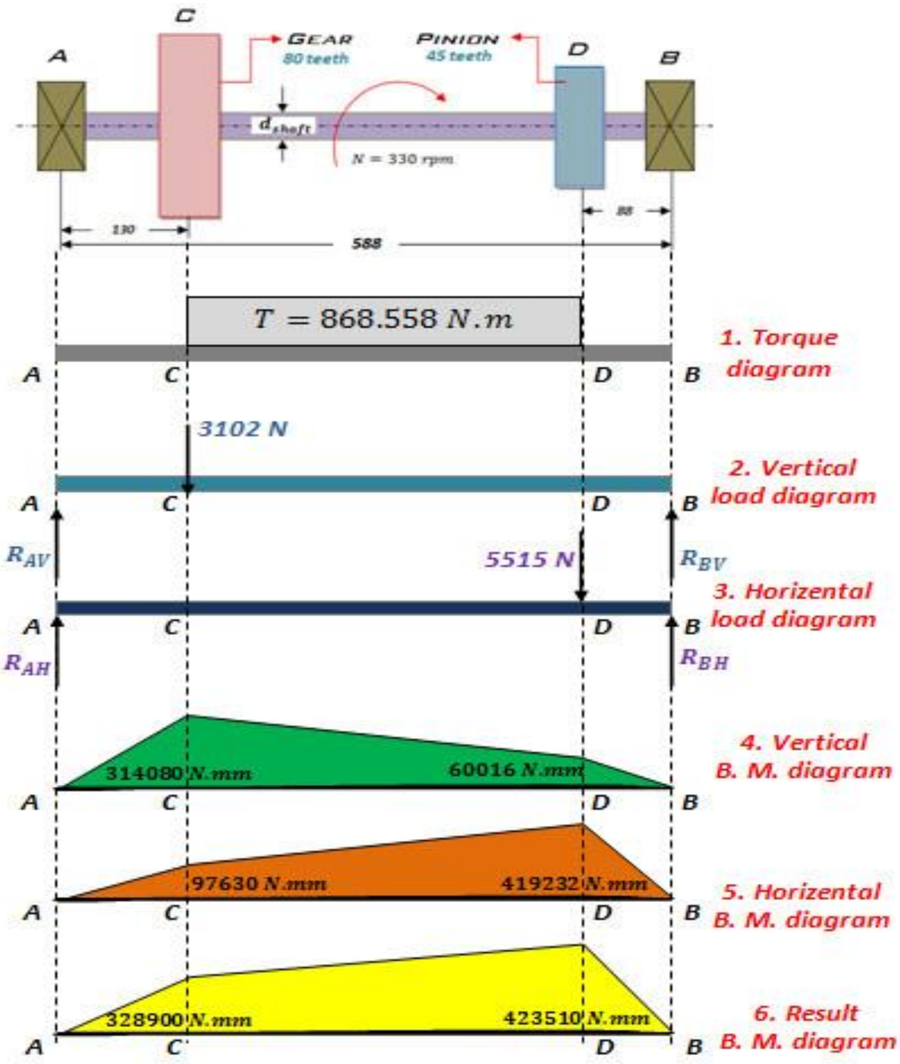

Figure (9-5): The steel solid shaft diagram, Torque diagram, Vertical load diagram, Horizontal load diagram, Vertical B. M. diagram, Horizontal B. M. diagram, Result B. M. diagram

If a hollow shaft is to be used instead of a solid shaft, find the inside and outside from the following:

$$\because\ M_e = \frac{\pi . d_o^3 . (1 - k^4) . \sigma_{(b)max.}}{16} = \left[(\mathrm{M} + \sqrt{\mathrm{M}^2 + \mathrm{T}^2}) \right]$$

$$\therefore M_e = \left[(\mathrm{M} + \sqrt{\mathrm{M}^2 + \mathrm{T}^2}) \right] = 423510 + \sqrt{(423510)^2 + (868558)^2}$$

$$= 1389820\, N.mm$$

$$\therefore\ d = \sqrt[3]{\frac{16\ M_{equ.}}{\pi.\ (1-k^4).\sigma_{(b)max.}}} = \sqrt[3]{\frac{16 \times 1389820}{3.14 \times (1-0.5^4) \times 90}} = \sqrt[3]{\frac{16 \times 1389820}{3.14 \times (0.9375) \times 90}}$$

$$= 43.78\ mm = 45\ mm$$

$$k = \frac{d_i}{d_o} \quad \Rightarrow \quad d_i = k\ .\ d_o = 0.5 \times 45 = 22.5\ mm = 20\ mm$$

9-12. Design shaft under Rigidity and Stiffness

To Design of shaft under Rigidity and Stiffness using the torsion equation.

$$\frac{\tau}{r} = \frac{T}{J} = \frac{G\varphi}{L}$$

1. Torsional Rigidity

1. Torsional rigidity is an important factor in internal combustion engine cam shaft design,
2. Timing valves operated by cam shaft,
3. Allowable angle of twist not exceeded an angle (0.25°) in peer meter length of the shaft,
4. For line transmission shaft deflection (2.5° to 3°) can be used as limiting value,
5. The commonly used shaft deflection is one degree in a length equal to twenty times the diameter of the shaft..

$$\frac{T}{J} = \frac{G\varphi}{L} \quad \Rightarrow \quad \varphi = \frac{TL}{GJ} \leq \{\varphi\} \qquad\qquad (9-17)$$

Where:

$T\ =\ twisting\ moment\ or\ torque,$

$J\ =\ polar\ moment\ of\ inertia\ of\ the\ cross\ section.$

$J = \dfrac{\pi d^4}{32} \qquad\qquad ,\quad For\ solid\ circular\ section,$

$$J = \frac{\pi(d_o^4 - d_i^4)}{32} \quad , \quad For\ hollow\ circular\ section,$$

$\varphi\ =\ torsion\ angle,$

$G\ =\ modulus\ of\ rigidity,$

$L\ =\ length\ of\ shaft.$

2. Lateral Rigidity

The equation below determines the bending moment applied on any shaft:

$$\frac{d^2y}{dx^2} = \frac{M}{EI} \quad \Rightarrow \quad M = EI\,\frac{d^2y}{dx^2} \qquad (9-18)$$

Applying the boundary conditions and integrating this equation twice with respect to (x), then (y) can be calculated. (y) should be equal or less the allowable value of deflection, (y).

9-13. A.S.M.E. Code for Shaft Design

The bending moment and twisting moment are to be multiplied by factors (K_b and K_t), respectively, in accordance with the American Society of Mechanical Engineers regulation, to account for shock and fatigue in working state. Due to this, if the shaft experiences dynamic loading, equivalent torque and bending moment will change to:

$$T_{equ.} = \sqrt{K_b.M^2 + \ K_t.T^2} \qquad (9-19)$$

$$M_{equ.} = K_bM + \sqrt{K_b.M^2 + \ K_t.T^2} \qquad (9-20)$$

Table 9-1: Values of (K_b and K_t) for different types of loading

No.	Type of applied load	K_b	K_t
1.	Gradually applied load	1.5	1
2.	Suddenly applied load (minor shock)	1.5 – 2	1 – 1.5
3.	Suddenly applied load	2 - 3	1.5 - 3

Example 5:

Figure (9-6) illustrates a steel spindle transmitting ($15\ KW\ at\ 1500\ rpm$). The spindle's angular deflection shouldn't be greater than (0.33° per 1.5 meters). If the spindle's material's modulus of stiffness is ($105\ GPa$). Find the spindle's diameter and the shear stress that it has undergone?

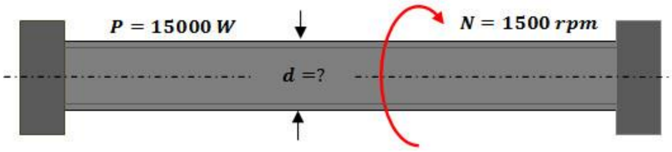

Figure (9-6): A steel spindle solid shaft

<u>Solution</u>

Given

$$P = 15000\ W, N = 1500\ rpm, L = 1500\ mm, G = 105000\ MPa\left(\frac{N}{mm^2}\right),$$

$$\varphi = 0.33°, \varphi = 33° \times \frac{\pi}{180°} = \frac{33 \times 3.14}{180} = 0.576\ rad$$

$$T = \frac{60\ P}{2\pi N} = \frac{60 \times 15000}{2 \times 3.14 \times 1500} \approx 95.541\ N.m = 95541\ N.mm$$

$$\because \frac{T}{J} = \frac{G\varphi}{L} \quad \Rightarrow \quad J = \frac{T\ L}{G\ \varphi}$$

$$J = \frac{T\ L}{G\ \varphi} = \frac{95541 \times 1500}{105000 \times 0.576} = 2369.568\ mm^4$$

Also: $J = \frac{\pi\ d^4}{32}$

$$\therefore\ d = \sqrt[4]{\frac{32\ J}{\pi}} = \sqrt[4]{\frac{32\ \times 2369.568}{3.14}} \approx\ 12.466\ mm = 15\ mm$$

The following equation can be used to determine the shear stress created in the spindle:

$$\frac{\tau}{r} = \frac{T}{J} \qquad \text{or} \qquad \frac{\tau}{r} = \frac{G\varphi}{L}$$

$$\tau = \frac{T.r}{J} = \frac{T.\frac{d}{2}}{\frac{\pi d^4}{32}} = \frac{95541 \times \frac{15}{2}}{\frac{3.14 \times (15)^4}{32}} = \frac{716557.5}{4967.58} = 144.245\ MPa, \qquad where, d = 15\ mm$$

9-14. Chapter questions

1. The material used to make shafts A and B is the same. Shaft A diameter is twice as large as shaft B. Shaft A will transfer a torque of:

8 times as much a B shaft.

twice as much as B shaft.

16 times as much as B shaft.

4 times as much as B shaft.

2. The shafts will have same strength on the basis of tensional rigidity, if

a. diameter and length of both shafts is same

b. material of both shafts is same

c. angle of twist for both shafts is same

d. all of above conditions are satisfied

3. The torsion angle of the drive shaft is:

a. d^2

b. d^3

c. $\boldsymbol{d^4}$

d. d

4. The design of a drive shaft for pure bending moments should be based on:

a. distortion energy theory.

b. maximum shear stress theory

c. Goodman or Soderberg diagrams.

d. maximum principal stress theory.

5. Which of the following statement is the right?

a. Cold rolling and hot moving produce areas of strength for similarly.

b. Hot moving produces more grounded shafts than cold rolling.

c. Strength of shaft is autonomous of moving cycles.

d. Cold moving produces more grounded shafts than hot rolling.

6. Most extreme maximum shear stress hypothesis is utilized for:

a. Shafts made of a cast iron. **b. Shafts made of a steel.**

c. Shafts made of an aluminum. d. Shafts made of a plastic.

7. Most extreme maximum principal stress hypothesis is utilized for:

a. **Shafts made of a cast iron.** b. Shafts made of a steel.

c. Shafts made of an aluminum. d. Shafts made of a plastic.

8. The shafts' function is to transmit power from one ——————- member to another that is supported by it or connected to it.

a. Fixed and rotating b. Fixed

b. Rotating or fixed **d. Rotating**

9. As a result, they are only subjected to bending loads and not torque.

a. **Axles** b. Beam

b. Shaft d. link

10. Are intended to support rotating parts that do not transmit torques and are subjected to bending only

a. Shaft b. Beam

b. **Axles** d. Link

11. The shaft can be meant for balancing or transferring torque while the ------- is meant for balancing or transferring bending moment.

a. Colum b. Beam c. **Axles** d. Link

12. Shafts can be classified according to the purpose of the shaft:

a. **flywheels** b. Cam

b. Spline shaft d. Shafts with keyways

13. Shaft used in vehicles

a. Transmission shafts b. Machine shafts

b. A spindle shaft **d. Axle shafts**

14. The material used for ordinary shafts is

a. nickel b. nickel-chromium

b. **mild steel** d. chromium

15. Machine shaft standard sizes are as follows:

a. **Up to 25 mm steps of 0.5 mm** b. Up to 25 mm steps of 0.3 mm

b. Up to 25 mm steps of 0.6 mm d. Up to 25 mm steps of 0.2 mm

16. Shafts are typically produced by —————- rolling and are shaped by cold drawing, turning, and grinding.

a. hot b. warm

b. **cold** d. very hot

17. For the same torque transmission, a hollow shaft is lighter than a solid shaft.

a. **same torque transmission** b. less torque transmission

b. different torque transmission d. more torque transmission

18. Stress in Shafts

a. only Shear stresses b. only bending stresses

c. tensile and compression stresses d. shear and bending stresses

19. Equation to find diameter of the solid shaft subjected to twisting moment only is:

a. $d = \sqrt[3]{\frac{32T}{\pi.\tau}}$ b. $d = \sqrt[3]{\frac{64T}{\pi.\tau}}$

b. $d = \sqrt[3]{\frac{48T}{\pi.\tau}}$ **d.** $\boldsymbol{d = \sqrt[3]{\frac{16T}{\pi.\tau}}}$

20. Equation to find outer diameter of the hollow shaft subjected to twisting moment only is:

a. $\boldsymbol{d = \sqrt[3]{\frac{32T}{\pi.\tau.(1-k^4)}}}$ b. $d = \sqrt[3]{\frac{64T}{\pi.\tau.(1-k^2)}}$

b. $d = \sqrt[3]{\frac{48T}{\pi.\tau.(1-k)}}$ d. $d = \sqrt[3]{\frac{16T}{\pi.\tau.(1-k^3)}}$

21. Equation to find diameter of the solid shaft subjected to bending moment only is:

c. $d = \sqrt[3]{\frac{16\,M}{\pi\,.\sigma_b}}$ **b.** $\boldsymbol{d = \sqrt[3]{\frac{32\,M}{\pi\,.\sigma_b}}}$

d. $d = \sqrt[3]{\frac{48\,M}{\pi\,.\sigma_b}}$ d. $d = \sqrt[3]{\frac{64\,M}{\pi\,.\sigma_b}}$

22. Equation to find outer diameter of the hollow shaft subjected to bending moment only is:

a. $d = \sqrt[3]{\frac{16\,M}{\pi\,.\sigma_{b.}(1-k^4)}}$ **b.** $\boldsymbol{d = \sqrt[3]{\frac{32\,M}{\pi\,.\sigma_{b.}(1-k^4)}}}$

b. $d = \sqrt[3]{\frac{48\,M}{\pi\,.\sigma_{b.}(1-k^4)}}$ d. $d = \sqrt[3]{\frac{64\,M}{\pi\,.\sigma_{b.}(1-k^4)}}$

23. Equation to find Equivalent twisting moment of the solid shaft subjected to combined twisting and bending moment is:

a. $\boldsymbol{T_{equ.} = = \frac{\pi d^3}{16}\,.\tau_{max.} = \sqrt{M^2 + T^2}}$ b. $T_{equ.} = = \frac{\pi d^3}{32}\,.\tau_{max.} = \sqrt{M^2 + T^2}$

c. $T_{equ.} = = \frac{\pi d^3}{48}\,.\tau_{max.} = \sqrt{M^2 + T^2}$ d. $T_{equ.} = = \frac{\pi d^3}{64}\,.\tau_{max.} = \sqrt{M^2 + T^2}$

24. Equation to find Equivalent bending moment of the solid shaft subjected according to normal stress theory is:

a. $\boldsymbol{M_e = \frac{\pi\,.d^3}{16}\,\tau_{max.} = \left[\,(M + \sqrt{M^2 + T^2})\right]}$ b. $M_e = \frac{\pi\,.d^3}{16}\,\tau_{max.} = \left[\,(T + \sqrt{M^2 + T^2})\right]$

c. $M_e = \frac{\pi\,.d^3}{16}\,\tau_{max.} = \left[\,(\sqrt{M^2 + T^2})\right]$ d. $M_e = \frac{\pi\,.d^3}{16}\,\tau_{max.} = \left[\,(M.T + \sqrt{M^2 + T^2})\right]$

25. Equation to find Equivalent bending moment of the hollow shaft subjected according to maximum shear stress theory is:

a. $T_{equ.} = = \frac{\pi \cdot d_o^6 \cdot (1-k^4)}{16} \cdot \tau_{max.} = \sqrt{M^2 + T^2}$ b. $T_{equ.} = = \frac{\pi \cdot d_o^6 \cdot (1-k^3)}{16} \cdot \tau_{max.} = \sqrt{M^2 + T^2}$

c. $T_{equ.} = = \frac{\pi \cdot d_o^6 \cdot (1-k^2)}{16} \cdot \tau_{max.} = \sqrt{M^2 + T^2}$ d. $T_{equ.} = = \frac{\pi \cdot d_o^6 \cdot (1-k)}{16} \cdot \tau_{max.} = \sqrt{M^2 + T^2}$

26. Equation to find Equivalent bending moment of the hollow shaft subjected according to normal stress theory is:

a. $M_e = \frac{\pi . d_o^3 .(1-k^4). \sigma_{(b)max.}}{16} = [\,(M + \sqrt{M^2 + T^2})]$ b. $M_e = \frac{\pi . d_o^3 .(1-k^4). \sigma_{(b)max.}}{16} = [\,(T + \sqrt{M^2 + T^2})]$

c. $M_e = \frac{\pi . d_o^3 .(1-k^4). \sigma_{(b)max.}}{16} = [\,(\sqrt{M^2 + T^2})]$ d. $M_e = \frac{\pi . d_o^3 .(1-k^4). \sigma_{(b)max.}}{16} = [\,(M.T + \sqrt{M^2 + T^2})]$

27. Equation to find solid shaft subjected to axial load in addition to combine torsion and bending loads:

a. $\sigma_R = \frac{32\, M}{\pi\, d^3}(M + \frac{F \times d}{8})$ b. $\sigma_R = \frac{16\, M}{\pi\, d^3}(M + \frac{F \times d}{8})$

c. $\sigma_R = \frac{32\, M}{\pi\, d^3}(M + \frac{F \times d}{16})$ d. $\sigma_R = \frac{16\, M}{\pi\, d^3}(M + \frac{F \times d}{16})$

28. Equation to find hollow shaft subjected to axial load in addition to combine torsion and bending loads:

a. $\sigma_R = \frac{32\, M}{\pi d^3\,(1-k^4)}[M + \frac{F.d\,(1-k^2)}{8}]$ b. $\sigma_R = \frac{16\, M}{\pi d^3\,(1-k^4)}[M + \frac{F.d\,(1-k^2)}{8}]$

c. $\sigma_R = \frac{32\, M}{\pi d^3\,(1-k^4)}[M + \frac{F.d\,(1-k^2)}{16}]$ d. $\sigma_R = \frac{32\, M}{\pi d^3\,(1-k^4)}[M + \frac{F.d\,(1-k^2)}{32}]$

29. A solid circular shaft is subjected to a bending moment of ($3000\, N.m$) and a torque of ($10000\, N.m$). The shaft is made of 45 C 8 steel having ultimate tensile stress of ($700\, MPa$) and a ultimate shear stress of ($500\, MPa$). Assuming a factor of safety as (6), determine the diameter of the shaft. [Ans: $d = 86\, mm$].

30. Figure (9-6) shows a mild steel shaft supported at both ends by ball bearings, carrying a straight tooth spur gear and transmitting (7.5 kW) at (300 rpm). The gear's pitch circle diameter is (150 mm). The distances between the center lines of the bearings and the gear are each (100 mm). Determine the diameter of the shaft if it is made of steel and the allowable shear stress is (45 MPa). Show how the gear will be mounted on the shaft in a sketch, as well as the ends where the bearings will be mounted. The gear's pressure angle can be used as (20°). [Ans: $d = 35\, mm$].

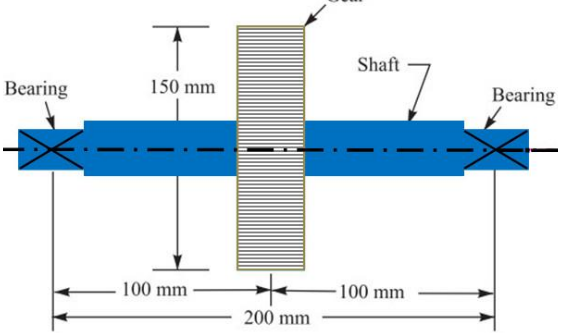

Figure (9-6): The mild steel solid shaft diagram

31. To transmit ($100\ kW$) at ($300\ rpm$), a mild steel shaft is needed, as shown in figure (9-7). The shaft's supported length is ($3000\ mm$). It supports two pulleys with a combined weight of ($1.5\ N$), each supported at a distance of ($1000\ mm$) from the ends. Determine the diameter of the shaft assuming the safe value of tension. **[Ans: $d = 70\ mm$].**

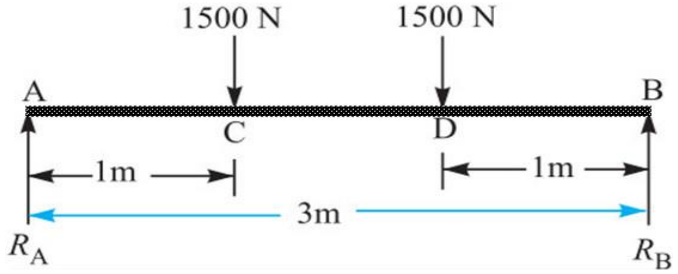

Figure (9-7): The mild steel solid shaft diagram

32. Figure (9-8) shows a mild shaft supported by two bearings spaced (1000 mm) apart. A (600 mm) diameter pulley is mounted (300 mm) to the right of the left hand bearing and drives a pulley directly below it with the aid of a belt with a maximum tension of (2.25 kN). Another pulley (400 mm) diameter is placed (200 mm) to the left of the right hand bearing and is powered by an electric motor and a belt that is horizontally to the right. Both pulleys have contact angles of (180°) and ($\mu = 0.24$). Determine the appropriate diameter for a solid shaft that allows for a working stress of (63 MPa) in tension and (42 MPa) in shear for the shaft material. Assume the torque on one pulley equals the torque on the other.

[Ans: $d = 55\ mm$].

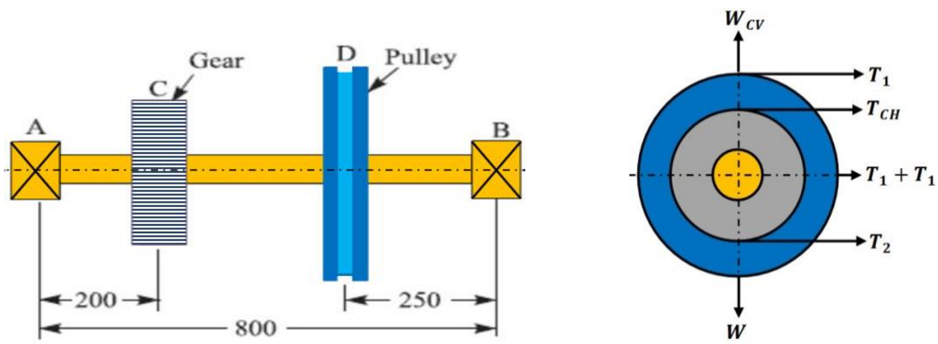

Figure (9-8): The mild steel solid shaft diagram

33. Two bearings spaced (1000 mm) apart support a mild shaft. A (600 mm) diameter pulley is mounted (300 mm) to the right of the left hand bearing and drives a pulley directly below it with the aid of a belt with a maximum tension of (2.25 kN). Another pulley (400 mm diameter) is placed (200 mm) to the left of the right hand bearing and is powered by an electric motor and a belt that is horizontally to the right. The contact angles for both pulleys are (180°) and ($\mu = 0.24$). Determine the appropriate diameter for a solid shaft with a working stress of (63 MPa) in tension and (42 MPa) in shear for the shaft material. Assume that the torque on one pulley equals the torque on the other**.**

[Ans: $d = 55\ mm$].

Chapter 10

Design of Journal Bearings

10. Design of Journal Bearings

10-1. Introduction

Bearings are technical devices that are part of the bearings of rotating axles and shafts. They detect radial and axial forces imparted to the shaft or axle and transmit those forces to the frame, body, or other components of the structure, figure (10-1). At the same time, they must also hold the shaft in space, provide rotation, swing or linear movement with minimal energy loss. The efficiency, performance and durability of the machine largely depend on the quality of the bearings. Currently, bearings are widely used:

1. Contact (having friction surfaces) - rolling and sliding bearings,
2. Contactless (not having rubbing surfaces) - magnetic bearings.

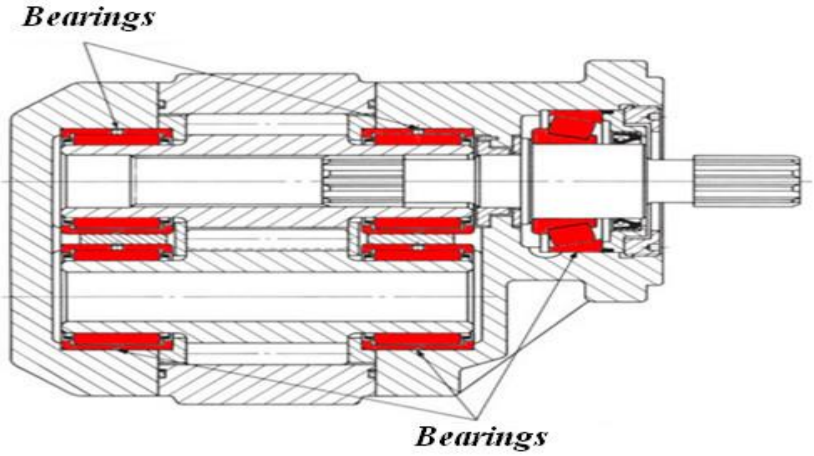

Figure 10-1: Bearings act as supports for axles and shafts

Bearings are designed to carry axial, radial or combined loads. The purpose determines the classification.

1. **Radial.** They perceive a load perpendicular to the axis of the shaft,
2. **Persistent.** Perceive axial load,
3. **Combined.** Designed to accommodate both radial and axial loads.

10-2. Journal bearing (Plain bearing)

Bearings operating on the sliding friction principle are called journal bearings. Journal bearings (more commonly known as plain bearings or sliding bearings) are constructed with a shaft (journal) that rotates in a supporting sleeve or housing. There is no rolling element in these bearings. Their design and construction are relatively simple, but the operation can be quite complex, figure (10-2).

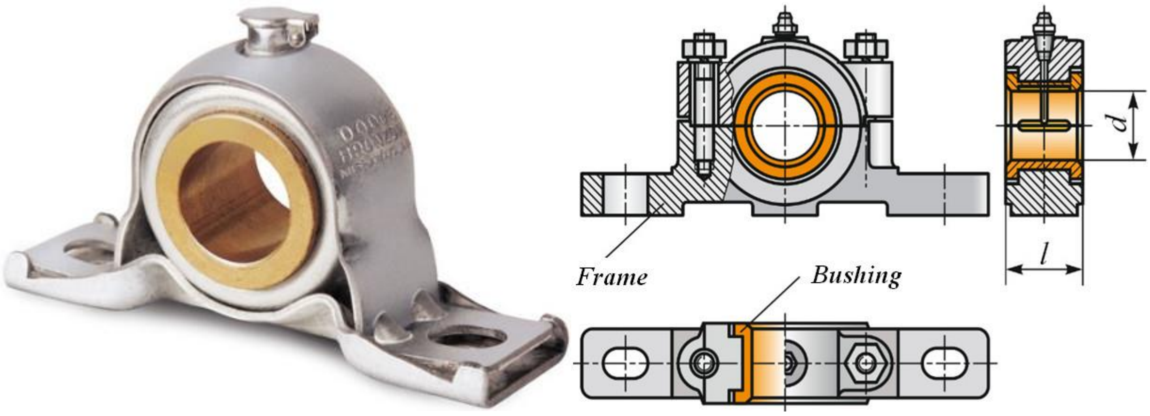

Figure 10-2: Journal bearing (Plain bearing)

Journal bearings are used to transmit torque in generators and internal combustion engines. The journal bearing design is simple. It consists of a body with a cylindrical bore into which the sleeve is placed. Slip in a pair of body-sleeve is provided by an anti-friction insert or a lubricating fluid. Water, gasoline, oils, oil emulsions, as well as solid lubricants of various consistencies can be used as a lubricant.

Journal bearing sliding action is along the circumference of a circle or an arc of a circle and carrying radial loads.

Journal bearings, depending on the method of lubricant supply and its type, are classified into:

1. Hydrostatic,
2. Hydrodynamic,
3. Gas Static,
4. Gas-Dynamic,
5. Solid Lubrication.

Hydrostatic journal bearings are lubricated from the outside, so the sliding ability of the bearing is due to the external pressure of the lubricating fluid.

In hydrodynamic journal bearings, the anti-friction effect is created by an oil insert located in the gap between the housing and sleeve.

Air or inert gas is used as a lubricant in gas-static and gas-dynamic journal bearings.

10-3. Spherical plain bearings

The main difference between ordinary bearings is that the inner and outer sliding surfaces are spherical.

Spherical bearings are used to transmit torque in the joints of fixed or movable mechanisms.

In fixed joints, there are single periodic displacements of a spherical bearing ring relative to another fixed ring.

Ordinary spherical bearings for movable links are distinguished by the fact that both spherical surfaces rotate relative to each other. The sliding speed of these bearings is limited.

10-4. Components of a Journal bearing (Plain bearing)

The most common design is plain bearings, which include a housing part (3) with an installed anti-friction liner (2). In the liner bore with a gap, the shaft journal (5) rotates or the rod moves linearly. Through the system of holes (1) and distribution grooves, grease (4) is supplied into the gap, separating the contacting surfaces, figure (10-3):

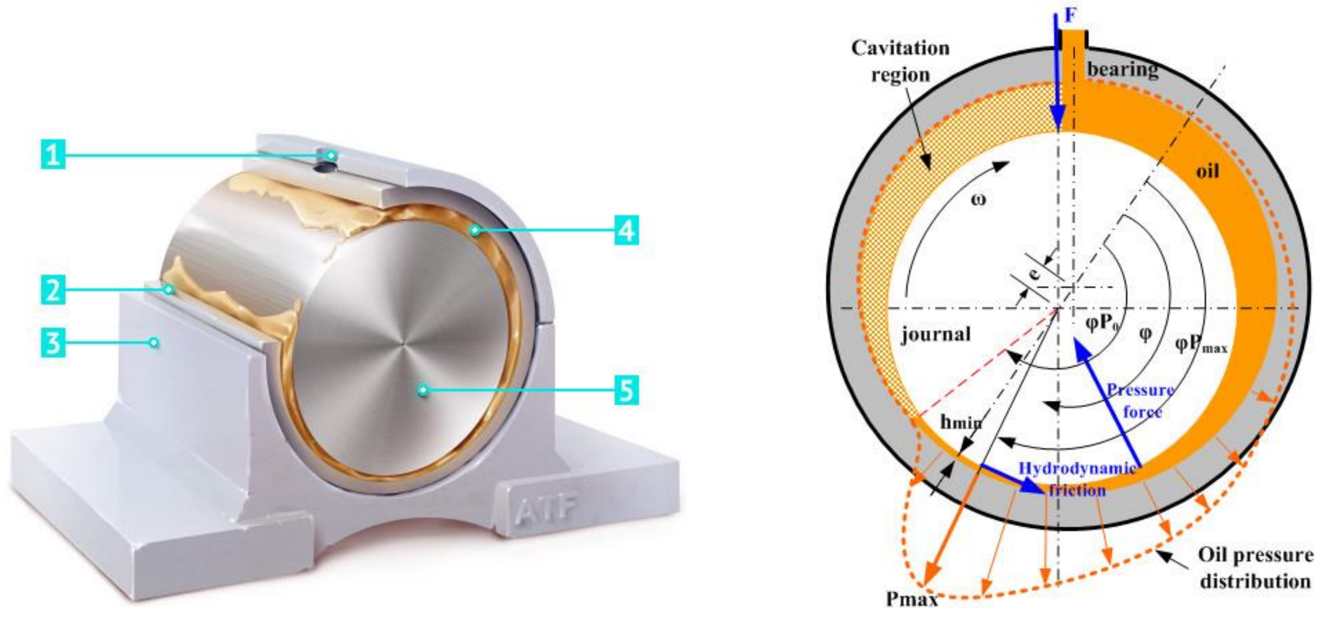

Figure 10-3: Journal bearing component

The following the components of a Journal bearing are:

1. Oil inlet,
2. Bearing liner,
3. Housing,
4. Clearance,
5. Shaft or Journal.

10-5. Materials used in the manufacture of a journal bearing

Journal bearing are any two materials rubbing each other and the materials it is made of depend on the application, the load, the speed and the materials, plain bearings adjust to the sliding counterpart and made of the following:

1. First part is typically structural material (Steel, Cast Iron, etc.),
2. Second part is made of bearing material (Bronze, Babbitt Non-metallic). Polymers.

The proportions of materials used in each alloy

1. Babbitt metal

Tin base Babbitt: Tin 90 %, Copper 4.5 %, Antimony 5 %, Lead 0.5 %. Lead base Babbitt: Lead 84 %, Tin 6 %, Antimony 9.5 %, Copper 0.5 %.

2. Bronzes

a. For high-grade bearings subjected to high pressures and speeds (no more than 10 N/mm^2 of predicted area), gun metal (copper 88 %, tin 10 %, and zinc 2 %) is utilized.
b. Phosphor bronze, which has a composition of 80 % copper, 10 % tin, 9 % lead, and 1 % phosphorus, is used for bearings that must withstand exceptionally high pressures and speeds (not more than 14 N/mm^2 of predicted area).

3. Cast iron

Steel journals are frequently used with cast iron bearings. When lubrication is sufficient, pressure is restricted to 3.5 N/mm^2, and speed is restricted to 40 meters per minute, these bearings are comparatively successful.

4. Silver

The majority of silver and silver lead bearings are used in aircraft engines, where fatigue strength is crucial.

5. Non-metallic bearings

Carbon-graphite, rubber, wood, and plastics are used to make the various non-metallic bearings. Under a variety of operating circumstances, carbon-graphite bearings are self-lubricating and dimensionally stable.

10-6. Uses of the journal bearing

It should be noted that sleeve bearings are not as widely used as rolling bearings. However, in modern mechanical engineering there are areas of application in which sleeve bearings cannot be dispensed with. T following some of them:

1. Bearings designed for particularly high speeds,
2. Bearings for uniform and very precise rotation and rotation,
3. Split bearings (for example, for crankshafts),
4. Bearings for heavy shafts, where the use of rolling bearings is not economically viable,
5. Bearings operating under high vibration or shock loading,
6. Bearings of very small diameters (for example, for closely spaced shafts),
7. Bearings for low-critical low-speed and auxiliary mechanisms.

10-7. Advantages and disadvantages of a Journal bearing (Plain bearing)

Journal bearings have advantages and disadvantages when compared to rolling bearings, some of them are:

a. Advantages:

1. They are lighter,
2. Lower costs,
3. Longer fatigue-free service life of their elements,
4. Lighter weight,
5. Less noisy,
6. They require less radial space, since they are built with thin walls,

7. Their installation is simpler,
8. Using self-lubricating bearings, lubrication procedures are not required,
9. They allow for higher rotational speeds, and Greater shock resistance.

b. Disadvantages:

1. Higher friction during transient processes (especially during startup),
2. They require more axial space,
3. The use of friction resistant materials in their manufacturing is indispensable,
4. Greater wear when compared to rolling bearings, since there is a direct friction between the bush and the shaft.

10-8. Design of a Journal bearing (Plain bearing)

The following steps to design Journal bearing, figure (10-4).

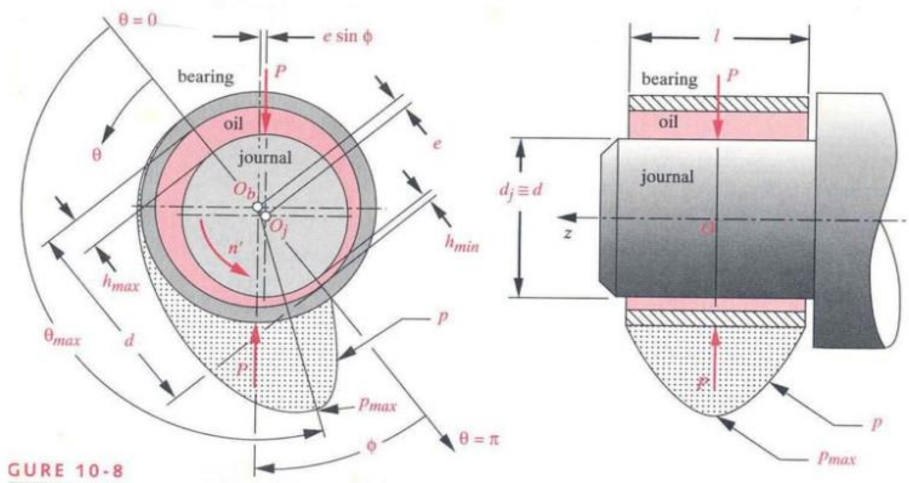

Figure 10-4: Basic terms of a hydro dynamic bearing design radial load

1. Calculate the diameter of journal bearing (d) from power, speed, torque and shear stress relationships such as:

$$T = \frac{60\,P}{2\,\pi\,N} \qquad (10-1)$$

$$d = \sqrt[3]{\frac{16\,T}{\pi\,\tau}} \qquad (10-2)$$

2. Find the length of journal bearing, from an appropriate $(\frac{l}{d})$ ratio from the following tables (10-1).

Dr. Emad T. Karash Dr. Jamal N. Sultan Ahmed M. Mosa Iman Z. Alshih Yahya

Table 10-1: Design value for journal bearings

Machinery	Bearing	Maximum bearing pressure (p) in N/mm^2	Operating values			
			Absolute Viscosity (Z) in kg/m-s	ZN/p Z in kg/m-s p in N/mm^2	$\frac{c}{d}$	$\frac{l}{d}$
Railway cars	Axle	3.5	0.1	7	0.001	1.8 – 2
Steam turbines	Main	0.7 – 2	0.002 – 0.016	14	0.001	1 – 2
Generators, motors, centrifugal pumps	Rotor	0.7 – 1.4	0.025	28	0.0013	1 – 2
Transmission shafts	Light, fixed Self -aligning Heavy	0.175 1.05 1.05	0.025- 0.060	7 2.1 2.1	0.001	2 – 3 2.5 – 4 2– 3
Machine tools	Main	2.1	0.04	0.14	0.001	1–4
Punching and shearing machines	Main Crank pin	28 56	0.10	—	0.001	1–2
Rolling Mills	Main	21	0.05	1.4	0.0015	1–1.5
Automobile and air-craft engines	Main Crank pin Wrist pin	5.6 – 12 10.5 – 24.5 16 – 35	0.007 0.008 0.008	2.1 1.4 1.12	—	0.8 – 1.8 0.7 – 1.4 1.5 – 2.2
Four stroke-Gas and oil engines	Main Crank pin Wrist pin	5 – 8.5 9.8 – 12.6 12.6 – 15.4	0.02 0.04 0.065	2.8 1.4 0.7	0.001	0.6 – 2 0.6 – 1.5 1.5 – 2
Two stroke-Gas and oil engines	Main Crank pin Wrist pin	3.5 – 5.6 7 – 10.5 8.4 – 12.6	0.02 0.04 0.065	3.5 1.8 1.4	0.001	0.6 – 2 0.6 – 1.5 1.5 – 2
Marine steam engines	Main Crank pin Wrist pin	3.5 4.2 10.5	0.03 0.04 0.05	2.8 2.1 1.4	0.001	0.7 – 1.5 0.7 – 1.2 1.2 – 1.7
Stationary, slow speed steam engines	Main Crank pin Wrist pin	2.8 10.5 12.6	0.06 0.08 0.06	2.8 0.84 0.7	0.001	1 – 2 0.9 – 1.3 1.2 – 1.5
Stationary, high speed steam engine	Main Crank pin Wrist pin	1.75 4.2 12.6	0.015 0.030 0.025	3.5 0.84 0.7	0.001	1.5 – 3 0.9 – 1.5 13 – 1.7
Reciprocating pumps and compressors	Main Crank pin Wrist pin	1.75 4.2 7.0	0.03 0.05 0.08	4.2 2.8 1.4	0.001	1 – 2.2 0.9 – 1.7 1.5 – 2.0
Steam locomotives	Driving axle Crank pin Wrist pin	3.85 14 28	0.10 0.04 0.03	4.2 0.7 0.7	0.001	1.6 – 1.8 0.7 – 1.1 0.8 – 1.3

3. Calculate the bearing pressure (P) by using the following equation:

$$P = \frac{W}{l.d} \qquad (10-3)$$

$Where: \quad W = applied\ load.$

Can calculate the critical pressure or minimum operating pressure from the following equation:

$$P = \frac{Z.N}{475 \times 10^3} \left[\frac{d}{c}\right]^2 \left[\frac{1}{d+1}\right], \qquad \left\{\frac{N}{mm^2}\right\} \qquad (10-4)$$

$Where: \qquad (Z)\ in\ \left\{\frac{Kg}{m.s}\right\}$

After that check this pressure with allowable pressure, given in table (10-1). If the pressure not be within the range, change ($\frac{l}{d}$) ratio suitable.

4. Consider the operating temperature properly. Commonly it may be assumed from (60°) to (90°).

5. Choose the diametric clearance (c), [$\frac{c}{d} = 0.001$]. Can calculate it from difference between the diameter of the bearing (D) and diameter of the journal (d).

$$c = D - d \qquad (10-5)$$

6. Select the value of bearing characteristic number ($\frac{Z\,N}{P}$) from table (10-1) and from that parameter determine (Z) and find corresponding lubricant.

$$Unit\ of\ (Z)\ \ is\ \ Centipoise, \qquad where:$$

$$\left\{Centipoise = 0.01\ Poise = 0.001\ \frac{N.s}{m^2} = 0.001\ \frac{Kg}{m.s}\right\}.$$

7. Calculate the coefficient of friction (μ) by using appropriate relation.

The formula for the coefficient of friction is:

$$\mu = \frac{33}{10^8}\left\{\frac{Z.N}{P}\right\}\left\{\frac{d}{c}\right\} + K \qquad (10-6)$$

$Where:$

$$\mu = \text{Coefficient of friction, N} = \text{Speed of the journal in (rpm)},$$

$$\text{K} = \text{Factor to correct for end leakage an depends on } \left(\frac{l}{d}\right),$$

$$\text{K} = 0.002 \text{ for } \left(\frac{l}{d} = 0.75 \text{ to } 2.8\right),$$

$$\text{Z} = \text{Absolute viscosity of the lubricant in } \left(\frac{\text{Kg}}{\text{m. s}}\right),$$

$$\text{P} = \text{Bearing pressure on the projected bearing area } \left(\frac{\text{N}}{\text{mm}^2}\right),$$

$$d = Diameter\ of\ the\ journal,$$

$$l = Length\ of\ the\ bearing, \qquad and\ \ c = Diameter\ clearance.$$

8. Determine the heat generated (H_g) and heat dissipated (H_d).

9. If ($H_g > H_d$) provide artificial cooling arrangements.

$$H_g = \mu.W.V \ , \ \left\{\frac{N.m}{s} \ or \ \frac{J}{s}\right\}. \qquad (10-7)$$

$$H_d = C.A\,(t_b - t_a) \ , \ \left\{\frac{J}{s} \ or \ watt\right\} \qquad (10-8)$$

Were,

$$V = Rubbing\ velocity\ in \left\{\left(\frac{m}{s}\right) \ \& \left(V = \frac{\pi\, d.N}{60}\right)\right\},$$

$$C = Heat\ dissipation\ coefficient\ \ in \left\{\frac{W}{m^2.C^O}\right\},$$

$$A = Projected\ area\ of\ bearing\ in\{m^2\},$$

$$t_b = Temperature\ of\ bearing\ surface\ in\ \{C^O\},$$

$$t_a = Temperature\ of\ surrounding\ in\ \{C^O\}.$$

10. Decide proper bearing material and other required dimensions for the journal bearing.

10-9 Solve examples

Example 1:

Design a journal bearing mounted on shaft reciprocating pump and calculate mass of the lubricating oil required for artificial cooling from the following data:

Load of journal ($W = 252\ KN$), diameter of journal ($d = 300\ mm$), speed of journal ($N = 2100\ rpm$), bearing pressure ($P = 4.2\ MPa$), type of oil is SAE 12, operating temperature ($t_o = 66\ C^o$), have viscosity of oil ($Z = 50\ cp$) and specific heat of oil $\left(S = 2000\ \frac{J}{Kg.C^o}\right)$, bearing clearance ($c = 0.33$), rise temperature of oil be limited to ($t = 7\ C^o$), and heat dissipation coefficient $\left(C = 1232\ \frac{W}{m^2.C^o}\right)$. If atmospheric temperature of oil ($t_a = 18\ C^o$).

<u>Solution</u>

1. Length of bearing (l),

$$\because \quad P = \frac{W}{l.d}$$

$$\therefore\ l = \frac{W}{P.d} = \frac{252000}{4.2 \times 300} = 200\ mm$$

2. Coefficient of friction (μ),

$$\because \quad \mu = \frac{33}{10^8}\left\{\frac{Z.N}{P}\right\}\left\{\frac{d}{c}\right\} + K$$

$$\frac{Z.N}{P} = \frac{0.05 \times 2100}{4.2} = 25\ , \quad \frac{d}{c} = \frac{300}{0.33} = 909.09\ , \quad \frac{l}{d} = \frac{200}{300} = 0.87$$

$$\therefore\ K = 0.002\quad for \quad \left(\frac{l}{d} = 0.75 \quad to \quad 2.8\right)$$

$$\therefore\ \mu = \frac{33}{10^8}\,(25).(909.09) + 0.002 = 0.0095$$

3. Heat generated,

$$H_g\ =\ \mu.W.V\ ,\ \left\{\frac{N.m}{s}\ \ or\ \ \frac{J}{s}\ or\ \ Watt\right\}$$

$$V = \frac{\pi\, d.N}{60} = \frac{3.14 \times 0.3 \times 2100}{60} = 32.97\ m/s$$

$$\therefore\ H_g = 0.0095 \times 252000 \times 32.97 = 78930.18\, Watt \approx 78.93\, KW$$

4. Heat dissipated,

$$H_d = C.A\,(t_b - t_a)\ ,\ \left\{\frac{J}{S}\ or\ watt\right\}$$

$$A = l.d = 200 \times 300 = 60000\, mm^2 = 0.06\ m^2$$

$$(t_b - t_a) = \frac{1}{2}\,(t_o - t_a) = \frac{1}{2}\,(66 - 18) = 42\ C^o$$

$$H_d = 1232 \times 0.06 \times 42 = 3104.64\ Watt$$

5. Mass of the lubricating oil required (m),

Amount of artificial heat cooling (H_t):

$$H_t = H_g - H_d = 78930.18 - 3104.64 = 75825.54\, Watt$$

$$Also, \qquad H_t = m.S.t = m \times 2000 \times 7 = 14000\, m$$

$$\therefore\ m = \frac{14000}{H_t} = \frac{14000}{75825.54} = 0.185\, Kg/s$$

Example 2:

Design a journal bearing mounted on shaft a steam turbine, as show in figure (10-5) from the following data:

Weight of turbine with shaft ($W = 133\, KN$), speed of shaft ($N = 2000\, rpm$) and diameter of shaft ($d = 120\, mm$). If operating temperature of bearing ($t_b = 65\ C^o$), and atmospheric temperature of oil ($t_a = 28\ C^o$), specific heat of oil $\left(S = 1999\ \frac{J}{Kg.C^o}\right)$, rise temperature of oil be limited to ($t = 10\ C^o$), and heat dissipation coefficient $\left(C = 1232\ \frac{W}{m^2.C^o}\right)$. Assuming $\left(\frac{l}{d} = 1.35\ ,\ \frac{c}{d} = 0.001\right)$.

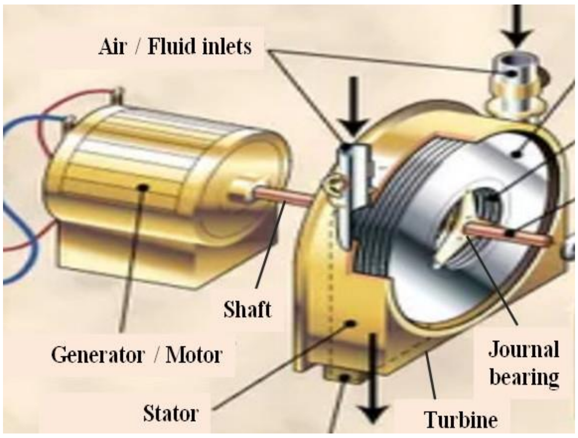

Figure 10-4: Journal bearing mounted on shaft a steam turbine

Solution

1. Calculate journal bearing dimension,

$$\because\ \frac{l}{d} = 1.33 \quad \Rightarrow \therefore l = 1.35d = 1.35 \times 120 = \ 162\ \ mm$$

$$P = \frac{W}{l.d} = \frac{33000}{120 \times 162} \approx 1.7 \quad N/mm^2 (MPa)$$

This pressure ($P = 1.7 \ \ N/mm^2$) is within the safe range ($0.7\ to\ \ 2\ \ N/mm^2$), table (10-1).

2. Calculate the viscosity of lubricant (Z),

From table (10-1), $\frac{Z.N}{P} = 14$

$$\therefore Z = \frac{14\ P}{N} = \frac{14 \times 1.7}{2000} = 0.019\ \frac{Kg}{m.s}$$

3. Heat generated,

$$H_g \ = \ \mu.W.V \ \ , \ \ \left\{\frac{N.m}{s} \ \ or \ \ \frac{J}{s} \ or \ \ Watt\right\}$$

$$\therefore \quad \mu = \frac{33}{10^8}\left\{\frac{Z.N}{P}\right\}\left\{\frac{d}{c}\right\} + K$$

$$\because \frac{c}{d} = 0.001 \quad \Rightarrow \quad \therefore\ c = 0.001 \times 120 = 0.12\,mm$$

$$\therefore\ K = 0.002 \ \ for \quad \left(\frac{l}{d} = 0.75 \ \ to \ \ \ 2.8\right)$$

$$\because \quad \mu = \frac{33}{10^8}\left\{\frac{0.019 \times 2000}{1.7}\right\}\left\{\frac{120}{0.12}\right\} + 0.002 = 0.0094$$

$$V = \frac{\pi\, d.N}{60} = \frac{3.14 \times 0.12 \times 2000}{60} = 12.56\ m/s$$

$$\therefore\ H_g\ =\ 0.0094 \times 33000 \times 12.56 = 3896.11\, Watt\ \approx 3.896\ KW$$

4. Heat dissipated,

$$H_d\ =\ C.A\,(t_b - t_a)\ ,\ \left\{\frac{J}{s}\ or\ watt\right\}$$

$$A = l.d = 162 \times 120 = 19440\, mm^2$$

$$H_d\ =\ 1232 \times 0.01944 \times (65 - 28) \approx\ \ 886.15\ Watt$$

5. Mass of the lubricating oil required (m),

Amount of artificial heat cooling (H_t):

$$H_t\ = H_g\ - H_d\ = 3896.11 - 886.15 = 3009.96\ Watt$$

$$Also,\qquad H_t\ = m.S.t = m \times 1999 \times 10 = 19990\, m$$

$$\therefore\ m = \frac{19990}{H_t} = \frac{19990}{3009.96} = 6.64\, Kg/s$$

6. Diameter of bearing (D),

$$D = d + c = 120 + 0.12 = 120.12\, mm$$

Resulting design dimension journal bearing for steam turbine				
No.	**Paragraph name**	Symbol	**Amount**	**Unit**
1.	diameter of shaft (journal)	d	*120*	*mm*
2.	Length of bearing journal	l	*162*	*mm*
3.	Diameter of bearing	D	*120.12*	*mm*
4.	Diameter clearance	c	*0.12*	*mm*
5.	Viscosity of lubricant	Z	*0.019*	*Kg/m.s*
6.	Operating temperature of bearing	t_b	*65*	*C°*
7.	Atmospheric temperature	t_a	*28*	*C°*
8.	Heat generated	H_g	3896.11	$Watt$
9.	Heat dissipated	H_d	886.15	$Watt$
10.	Mass of the lubricating oil required	m	6.64	Kg/s
11.	Material selected	*Bronze bushing or Cast iron*		

10-10 Chapter Questions

1. Bearings are widely used:

a. sliding bearings.
b. rolling bearings.
c. magnetic bearings.
d. rolling, sliding, and magnetic bearings.

2. Bearings are designed to carry:

a. axial loads
b. radial loads
c. tension loads
d. axial, radial or combined loads

3. Bearings are designed to carry axial, radial or combined loads. The purpose determines the classification.

a. Radial
b. Persistent
c. Combined
d. Radial, persistent, and combined

4. Bearings operating on the sliding friction principle are called ------------- bearings.

a. journal
b. ball
c. roller
d. thrust

5. The main difference between ordinary bearings is that the inner and outer sliding surfaces are:

a. spherical
b. square
c. convex
d. concave

6. Journal bearing are any two materials rubbing each other and the materials it is made of depend on the:

a. type
b. application
c. shape
d. dimension

7. Tin base Babbitt:

a. Tin 890%, Copper 9.5%, Antimony 5.8 %, Lead 0.7% Lead

b. Tin 90%, Copper 4.5%, Antimony 5%, Lead 0.5% Lead

c. Tin 95%, Copper 6.5%, Antimony 7%, Lead 2.5% Lead

d. Tin 92%, Copper 5.5%, Antimony 5.5%, Lead 1.0% Lead

8. To calculate the bearing pressure (P) by using the following equation:

a. $P = \frac{W}{c.d}$ b. $P = \frac{W}{l.c}$

c. $P = \frac{W}{l.\mu}$ d. $\boldsymbol{P = \frac{W}{l.d}}$

9. The coefficient of friction ($\boldsymbol{\mu}$) can be written as:

a. $\boldsymbol{\mu = \frac{33}{10^8}\left\{\frac{Z.N}{P}\right\}\left\{\frac{d}{c}\right\} + K}$

b. $\mu = \frac{30}{10^8}\left\{\frac{Z.N}{P}\right\}\left\{\frac{d}{c}\right\} + K$

c. $\mu = \frac{31}{10^8}\left\{\frac{Z.N}{P}\right\}\left\{\frac{d}{c}\right\} + K$

d. $\mu = \frac{32}{10^8}\left\{\frac{Z.N}{P}\right\}\left\{\frac{d}{c}\right\} + K$

10. Design a journal bearing mounted on shaft centrifugal pump and calculate mass of the lubricating oil required for artificial cooling from the following data:
Load of journal ($W = 20\ KN$), diameter of journal ($d = 300\ mm$), speed of journal ($N = 900\ rpm$), bearing pressure ($P = 1.5\ MPa$), type of oil is SAE 12, operating temperature ($t_o = 55\ C^o$), have viscosity of oil ($Z = 50\ cp$) and specific heat of oil $\left(S = 2000\ \frac{J}{Kg.C^o}\right)$, bearing clearance ($c = 0.33$), rise temperature of oil be limited to ($t = 10\ C^o$), and heat dissipation coefficient $\left(C = 1232\ \frac{W}{m^2.C^o}\right)$. If atmospheric temperature of oil ($t_a = 15\ C^o$). $[Ans: l = 160\ mm, \mu = 0.005, H_g = 480.7\ W, H_d = 91.4\ W,\ and\ m = 0.288 \frac{Kg}{min}]$.

11. Design a journal bearing mounted on shaft centrifugal pump and calculate mass of the lubricating oil required for artificial cooling from the following data:
Load of journal ($W = 7\ KN$), diameter of journal ($d = 50\ mm$), speed of journal ($N = 900\ rpm$), bearing pressure ($P = 1.4\ MPa$), operating temperature ($t_o = 75\ C^o$), have viscosity of oil ($Z = 11\ cp$) and specific heat of oil $\left(S = 1850\ \frac{J}{Kg.C^o}\right)$, bearing clearance ($c = 0.33$), rise temperature of oil be limited to ($t = 10\ C^o$), and heat dissipation coefficient $\left(C = 1232\ \frac{W}{m^2.C^o}\right)$. If atmospheric temperature of oil ($t_a = 35\ C^o$). $[Ans: l = 100\ mm, \mu = 0.00433, H_g = 71.5\ W, H_d = 28\ W,\ and\ m = 0.23 \frac{Kg}{min}]$.

Chapter 11

Selection of Ball Bearings

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11. Selection of Ball Bearings

11-1. Introduction

Bearings are used to help reduce friction. Metal-upon-metal contact produces large amounts of friction. The friction adds to wear and tear of the metal, producing grinding that slowly degrades the metal. Bearings reduce friction by having the two surfaces roll over each other, reducing the amount of friction produced. They consist of a smooth metal ball or roller that rolls against a smooth inner and outer metal surface. The rollers or balls take the load, allowing the device to spin.

- A machine component that supports another moving machine component is a bearing (known as journal). It permits relative movement between the contact surfaces of the components while supporting the load.
- Due to the relative motion between the contact surfaces, some power is lost in overcoming frictional resistance, and rapid wear results if the rubbing surfaces are in direct contact.
- To lessen wear and friction, as well as in some circumstances to carry away the heat produced, a layer of liquid (sometimes referred to as lubricant) may be applied.
- Vegetable oils, silicon oils, greases, and other substances may also be used in place of the traditional mineral oil refined from petroleum as the lubricant to keep the journal and bearing apart.

11-2. Factors required before determining bearing

When looking for a suitable bearing in the SKF catalogs, there are many factors to bear in mind:

1. Available space,
2. Acting loads (their magnitude and direction),
3. Skew,
4. Precision manufacturing and rigidity of structural elements,
5. Rotation frequency,
6. Working temperatures,

7. Vibration levels,
8. Contamination levels,
9. Type and method of lubrication.

11-3. Factors required after determining bearing

After determining a suitable bearing, you need to pay attention to a number of other equally important factors:

1. Suitable shape and design of other parts of the assembly,
2. Correct selection of ring seating, ensuring correct internal clearance or bearing preload,
3. Latching devices,
4. The correct choice of seal,
5. Type and quantity of lubricant,
6. Methods of installation and dismantling.

11-4. Classification of bearings

Rolling bearings are classified based on the following criteria:

I. By the type of rolling bodies

a. Ball bearing

There are many different of ball bearing, the most important:

1. Deep groove,
2. Filling notch,
3. Angular contact,
4. Sealed,
5. External,
6. Double row,
7. Self – aligning,
8. Thrust,
9. Self – aligning thrust.

b. Roller bearing

There are many different of ball bearing, the most important:

1. Straight cylindrical,

2. Spherical roller thrust,
3. Taper roller thrust,
4. Needle,
5. Tapered roller,
6. Step angle - Taper roller.

II. **By type of perceived (direction) load, figure (11-1).**

a. Radial (load along the shaft axis is not allowed),
b. Radial thrust, thrust radial. They accept loads both along and across the axis of the shaft. Often the load along the axis is only one direction,
c. Thrust (load across the shaft axis is not allowed).
   1. Ball screws. Provide screw-nut interface through rolling elements.

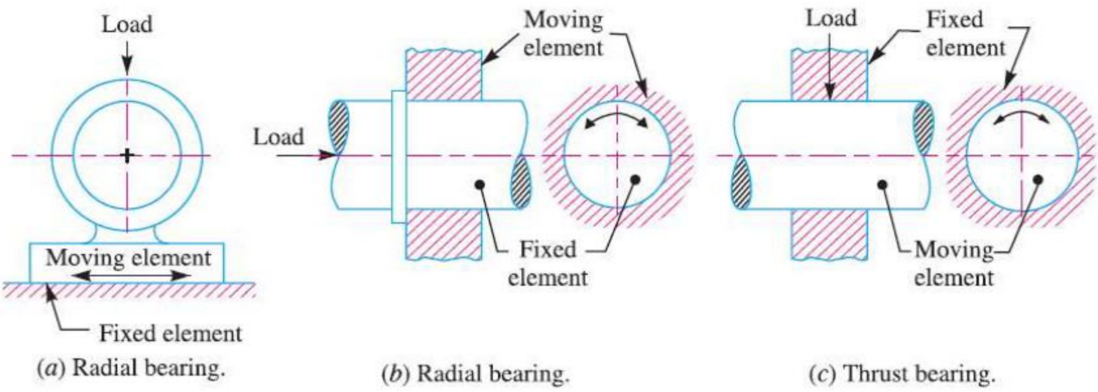

Figure 11-1: Types of bearing depending upon the direction of load to be supported

III. **By the number of rows of rolling elements**

a. Single row,
b. Double row,
c. Multi-row,
d. Self-aligning,
e. Non-self-aligning.

IV. **According to the material of the rolling elements:**

a. Completely steel,
b. Hybrid: steel rings, rolling elements, non-metallic, usually ceramic, used in a high-speed mechanism, most often in gas turbine engines,

V. Depending upon the nature of contact

Figure (11-2) shows the types of bearing depending upon the nature of contact:

a. Sliding contact bearings,

b. Rolling contact bearings.

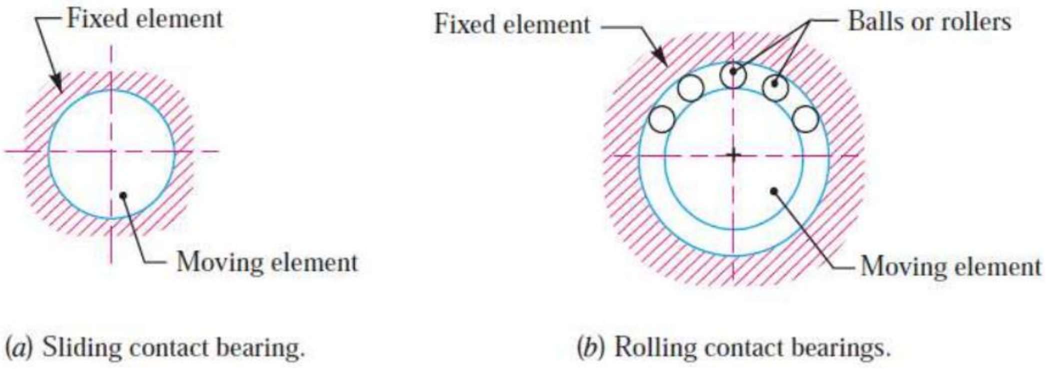

Figure 11-2: Various bearing types based on the type of contact

11-5. Types of Bearings

There are six most popular different types of bearings:

a. Plain bearings,
b. Rolling element bearings,
c. Jewel bearings,
d. Fluid bearings,
e. Magnetic bearings,
f. Flexure bearings.

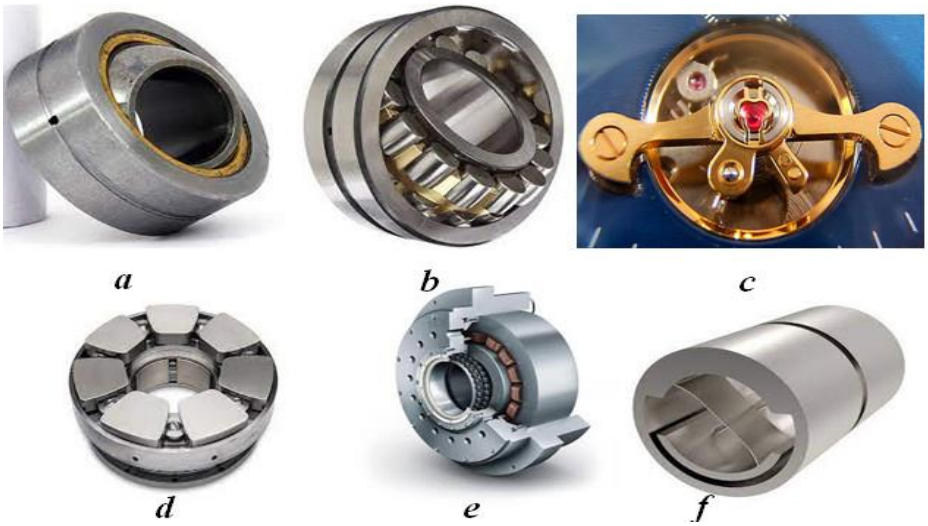

Figure 11-3: Types of bearing

11-6. Types of Sliding Contact Bearings

The following figure (11-4) shows the types of sliding contact bearing:

a. Full journal bearing,
b. Partial bearing,
c. Fitted journal bearing.

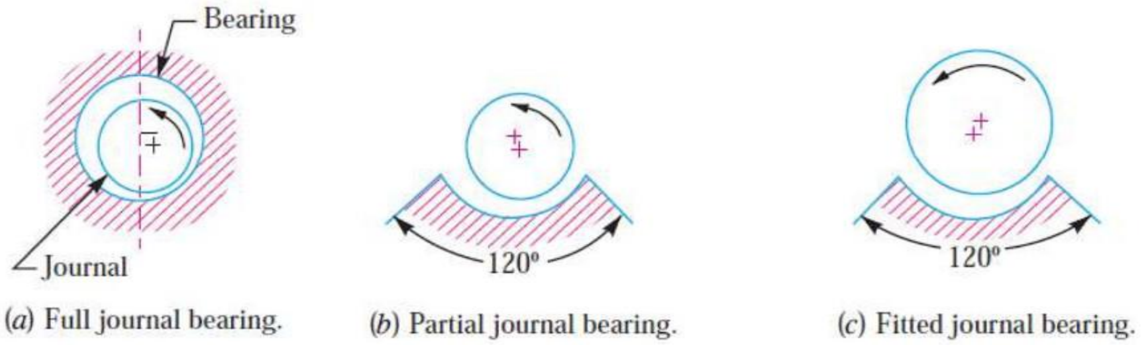

Figure 11-4: Types of Sliding Contact Bearings

11-7. Main parts of bearing

The bearing consists of five main parts, as shown in Figure (11-5).

1. Outer ring,
2. Ball (rolling body),
3. Separator,
4. Raceway,
5. Inner ring.

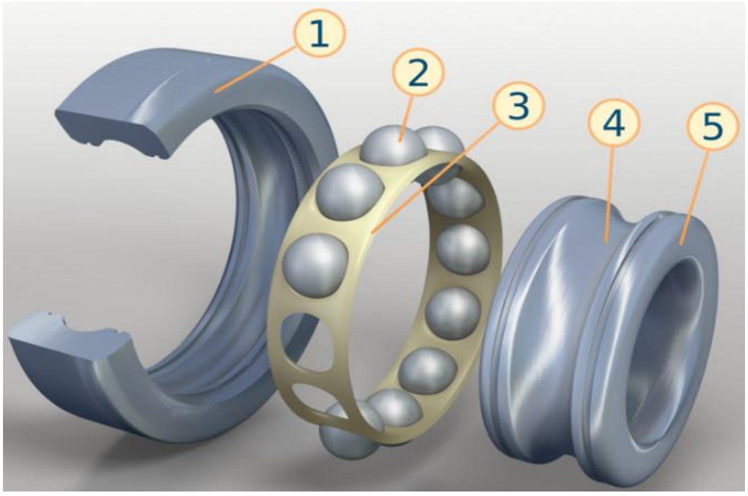

Figure 11-5: Main parts of bearing

11-8. Compounded of ball and rolling bearings.

Figure (11-6) shows the compounded of ball and rolling bearings.

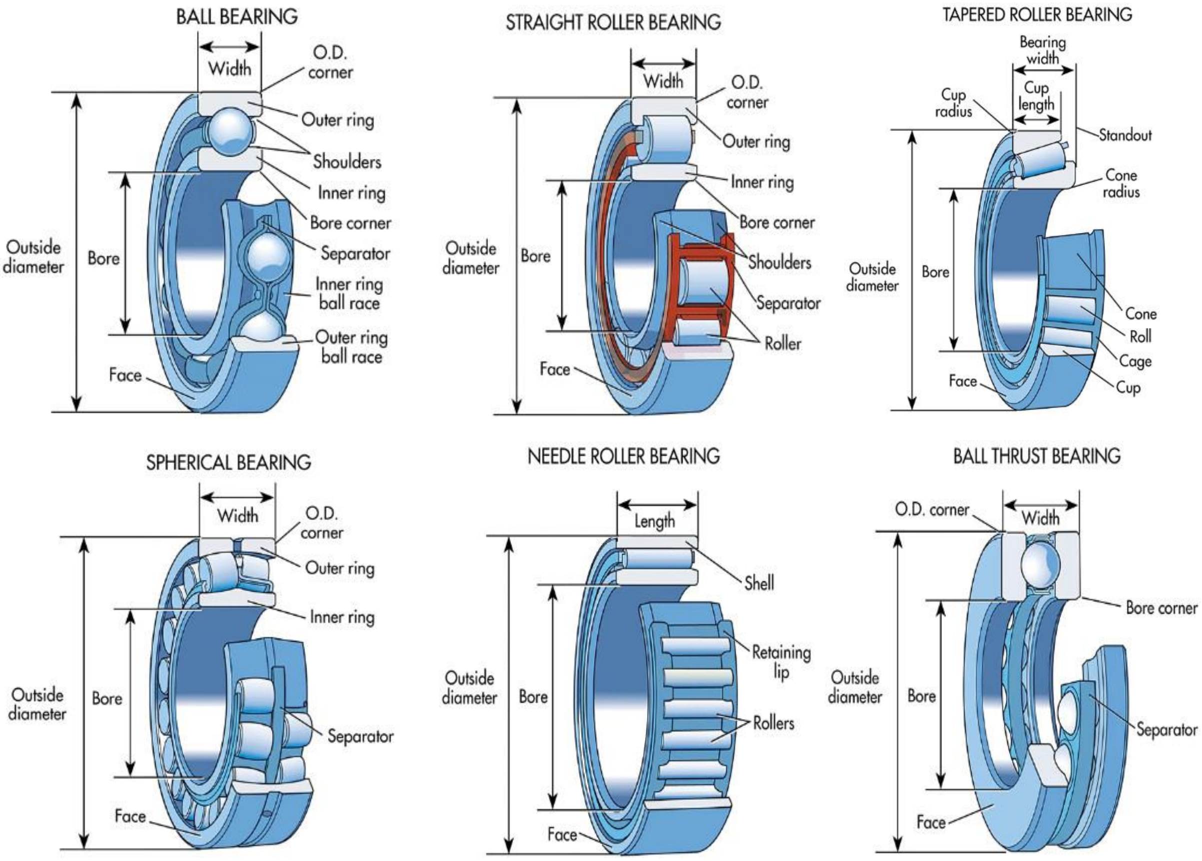

Figure 11-6: Compounded of ball and rolling bearings

11-9. Types of motion in the bearing

The movements allowed by the bearings are:

1. Axial rotation.
2. Linear motion, such as a drawer.
3. Spherical movement, such as a ball joint.
4. Articulated (hinge) movement, such as doors, joint and knee.

11-10. Friction in the bearing

Reducing friction in bearings is important for efficiency, in order to reduce wear and to allow prolonged use at high speeds to avoid overheating and premature bearing breakdown. A bearing can reduce friction by virtue of its shape, its materials, the presence of fluid between surfaces or the separation of surfaces with a magnetic field.

1. **Shape**, where the spherical or cylindrical shape gives the advantage of reducing friction, or by forming flexural bearings.
2. **Materials**, takes advantage of the nature of the material used in the bearing.
3. **Fluid,** Exploits the low viscosity of a layer of fluid as a lubricant or as a pressurized medium to keep the two solid surfaces out of contact, or to reduce the regulating forces between them.
4. **Field,** exploits electromagnetic fields such as magnetic fields to keep the two solid surfaces from touching.
5. **Air pressure,** exploits air pressure to keep solid parts from touching.

A combination of these means can be used in the same bearing.

11-11. Types of Rolling Contact Bearings

Rolling element bearings have been widely used in rotating equipment. They are designed to carry load and provide constrained motion between two or more parts.

Below are common types of rolling element bearings, figure (11-7):

a. Cylindrical roller bearings with cage

Single row cylindrical roller bearings with cage consist of inner ring, outer ring, cylindrical rollers, and cage.

b. Deep groove ball bearings

Deep groove ball bearings are probably the most common rolling element bearings used since they are durable and easy to maintain.

c. Needle roller bearings

Needle roller bearings can be single or double row units. The rolling elements are needle rollers. They are mostly used in applications which have small envelope in radial direction. They can carry very high load and can be fitted easily.

d. Self-aligning ball bearings

Self-aligning bearings are double row units, which consist of solid outer ring with a concave raceway, inner ring with a cylindrical or tapered bore, balls, and cage.

e. Spherical roller bearings

Spherical roller bearings are double row units which consist of solid outer ring with a concave raceway, solid inner ring, barrel rollers, and cage.

f. Roller thrust bearing

Roller thrust bearings can be used to support thrust loads – axial direction.

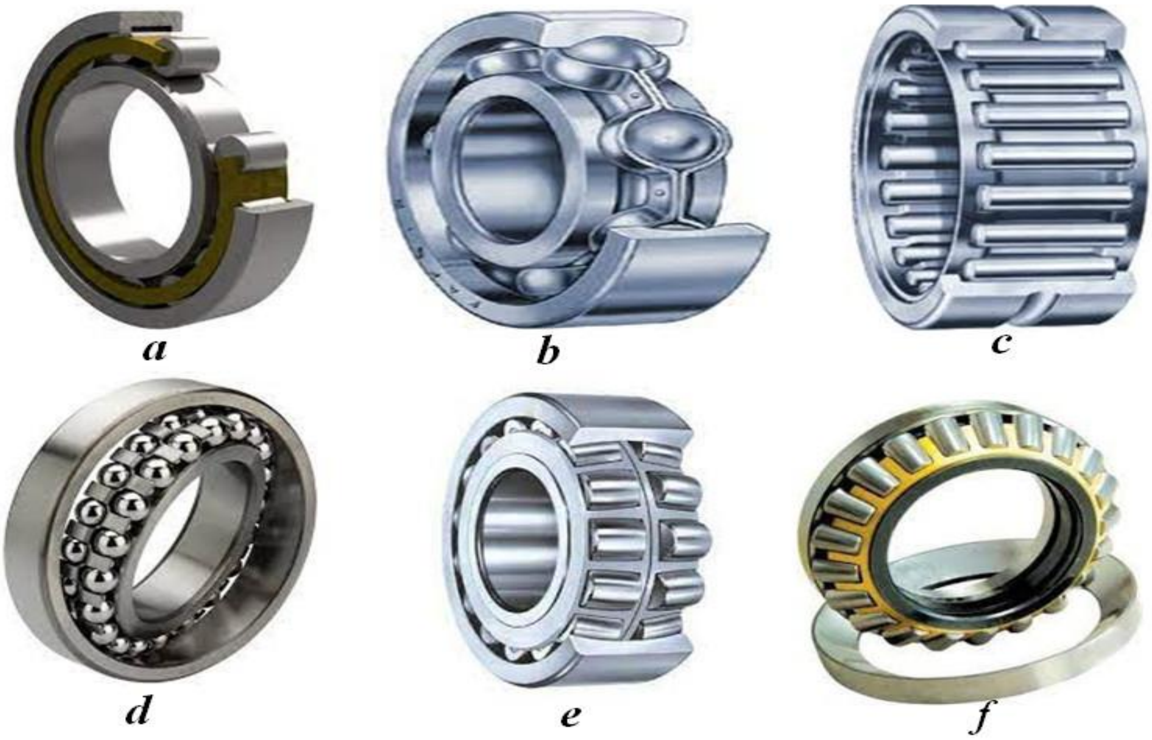

Figure 11-7: Types of ball and rolling bearings

11-12. Bearing series

There are four types of bearing series, figure (11-8).

1. Extra light bearing (less than 200),
2. Light bearing (200),
3. Medium bearing (300),
4. Heavy bearing (400).

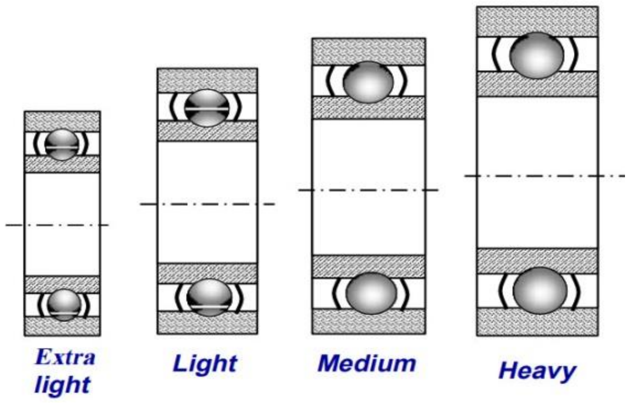

Figure 11-8: Types of bearing series

11-13. Standardization of ball bearing

The following chart shows the standard classification of ball bearing, figure (11-9).

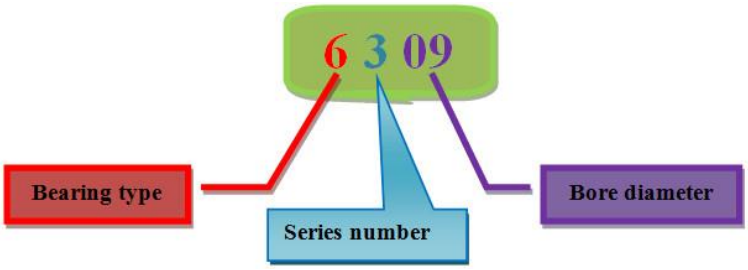

Figure 11-9: Standardization of ball bearing

11-14. Designation of Ball Bearing

Figure (11-10) shows the designation of ball bearing.

- Anti-friction Bearing Manufacturer's Association (AFBMA) has standardized the rolling Contact Bearing sizes. The ISO & BIS also Adopted this Standards.
- According to AFBMA roller bearing is designated by following.

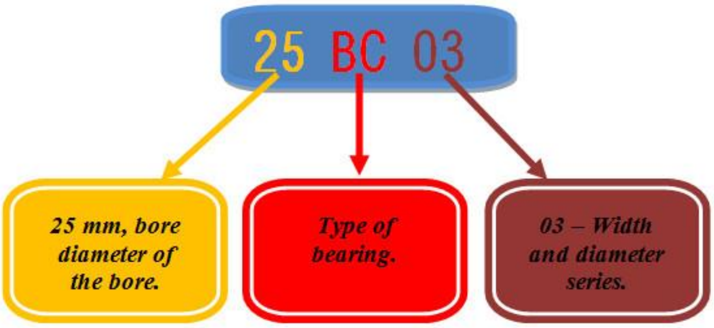

Figure 11-10: Designation of Ball Bearing

11-15. Direction of radial and thrust load of ball bearing

Figure (11-11) shows the direction of axial and radial loads in the ball bearing.

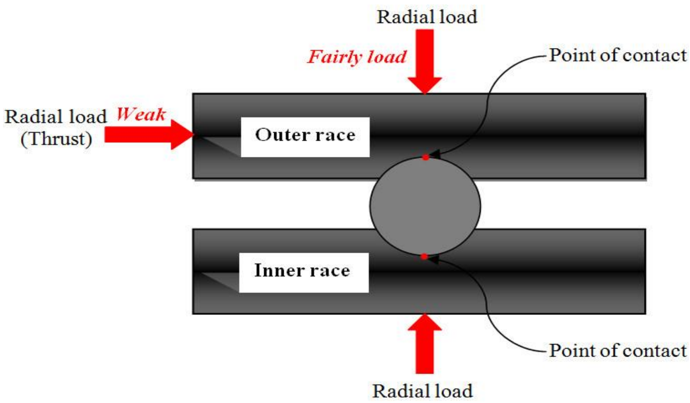

Figure 11-11: Direction of radial and thrust load of ball bearings

11- 16. Bearing Materials

Bearings are made of various materials; they can be:

1. Steel,
2. Ceramic,
3. Metal-ceramic,
4. Self-lubricating.

11- 17. Bearing selection

Bearing is selected based on:

1. Load,
2. Speed,
3. Temperature,
4. Environment,
5. Life expectancy.

11-17-1. Selection of bearing from the manufacturer's catalogue

The following procedure is followed in selecting the bearing from the manufacturer's catalog.

Step (1):

1. Calculate the radial and axial loads (F_r) and (F_a) acting on bearing.
2. Determine the diameter of the shaft where the bearing is to be fitted.
3. Select the type of bearing for the given application, figure (11-12).

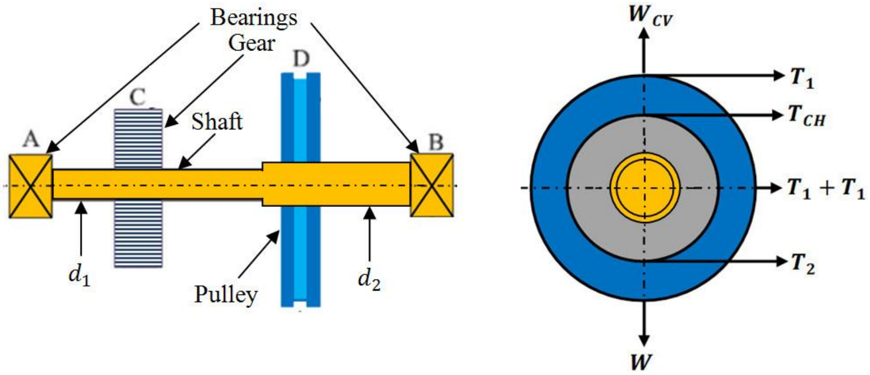

Figure 11-12: Classification of rolling bearings

Step (3):

1. Determine the values of the radial (X) and thrust (Y) factors, from the catalogue.
2. Single-row deep groove ball bearings of various series' static and dynamic load capabilities are listed.

Table (11-1): Relation between principal dimension with basic load rating

Principle dimension (mm)			**Basic load rating (N)**		**Designation**	**Notes**
d	***D***	***B***	***C***	***C_o***		
12	21	5	1430	695	61801	*First select a bearing of light series, then medium and finally heavy duty.* d = *Inner diameter of the bearing.* D = *Outer diameter of the bearing.* B = *Axial width of the bearing.* C_o = *Static load capacity.*
	28	8	5070	2240	6001	
	32	10	6890	3100	6201	
	37	12	9750	4650	6301	
15	24	5	1560	815	61802	
	32	9	5590	2500	6002	
	35	11	7800	3550	6202	
	42	13	11400	5400	6302	
17	26	5	1680	930	62803	
	35	10	6050	2800	6003	
	40	12	9560	4500	6202	
	47	14	13500	6550	6303	
	62	17	22900	11800	6403	

3. The information for single-row deep groove ball bearings' radial (X) and thrust (Y) parameters is provided below.

Table (11-2): Relation between $\left\{\left(\frac{F_a}{C_o}\right), \left(\frac{F_a}{F_r}\right) \text{ \& } (e)\right\}$

$\left(\frac{F_a}{C_o}\right)$	$\left(\frac{F_a}{F_r}\right) \le e$		$\left(\frac{F_a}{F_r}\right) > e$		***e***	***Notes***
	X	**Y**	**X**	**Y**		
0.025	1	0	0.56	2.0	0.22	The values depend upon two ratios,
0.040	1	0	0.56	1.8	0.24	
0.070	1	0	0.56	1.6	0.27	$\left[\left(\frac{F_a}{F_r}\right) \text{ \& } \left(\frac{F_a}{C_o}\right)\right]$
0.130	1	0	0.56	1.4	0.31	
0.250	1	0	0.56	1.2	0.37	
0.500	1	0	0.56	1.0	0.44	$C_o = is\ the\ static\ load\ capacity$

Step (4):

1. Calculate the equivalent dynamic load (P) from the equation:

$$\mathrm{P = X.V.F_r + Y.F_a} \qquad (11-1)$$

$Where,$

$P = Equivalent\ dynamic\ load\ (N),$

$(X) and\ \ (Y)\ are\ radial\ and\ thrust\ factors\ \ respectively\ ,$

$(F_r) and\ \ (F_a)\ are\ radial\ and\ thrust\ factors\ load\ respectively\ (N),$

$V =\ Race\ rotation\ factor$

$(where\ the\ inner\ race\ is\ rotating\ or\ the\ outer\ race)\ ,$

$V = 1\ , when\ the\ inner\ race\ rotating, and$

$V = 1.2\ , when\ the\ outer\ race\ rotating.$

Step (5):

Selection of bearing life:

1. In order to select the proper size of a bearing, the expected life of the bearing must be specified,

2. For vehicles, the speed of rotation is varying, therefore, life is expressed in terms of (Millions / Billion) of revolutions,
3. In some applications, the rotational speed is relatively constant, and the life is expressed in hours of service.

Table (11-3): Relation between wheel application and life (million rev.)

Wheel application	Life (million rev.)	Notes
Automobile cars	50	$Bearing\ life\ in\ million\ revlution$ $(L_{10}) = \frac{60\ \ N.L_{10h}}{10^6}$ $N = Rated\ bearing\ life\ (hours)$ $L_{10h=Speed\ of\ rotation\ (rpm)}$
Trucks	100	
Trolley cars	500	
Rail-road cars	1000	
Other application	Bearing life (hours)	
Machines that are used only occasionally, lifting, hand tools, and home furnishings are a few examples.	4000 - 8000	
Electric motors and gear devices are examples of machines that are used for eight hours of service per day.	12000 - 20000	
Pumps, compressors, and conveyors are examples of machines that run continuously (24 hours a day).	40000 - 60000	

Step (6):

1. Calculate the dynamic load capacity ($\boldsymbol{C}$) from the equation:

$$L_{10} = \left(\frac{C}{P}\right)^P$$

$$C = P\ (L_{10})^{\frac{1}{P}} \qquad (11-2)$$

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$Where,$

$L_{10} = rated\ bearing\ life(n\ million\ revolutions).$

$P = 3\ (for\ ball\ bearing), and\ P = \dfrac{10}{3}(for\ roller\ bearing).$

2. Check whether the required dynamic capacity, if it is more than dynamic load capacity of bearing (chart) then select the bearing of the next series and go back to step (3) and continue.
3. Trial and error are consequently used to choose the bearing.

11-18. Solve examples

Example 1:

Calculation bearing load, as show in figure (10-6) from the following data: Length of the shaft from center pulley to center fan ($L = 290\ mm$), weight of shaft ($W = 12\ Kg$), weight of pulley ($W = 7\ Kg$) , weight of fan ($W = 5\ Kg$) , and tension force in pulley down and equal ($F = 1000\ N$).Neglecting weight of bearings, figure (11-13).

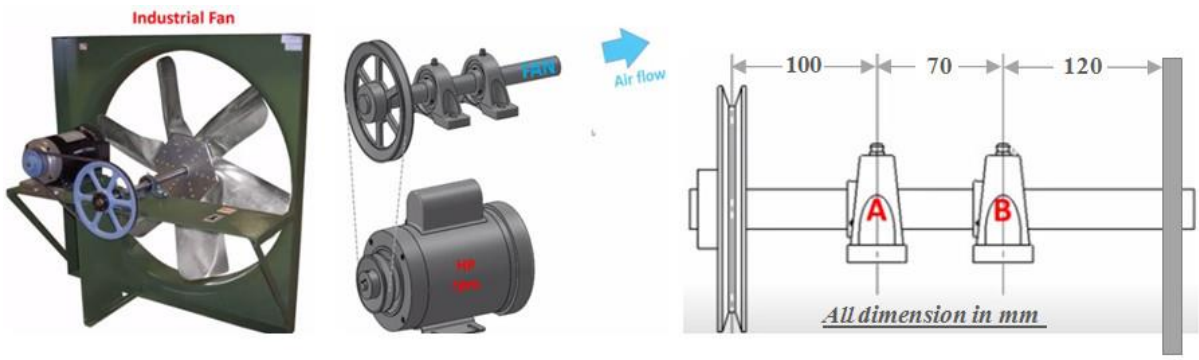
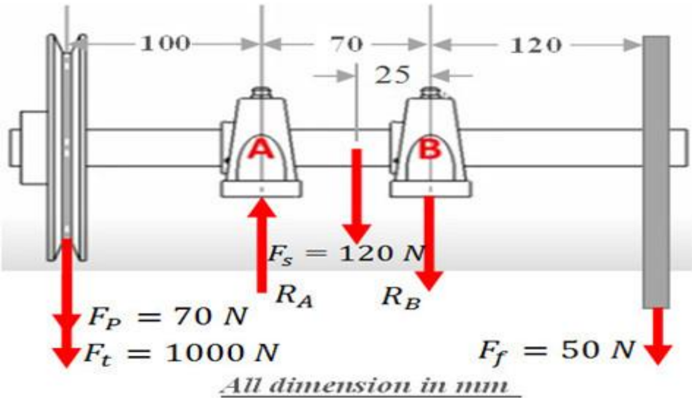


Figure 10-13: Industrial fan

Solution



1. Determine bearing reaction,

$$\sum M_B = 0$$

$$R_A \times (70) - 120 \times (25) - 70 \times (170) - 1000 \times (170) + 50 \times (120) = 0$$

$$R_A = \frac{179025}{70} = 2557.5\ N$$

$$\sum F_y = 0$$

$$2557.5 - 1000 - 70 - 120 - R_B - 50 = 0$$

$$R_B = 1317.5\ N$$

Example 2:

A single-row deep groove ball bearing is subjected to a pure radial force of ($F_r = 5000\ N$) from a shaft that rotates at ($N = 750\ rpm$). The expected life of the bearing ($L_{10h} = 33000\ hour$). The minimum acceptable diameter of the shaft is ($d = 17\ mm$. Choose an appropriate ball bearing for this application.

Solution

Given

$$F_r = 2000\ N, \qquad N = 750\ rpm, \qquad L_{10h} = 33000\ hour, \qquad d = 17\ mm.$$

Step (1): Dynamic load capacity

The bearing is subjected to pure radial load, $P = F_r = 2000\ N$.

$$Bearing\ life\ in\ million\ revlution\ (L_{10}) = \frac{60\ \ N.L_{10h}}{10^6}$$

$$= \frac{60 \times 750 \times 33000}{10^6} = 1485\ million\ rev.$$

$$Dynamic\ load\ capacity\ (C) = P\ (L_{10})^{\frac{1}{P}} = 2000(1485)^{\frac{1}{3}} = 22817.71\ N$$

Step (2): Selection of ball bearing

From table (11-1), we see the following dimension of the ball bearing

$$C = 22817.71\ N$$

$Bearing\ number\ (6403) is\ suitable\ for\ given\ application$:

$$d = 17\ mm, D = 62\ mm, B = 17\ mm, C = 22900\ N, C_o = 11800\ N.$$

Example 3:

A pure radial force of is applied to a single-row deep groove ball bearing ($F_r = 1000\ N$) and a thrust force of ($F_a = 700\ N$). The shaft that rotates at ($N = 500\ rpm$). The expected life of the bearing ($L_{10h} = 10000\ hour$). The minimum acceptable diameter of the shaft is($d = 15\ mm$) . Assume the inner race is rotating. Choose an appropriate ball bearing for this application.

<u>Solution</u>

Given

$$F_r = 1000\ N, \qquad F_a = 700\ N, \qquad N = 500\ rpm,$$

$$L_{10h} = 15000\ hour, \quad d = 15\ mm.$$

1. Step (1):

Determine the values of the radial (X) and thrust (Y) factors, from the table (11-2). The value of (Y) varies between (1.0 to 2.0). Assume the average value (1.5) as the factor's first trial value (Y).

$$X = 0.56 \quad \& \quad Y = 1.5$$

$$F_r = 1500\ N, \qquad F_a = 1000\ N$$

$$P = X.V.F_r + Y.F_a$$

$$V = 1\ , when\ the\ inner\ race\ rotating$$

$$P = 0.56 \times 1000 + 1.5 \times 700 = 1610\ N$$

$$Bearing\ life\ in\ million\ revlution\ (L_{10}) = \frac{60\ \ N.L_{10h}}{10^6}$$

$$= \frac{60 \times 500 \times 10000}{10^6} = 300\ million\ rev.$$

$$Dynamic\ load\ capacity\ (C) = P\ (L_{10})^{\frac{1}{P}} = 1610(300)^{\frac{1}{3}} = 10777.34\ N$$

2. Step (2):

Selection of bearing, $(C) = 10777.34\ N$ from table (11-2).

$Bearing\ number\ (6302) is\ suitable\ for\ given\ application$:

$$d = 15\ mm, D = 42\ mm, B = 13\ mm, C = 11400\ N, C_o = 5400\ N.$$

3. Step (3):

Using value of ($C_o = 11400\ N$)

from table (11-1).

$$\frac{F_a}{F_r} = \frac{700}{1000} = 0.7 \quad \& \quad \frac{F_a}{C_o} = \frac{700}{5400} = 0.12963$$

$$From\ table\ (11-1), \qquad e = 0.31 \quad \& \quad \frac{F_a}{F_r} > e$$

Linear interpolation is used to obtain the value of (Y), as the following:

$$Y = \left(\frac{F_a}{F_r}\right)_i - \frac{\left[\left(\frac{F_a}{F_r}\right)_i - \left(\frac{F_a}{F_r}\right)_{i+1}\right]}{\left[\left(\frac{F_a}{C_o}\right)_{i+1} - \left(\frac{F_a}{C_o}\right)_i\right]} \times \left[\left(\frac{F_a}{C_o}\right) - \left(\frac{F_a}{C_o}\right)_i\right]$$

$$Y = 1.4 - \frac{(1.4 - 1.2)}{(0.25 - 0.13)} \times (0.12963 - 0.13) = 1.401$$

$$From\ table, \ \ X = 0.56$$

$$P = X.V.F_r + Y.F_a$$

$$V = 1\ , when\ the\ inner\ race\ rotating$$

$$P = 0.56 \times 1000 + 1.401 \times 700 = 1540.7\ N$$

$$Bearing\ life\ in\ million\ revlution\ (L_{10}) = \frac{60\ \ N.L_{10h}}{10^6}$$

$$= \frac{60 \times 500 \times 10000}{10^6} = 300\ million\ rev.$$

$$Dynamic\ load\ capacity\ (C) = P\ (L_{10})^{\frac{1}{P}} = 1540.7\ \times (300)^{\frac{1}{3}} = 10313.45\ N$$

4. Step (4):

Selection of bearing, $(C) = 10313.45\ N$ from table (11-2).

$Bearing\ number\ (6302) is\ suitable\ for\ given\ application$:

$$d = 15\ mm, D = 42\ mm, B = 13\ mm, C = 11400\ N, C_o = 5400\ N.$$

11-18. Chapter Questions

1. In a particular application, the bearing is subjected to radial and axial loads. Would you suggest a particular type of rolling contact bearing?

a. **Taper roller.**
b. Needle roller.
c. Cylindrical roller.
d. Thrust ball.

2. In comparison to medium series bearings with the same bore diameter, heavy series bearings have ------------- a larger load carrying capacity.

a. 20 to 50 %
b. 20 to 40 %
c. 20 to 60 %
d. **20 to 30 %**

3. In order to classify rolling contact bearings, according to:

a. Rolling element type.
b. **Type of rolling element and load direction.**
c. magnitude of the load.
d. the load's direction.

4. When using radial bearings, the load:

a. along the rotational axis.
b. between the rotational axis and similar to the rotational axis.
c. Parallel to the rotational axis.
d. **Perpendicular to the rotational axis.**

5. The load carrying capacity of the medium series bearings is ------------- greater than the load carrying capacity of the light series bearings for the same bore diameter.

a. 10 to 40 %
b. 30 to 40 %
c. 20 to 40 %
d. **30 to 40 %**

6. The load acts in thrust bearings is:

a. Perpendicular to the axis of rotation and parallel to it.
b. parallel to the rotational axis
c. Perpendicular to the rotation axis.
d. **Along the rotational axis.**

7. Taper roller bearings are used to take one of the following measurements:

a. There is only radial load.
b. **Both axial and radial loads are supported.**
c. There is only torque.
d. There is only axial load.

8. In comparison to sliding contact bearings, rolling contact bearings have one of the following:

a. **Require considerable axial space.**
b. Costly
c. Generate less noise

d. Lower starting torque

9. Rolling contact bearings' balls are comprised of ------------ steel.

a. case hardened
b. plain carbon
c. high carbon chromium
d. free cutting

10. When the last two digits of the bearing designation are multiplied by, the bore diameter of rolling contact bearings is obtained:

a. Ten
b. Five
c. Hundred
d. π

11. An XX10 bearing number indicates that the bearing is to ------------- diameter.

a. 75 mm bore.
b. 50 mm bore.
c. 100 mm bore.
d. 25 mm bore.

12. Stress induced in the balls or rollers of rolling contact bearing is ------------- stress.

a. tensional shear
b. tensile
c. crushing
d. contact

13. The rollers of rolling contact bearings are made of a ------------ steel.

a. case hardened
b. plain carbon
c. high carbon chromium
d. free cutting

14. Bearing catalogue life is one of the following:

a. Average life
b. Maximum life for 90% of the bearings
c. **Minimum life that 90% of the bearings will reach or exceed**
d. Median life

15. Anti-friction bearings are used.:

a. Bush bearings.
b. Oil lubricated bearings.
c. Boundary lubricated bearings.
d. Ball and roller bearings.

16. The rolling contact bearing is also referred to as:

a. Bush bearing.
b. Sleeve bearing.
c. **Antifriction bearing.**
d. Thin film bearing.

17. The number 410 represents a bearing. It means that it is a manifestation of::

a. The bore diameter of the medium series is 50 mm.

b. **The bore diameter of the heavy series is 50 mm.**
c. The bore diameter of the light series is 10 mm.
d. The bore diameter of the light series is 50 mm.

18. Bearings are made of various materials; they can be:
a. steel
b. ceramic
c. metal-ceramic
d. **steel and ceramic**

19. A single-row deep groove ball bearing is subjected to a pure radial force of ($F_r = 4000\ N$) from a shaft that rotates at ($N = 000\ rpm$). The expected life of the bearing ($L_{10h} = 25000\ hour$). The minimum acceptable diameter of the shaft is ($d = 20\ mm$. Select a suitable ball bearing for this application.

20. A single-row deep groove ball bearing is subjected to a pure radial force of ($F_r = 2000\ N$) and a thrust force of ($F_a = 1000\ N$). The shaft that rotates at ($N = 700\ rpm$). The expected life of the bearing ($L_{10h} = 15000\ hour$). The minimum acceptable diameter of the shaft is ($d = 20\ mm$) . Select a suitable ball bearing for this application. Assume inner race rotating.

Chapter 12

Design of Gears by Lewis Equation

12. Design of Gears by Lewis Equation

12-1. Determination of gear

The gear or gear wheel: is the most important part, which is used in a meshing system and performs the main function - transfers rotational motion between the shafts, by means of engagement with the teeth of adjacent gears. Gear looks like a disc with conical or cylindrical surface on which are arranged equidistant teeth. In a meshing system, the bigger wheel in diameter is called a gear and the smaller is called a pinion. If the mechanism uses a pair of gears with equal numbers of teeth, called the leading pinion, and driven - gear. But most of all gears small and large called the gear.

12-2. Types of gears

The gears are mainly distinguished by the type of teeth. The gears can be straight, slanted or spiral teeth and also them to divide the cylindrical and conical gears.

1. Spur gears or straight-cut gears

Spur gears are the most commonly used type of gears. The teeth are located in radial planes, the contact line of the teeth of a pair of gears is parallel to the axis of rotation, and the axes of both gears (gears) are strictly parallel.

Figure 12-1: Spur gear

2. Helical gears

Helical gears are an upgraded version of spur gears. The teeth are then set at an angle to the axis of rotation. The meshing of the teeth of these gears is quieter and smoother than that of spur gears. They are used either in low-noise mechanisms, or in those that require the transmission of a large torque at high speeds. The disadvantages of this type of

Figure 12-2: Helical gear

gears include: increased contact area of the teeth, which causes significant friction and heating of parts, and as a result: loss of power and additional use of lubricants; similarly, the mechanical force directed along the axis of the gear forces the use of thrust bearings to install the shaft.

3. Double Helical Gear

When you need to move more force, we use the double helical gear, which consists of two helical gears on the same shift, but the direction of helical rotation of each is opposite to the other, and when the two gears are connected or connected.

Figure 12-3: Double Helical

4. Internal Gear

Gears of this type have teeth cut from the inside. When used, there is one-way rotation of the drive and slave gear. In this equipment, there is less frictional cost, which means higher efficiency. Gears with internal engagement are used in limited mechanisms, in planetary gears, in gear pumps, in the drive from the turret of the tank.

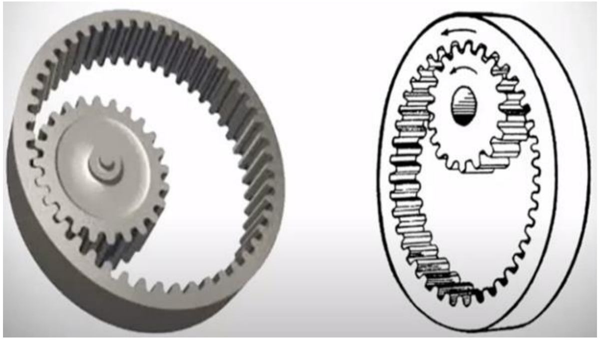


Figure 12-4: Internal Gear

5. Worm Gear

Gears have the shape of a cylinder with teeth located along the screw line. These gears are used on non-intersecting shafts, which are

Figure 12-5: Worm Gear

perpendicular to each other, and the angle between them is 90 degrees.

6. Bevel Gear

The bevel gear or bevel gear and transmits the movement from 0 degrees to 359 degrees, the most famous of which is the 90 degree angle, and moves the rotation axis and another axis perpendicular to it at an angle of 90 degrees, but we note that this type of gear when the meeting point of the gears is in the form of a straight line in length Clarification Once, clarify, illustrate, illustrate, illustrate, illustrate, or spiral or circular or spiral or twisted conical gear makes the space of that image allow it to be brought out more likable than that, gradually, gradually.

Figure 12-6: Worm Gear

7. Hypoid Gear

Hypoid gears are a cross between spiral bevel gears and worm gears, the axes of a pair of hypoid bevel gears are non-intersecting, the distance between the 'axes' being called the offset.

Figure 12-7: Hypoid Gear

8. Rack and Pinion Gears

Rack and pinion Gears are used to convert rotation into linear motion or linear motion into rotation. The diameter of the gear determines the speed that the rack gear moves as the pinion turns.

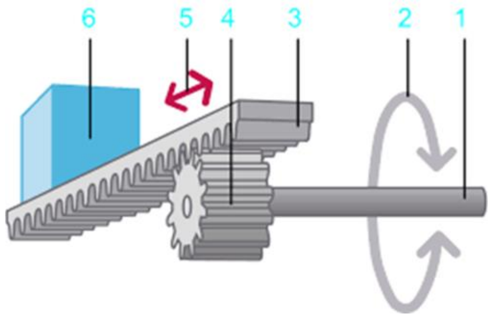


Figure 12-7: Rack and pinion

Rack & Pinion Gears a perfect example of rack gears and pinion gear systems is the steering system on many cars.

12-3. Materials of gears

Until the eighteenth century, gears were either made of wood if they were large; For use in mills, water wheels and cranes, for example, and either of bronze or copper, if they are small, intended for use in precision machines.

As for nowadays gears can be made of all sorts of materials, including many:

1. Types of Steel,
2. Brass,
3. Bronze,
4. Cast Iron,
5. Ductile Iron,
6. Aluminum,
7. Powdered Metals,
8. Plastics.

Steel is the most common material overall, although over the years, we've worked with all of the material types mentioned.

12-4. Advantages and disadvantages of gears

1. Advantages

1. It may be utilized to transfer huge power,
2. It transmits a precise velocity ratio,
3. It might be applied to shafts with short center distances,
4. It is quite effective,
5. The service is dependable,
6. Its layout is condensed.

2. Disadvantages

1. They are not suitable for large velocities,
2. They are not suitable for transmitting motion over a large distance,
3. Due to the engagement of toothed wheel of gears, some part of machine may get permanently damaged in case of excessive loading,

4. They have no flexibility,
5. Gear operation is noisy.

12-5. Gear Terminology

Following are the gear symbol and terminology and gear terms used in the description of gears:

1. $Pitch\ circle,$
2. $Pitch\ circle\ diameter$
3. $Pressure\ angle, \emptyset$
4. $Pitch\ point$
5. $Pitch\ surface$
6. $Addendum$
7. $Dedendum$
8. $Addendum\ circle$
9. $Dedendum\ circle$
10. $Base\ circle$
11. $Circular\ pitch, P_c = \frac{\pi D}{N}, D = Diameter\ of\ circular\ pitch\ \&$

$$N = Number\ of\ teeth.$$

$\boldsymbol{Note}:\ If\ D_1\ and\ D_2\ are\ the\ diameters\ of\ the\ two\ meshing\ gears\ having\ the\ teeth\ T_1\ and\ T_2\ respectively;\ then\ for\ them\ to\ mesh\ correctly,$

$$P_c = \frac{\pi D_1}{N_1} = \frac{\pi D_2}{N_2}, or\ \frac{D_1}{N_1} = \frac{D_2}{N_2}$$

12. $Diameter\ pitch, P_d = \frac{N}{D} = \frac{\pi}{P_c}.$
13. $Module, m = \frac{D}{N}\ \&\ \mathrm{m} = \frac{P_c}{\pi}.$

$Clearance$; $Whole\ Depth$; $Working\ Depth$; $Tooth\ thickness$
$Tooth\ space$; $The\ face\ of\ the\ tooth$; $The\ flank\ of\ the\ tooth$; $Top\ land$
$Face\ width$; $Profile$; $Backlash$; $Pressure\ angle\ (\emptyset).$

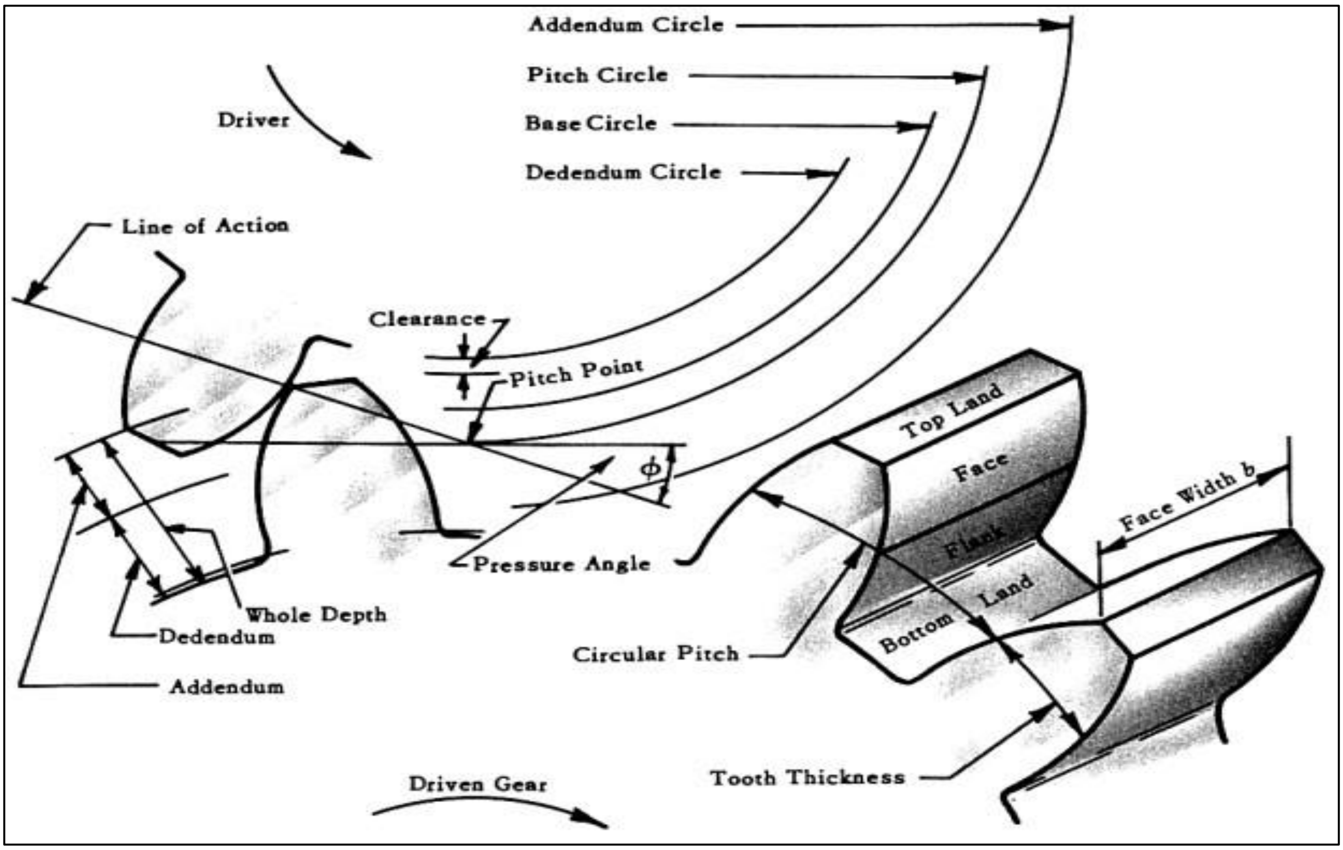


Figure 12-8: Gear Terminology

12-6. Standard gear tooth sizes

Table (12-2) shown the standard gear tooth sizes and table (12-3) Shown Standard modules taken from (ISO/R54).

Table (12-2): Standard gear tooth sizes

Term	14.5° Composite	14.5° Full depth involute	20° Full depth involute	20° Stub involute
Addendum	m	m	m	0.8 m
Minimum dedendum	1.157 m	1.157 m	1.157 m	m
Whole depth	2.157 m	2.157 m	2.157 m	1.8 m
Clearance	0.157 m	0.157 m	0.157 m	0.2 m

Table (12-3): Standard modules taken from (ISO/R54)

Preferred	1	1.25	1.5	2	2.5	3	4	5	6	8	10	12	16	20	25	32	40	50
Second choice	1.125	1.375	1.75	2.25	2.75	3.5	4.5	5.5	7	9	11	14	18	22	28	36	45	

12-7. Objective of design

The gear tooth design includes setting the appropriate step and width of the gear face for sufficient durability. Durability in use and economy in manufacturing.

12-8. Design of Gears by Strength of Gear Teeth – Lewis Equation

Consider each tooth as a cantilever beam loaded by a normal load (W_n) as shown in Fig. 12-8. It is resolved into two components i.e., tangential component (W_t) and radial component (W_r) acting perpendicular and parallel to the center line of the tooth respectively. The tangential component (W_t) induces a bending stress which tends to break the tooth. The radial component (W_r) induces a compressive stress of relatively small magnitude, therefore its effect on the tooth may be neglected. Hence, the bending stress is used as the basis for design calculations. The critical section or the section of maximum bending stress may be obtained by drawing a parabola through (K) and tangential to the tooth curves at (L) and (M). This parabola, as shown dotted in Figure 12-8, outlines a beam of uniform strength, i.e., if the teeth are shaped like a parabola, it will have the same stress at all the sections. But the tooth is larger than the parabola at every section except (LM). We therefore, conclude that the section (LM) is the section of maximum stress or the critical section. The maximum value of the bending stress (S), at the section (LM) is given by:

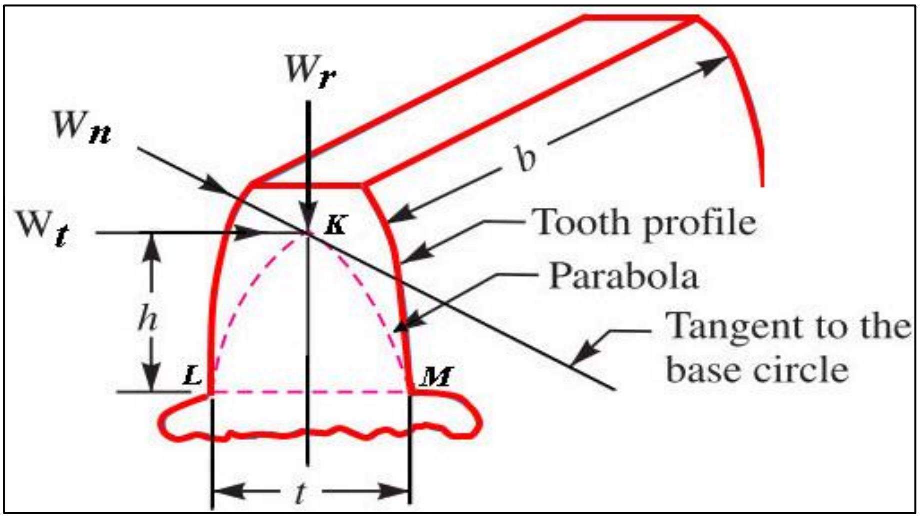


Figure 12-9: Tooth of a gear

$$S = \frac{M.c}{I} \qquad (12-1)$$

Where:

$$M = Maximum\ bending\ moment\ at\ the\ critical\ sction, (\ LM = W_t.h) in\ (MPa),$$

$$c = Half\ the\ thickness\ of\ the\ tooth\ (t)\ at\ critical\ section, \left(\ LM = \frac{t}{2}\right) in\ (mm),$$

$$I = Moment\ of\ inertia\ about\ the\ center\ line\ of\ the\ tooth, \left(\ I = \frac{b.t^3}{12}\right) in\ (mm^4),$$

$$h = Length\ of\ the\ tooth\ in\ (mm),$$

$$b = Width\ of\ gear\ face\ in\ (mm),$$

$$W_t = Tangential\ load\ acting\ at\ the\ tooth\ in\ (Newton),$$

Substituting the above values in equation (12-1), we get the following:

$$S = \frac{(W_t \times h).\frac{t}{2}}{\frac{b.t^3}{12}} = \frac{(W_t \times h).6}{b.t^2}$$

$$\therefore\ \ W_t = \frac{S.b.t^2}{6\ h} = \frac{S.b.t^2.P_c}{6\ h.P_c} \qquad (12-2)$$

$$Where, \frac{t^2}{6\ h\ .P_c} = y$$

The quantity (y) is known as Lewis's form factor or tooth form factor. The form factor is a function of the shape of the tooth, which depends on the tooth system and the number of teeth on the gear.

$Tangential\ load$ (W_t) is rounded by transmitted force (F). Where ($F = \frac{2M}{D}$).

compensate all of form factor and transmitted force by equation (12-2). We get the usual form of the Lewis equation,

$$F = S.b.P_c\ .y \qquad\qquad (12-3)$$

For normal design conditions, the width of the face (b) is limited to a maximum of four times the circular pitch.

$$F = K.P_c\ , \quad Where\ \ K \leq 4$$

$$\therefore F = S.K.{P_c}^2\ .y = S.\ \ \pi^2.K\ .y\ .m^2$$

In a gear design for durability, the pitch diameter is either known or unknown.

1. If the pitch diameter is known, the following form of the Lewis equation can be used,

$$\frac{1}{m^2.y} = \frac{S.K.\pi^2}{F}; \quad or \qquad m = \sqrt{\frac{F}{S.K.\pi^2.y}}$$

2. If the pitch diameter is unknown, the following form of the Lewis equation can be used,

$$S = \frac{2\ M_t}{m^3.K.\pi^2.y.T}$$

Where,

$$M_t = Torque\ for\ weaker\ gear$$

12-9. Allowable tooth stresses

The allowable stress of the gear tooth design depends on the material chosen and the speed of the stepping line. For spur gears, Barth's equation is:

Allowable stress (S):

1. For (V) less than (10 m/s), ($V < 10\ m/s$), can use the following equation:

$$S = S_o\left(\frac{3}{3+V_c}\right)$$

2. For (V) between $(10 - 20\, m/s)$, $(V = 10\, to\, 20\, m/s)$. Use the following equation:

$$S = S_o\left(\frac{3}{6+V_c}\right)$$

3. For (V) more than $(20\, m/s)$, $(V > 20\, m/s)$. We use the following equation:

$$S = S_o\left(\frac{5.6}{5.6+\sqrt{V_c}}\right)$$

12-10. Base design on weaker gear

According to the equation Louis, the amount of force that can be applied to a gear tooth depends on the $(S_o.y)$ product. The smaller $(S_o.y)$ value will be found in the weaker of two gears. The smaller (Pinion) gear, which controls the design when the two mated (interlocked) gears are formed of the same material, will be the weakest.

12-11. Solve example

Example 1:
Pair of gears with full depth teeth (14.5°). Module (10), and diameter of the pitch circle for a smaller gear is ($180\, mm$). If the transmission ratio is $(3 : 2)$, calculate:

a. Number of teeth per gear,
b. The upper elevation (addendum),
c. Whole depth,
d. Clearance,
e. Outer diameters pinion and gear,
f. Root diameters,
g. Year root (dedendum),
h. Diameter of base circle pinion and gear, and
i. Check of interference.

Solution

1. To calculate number of teeth per gear,

$$D_p = 180\,mm\,;\; D_g = 180 \times \frac{3}{2} = 270\,mm$$

$$m = \frac{D}{T};\; m = \frac{D_p}{T_p} = \frac{D_g}{T_g}$$

$$\therefore\;\; N_p = \frac{D_p}{m} = \frac{180}{10} = 18\,teeth$$

$$\because\;\; \frac{D_g}{D_p} = \frac{T_g}{T_p} = \frac{3}{2}$$

$$\therefore\;\; T_g = \frac{3}{2} \times T_p = \frac{3}{2} \times 18 = 27\,teeth$$

From table (12-1)

2. To calculate addendum distance,

$$Addendum = Module\ (m) = 10\,mm$$

3. To calculate whole depth,

$$Whole\ depth = 2.157 \times m = 2.157 \times 10 = 21.57\ \ mm$$

4. To calculate Clearance,

$$Clearance = 1.157 \times m = 1.157 \times 10 = 11.57\ \ mm$$

5. To calculate outside diameter pinion and gear,

$$Outside\ diameter\ \ = Pitch\ diameter + 2 \times addendum$$

$$Outside\ diameter\ of\ pinion\ \ = 180 + 2 \times 10 = 200\,mm$$

$$Outside\ diameter\ of\ gear = 270 + 2 \times 10 = 290\,mm$$

6. To calculate root diameter of pinion and gear,

$$Root\ diameter\ \ = Outside\ diameter - 2 \times Whole\ depth$$

$$Root\ diameter\ of\ pinion\ \ = 200 - 2 \times 21.57 = 156.86\,mm$$

$$Root\ diameter\ of\ gear\ = 290 - 2 \times 21.57 = 246.86\ mm$$

7. To calculate dedendum distance,

$$Dedendum\ = 1.157\ m = 1.157 \times 10 = 11.57\ mm$$

8. To calculate diameter of base circle,

$$Radius\ of\ base\ circle\ = Pitch\ radius\ \times \cos 14.5^{\circ}$$

$$Radius\ of\ base\ circle\ (R_b)\ of the\ pinion\ = \frac{180}{2} \times \cos 14.5^{\circ} = 89.59\ mm$$

$$\therefore Diameter\ of\ base\ circle\ (D_b)\ of the\ pinion\ = 89.59\ \times 2 = 179.18\ mm$$

$$Radius\ of\ base\ circle\ \left(R_g\right)\ of the\ gear\ = \frac{270}{2} \times \cos 14.5^{\circ} = 130.7\ mm$$

$$\therefore Diameter\ of\ base\ circle\ \left(D_g\right)\ of the\ gear\ = 130.7\ \times 2 = 261.4\ mm$$

9. To Check of interference, Overlap is avoided if the radius of the upper gear height is.

$$adedendum\ radius \leq \sqrt{(base\ cicle\ radius)^2 + (center\ distance)^2.(sin\ \emptyset)^2}$$

$$adedendum\ radius \leq \sqrt{(130.7)^2 + (0.5\ (180 + 270))^2.(sin\ 14.5°)^2}$$

$$adedendum\ radius \leq \sqrt{17082.49 + 3173.69}$$

$$adedendum\ radius \leq \mathbf{142.32\ mm}$$

$$Since\ the\ radius\ of\ the\ top\ gear\ height\ is = \frac{290}{2} = \mathbf{145\ \ mm}$$

There will be no overlap and the design are acceptable.

Example-2:

Pair of gears with full depth teeth (14.5°). Module (10). Diameter of the pitch circle for a smaller gear is ($160\ mm$). If the transmission ratio is ($3 : 2$), calculate:

a. Number of teeth per gear,
b. The upper elevation (addendum),
c. Whole depth,
d. Clearance,
e. Outer diameters pinion and gear,
f. Root diameters,
g. Year root (dedendum),
h. Diameter of base circle pinion and gear, and
i. Check of interference

Solution

1. To calculate number of teeth per gear,

$$D_p = 160\ mm\ ;\ D_g = 160 \times \frac{3}{2} = 240\ mm$$

$$m = \frac{D}{T};\ m = \frac{D_p}{T_p} = \frac{D_g}{T_g}$$

$$\therefore\ T_p = \frac{D_p}{m} = \frac{160}{10} = 16\ teeth$$

$$\because\ \frac{D_g}{D_p} = \frac{T_g}{T_p} = \frac{3}{2}$$

$$\therefore\ T_g = \frac{3}{2} \times T_p = \frac{3}{2} \times 16 = 24\ teeth$$

From table (12-1)

2. To calculate addendum distance,

$$Addendum = Module\ (m) = 10\ mm$$

3. To calculate whole depth,

$$Whole\ depth = 2.157 \times m = 2.157 \times 10 = 21.57\ \ mm$$

4. To calculate Clearance,

$$Clearance = 0.157 \times m = 0.157 \times 10 = 1.57\ \ mm$$

5. To calculate outside diameter pinion and gear,

$$Outside\ diameter\ \ = Pitch\ diameter + 2 \times addendum$$

$$Outside\ diameter\ of\ pinion\ \ = 160 + 2 \times 10 = 180\, mm$$

$$Outside\ diameter\ of\ \ gear = 240 + 2 \times 10 = 260\, mm$$

6. To calculate root diameter of pinion and gear,

$$Root\ diameter\ \ = Outside\ diameter - 2 \times Whole\ depth$$

$$Root\ diameter\ of\ pinion\ \ = 180 - 2 \times 21.57 = 136.86\, mm$$

$$Root\ diameter\ of\ \ gear\ \ = 260 - 2 \times 21.57 = 216.86\, mm$$

7. To calculate dedendum distance,

$$Dedendum\ \ = 1.157\, m = 1.157 \times 10 = 11.57\, mm$$

8. To calculate diameter of base circle,

$$Radius\ of\ \ base\ circle\ \ = Pitch\, radius\ \ \times \cos 14.5^{\circ}$$

$$Radius\ of\ \ base\ circle\ (R_b)\ of the\ pinion\ \ = \frac{160}{2} \times \cos 14.5^{\circ} = 77.45\, mm$$

$$\therefore Diameter\ of\ \ base\ circle\ (D_b)\ of the\, pinion\ \ = 77.45\ \ \ \times 2 = 154.90\, mm$$

$$Radius\ of\ \ base\ circle\ \left(R_g\right)\ of the\ gear\ \ = \frac{240}{2} \times \cos 14.5^{\circ} = 116.18\, mm$$

$$\therefore Diameter\ of\ \ base\ circle\ \left(D_g\right)\ of the\ gear\ \ = 116.18\ \ \times 2 = 232.36\, mm$$

9. To Check of interference, Overlap is avoided if the radius of the upper gear height is.

$$adedendum\ radius \leq \sqrt{(base\ cicle\ radius)^2 + (center\ distance)^2.(sin\ \emptyset)^2}$$

$$adedendum\ radius \leq \sqrt{(116.18)^2 + (0.5\ (160 + 240))^2.(sin\ 14.5°)^2}$$

$$adedendum\ radius \leq \sqrt{13497.79 + 2507.61}$$

$$adedendum\ radius \leq 126.51\ mm$$

$$Since\ the\ radius\ of\ the\ top\ gear\ height\ is = \frac{260}{2} = 130\ mm$$

There will be overlap and therefore some modifications to the design will have to be made. Reducing the module to (8) and increasing the diameter of the pitch circle for the pinion ($210\ mm$) will be sufficient to avoid interference.

Example 3:

Bronze spur pinion ($S_o\ =\ 83\ MN/m^2$) rotating at speed ($600\ rev/min$) and it is driven by an alloy steel spur gear ($S_o\ =\ 103\ MN/m^2$) with a power transmission ratio of ($4:1$). The pinion has ($16\ teeth$) by following specification: pressure angle (20°) full depth involute by module (8) and width of the face for pinion and gear are ($90\ mm$). What power can be transferred from the point of view of strength.

Solution

It is first necessary to determine which is weaker, the gear or the pinion.

Type of gear	Number of teeth	S_o MN/m^2	From face (y)	load capacity $S_o.y,\ MN/m$
Pinion	16	83	0.094	7.8
Gear	64	103	0.135	13.9

From the above table the load capacity of the pinion is less than the gear, so the pinion are the weakest.

The pitch line velocity is challenged in order to choose the correct velocity factor required to calculate the allowable stress:

$$V_c = \omega . r = \omega . \frac{D}{2}, \qquad D = T . m$$

$$\therefore V_c = \omega . \left(\frac{1}{2} \times \frac{T \times m}{1000}\right)$$

$$V_c = \left(\frac{V \times 2\pi}{60}\right) . \left(\frac{1}{2} \times \frac{N \times m}{1000}\right) = \left(\frac{600 \times 2\pi}{60}\right) . \left(\frac{1}{2} \times \frac{16 \times 8}{1000}\right) = 4.02\ m/s$$

Since the (V) is less than ($10\ m/s$),

Allowable $S = S_o \left(\frac{3}{3+V_c}\right) = 83. \left(\frac{3}{3+4.02}\right) = 35.5\ MN/m^2$

Therefore, the amount of force that can be transferred according to Lewis equation,

$$F = S.b.y.P_c = 35.5 \times 0.09 \times 0.094 \times \left(\frac{\pi \times 8}{1000}\right) = 7.54\ KN$$

The power can be transmitting is:

$$Power, P = F.V = 7.52 \times 4.02 = 30.3\ KW$$

Example 4:

Alloy steel spur pinion ($S_o = 140\ MN/m^2$). It is driven by a cast iron spur gear ($S_o = 55\ MN/m^2$) with a power transmission ratio of ($7/3:1$). Pinion diameter ($D = 105\ mm$). The power of ($P = 20\ KW$) is transferred at a speed of (900 rpm) to the pinion. Teeth shall have the following specifications: pressure angle (20°) full depth involute form. Design the largest number of teeth. Select the model and pinion and gear width in terms of strength. Assume that ($y = 0.1$).

<u>Solution</u>

Suppose that the number of pinion teeth is ($D_p = 30\ teeth$).

$$\frac{D_p}{D_g} = \frac{T_p}{T_g} = \frac{7/3}{1}$$

$$\therefore\ T_g = \frac{3}{7} \times T_p = \frac{7}{3} \times 30 = 70\ teeth$$

$$D_g = \frac{T_g.D_p}{T_p} = \frac{70 \times 105}{30} = 245\ mm$$

It is first necessary to determine which is weaker, the gear or the pinion.

Type of gear	Number of teeth	S_o MN/m^2	From face (y)	load capacity $S_o.y, MN/m$
Pinion	30	140	0.114	15.96
Gear	70	55	0.137	7.54

From the above table the load capacity of the gear is less than the pinion, so the gear are the weakest.

Since the diameters are known, we use the following form of the Lewis equation:

$$\frac{1}{m^2.y} = \frac{S.K.\pi^2}{F}$$

$$\therefore\ \ m = \sqrt{\frac{F}{S.K.\pi^2.y}}$$

Torque transmitted by pinion,

$$M_t = \frac{60\ P}{T_p.2\pi} = \frac{60 \times 20000}{900 \times 2 \times 3.14} = 212\ N/m$$

Transmitted force,

$$F = \frac{M_t}{r} = \frac{212}{0.105/2} = 4040\ N$$

Velocity of pitch line,

$$V_c = \omega.r = \omega.\frac{D}{2} = \left(900 \times \frac{2\pi}{60}\right) \times \frac{0.105}{2} = 4.95\ m/s$$

$$\because\ V_c < 10$$

Allowable: $S_o = S_o\left(\frac{3}{3+V_c}\right) = 55.\left(\frac{3}{3+4.95}\right) = 20.75\ MN/m^2$

$$\therefore\ \ m = \sqrt{\frac{F}{S.K.\pi^2.y}} = \sqrt{\frac{4040}{20.8 \times 4 \times (3.14)^2 \times 0.1}} = 7.018 \approx 7$$

$$\therefore\ \ T_g = \frac{D_g}{m} = \frac{245}{7} = 35\ teeth$$

From table (12-)

$$y = 0.119$$

$$\therefore\ \ \frac{1}{m^2.y} = \frac{1}{7^2.0.119} = 171.5 \times 10^3$$

The gear is strong.

Modules $(6, 8\ or\ 9)$ cannot be used due to the required speed ratio. Module (5) will give a very weak gear. Thus, reducing the value of (K) to:

$$K_N = \frac{K.\ \frac{1}{m^2.y}}{S_o} = \frac{4 \times 171.5}{202.75} = 3.383$$

Width of the pinion and gear,

$$b = K_N\ .m\ .\pi = 3.383 \times 7 \times 3.14 = 74.36 \approx 75\ mm$$

Largest number of teeth, the model and pinion and gear width in terms of strength are:

$$T_g = 35\ \&\ T_p = 15\ ;\ m = 7\ ; b = b_g = b_p = 75\ mm\ .$$

Final examination to determine the weakest of the gears on the selected number of teeth:

Pinion: $S_o.y = 140 \times 0.090 = 12.88\ MN/m$

Gear: $S_o.y = 55 \times 0.119 = 6.545\ MN/m$, the weakest

12-12. Chapter Questions

1. The gear tooth's area between the pitch circle and outer circle is referred to as:

- **a. Face**
- b. Top land
- c. Bottom land
- d. Flank

2. From the moment a pair of teeth first make contact until they make contact at the pitch point, a gear rotates at an angle called as:

- **a. Approach Angle.**
- b. Recess Angle.
- c. Contact Angle.
- d. Action Angle.

3. When compared between a stub tooth and a full depth tooth, stub tooth has.

- a. Dedendum and addendum are both lengthy.
- b. Short addendum and long dedendum.
- **c. Dedendum and addendum are both shortly.**
- d. Long addendum and short dedendum.

4. One of the following gear tooth systems will transmit extremely high loads:

- a. 14.5° full depth involute.
- **b. 20° full stub involute.**
- c. 20° full depth involute.
- d. 14.5° stub involute.

5. An item with a rack is a:

- a. Infinite module.
- **b. Infinite number of teeth.**
- c. Infinite circular pitch and Infinite module.
- d. Infinite circular pitch.

6. The smallest number of teeth needed on the pinion for a 20° full depth involute tooth system to prevent interference is:

- a. 18
- **b. 17**
- c. 16
- d. 15

7. **Gear tooth is seen as one of the following in Lewis' equation:**
   a. **Cantilever beam.**
   b. Both Curved and cantilever beams.
   c. Curved beam.
   d. Simply supported beam.
8. **According to the Lewis equation:**
   a. Pinion is always less powerful than gear.
   b. If both the gear and the pinion are made of the same material, the gear is weaker.
   c. **If made of the same material, the pinion is weaker than the gear.**
   d. A pinion is always the same as a gear.
9. **A pair of spur gears can have a maximum gear ratio of:**
   a. 4
   b. 8
   c. **10**
   d. 16
10. **Involute teeth gears have one of the following pressure angles:**
   a. Often changes.
   b. **Remains constant.**
   c. Unpredictable.
   d. Rarely changes.
11. **The formula for determining a pair of gears' working depth is:**
   a. Twice the dedendum.
   b. **Twice the addendum.**
   c. Addendum plus dedendum.
   d. Dedendum minus addendum.
12. **The thickness of gear tooth is measured:**
   a. Throughout the base circle.
   b. **Throughout the pitch circle.**
   c. Throughout the root circle.
   d. Throughout the entire addendum circle.
13. **A standard gear system's dimensions can all be expressed in terms of,**
   a. Circular and diametric pitch
   b. Pressure angle and number of teeth
   c. **Module and number of teeth**
   d. Module
14. **Lewis's form factor for spur gears is dependent on a:**
   a. **Pressure angle.**
   b. Module and Number of teeth.
   c. Module.
   d. Number of teeth.
15. **For a pair of gears, the total depth is given by:**
   a. Twice the addendum
   b. **Addendum plus dedendum**
   c. Dedendum minus addendum
   d. Twice the dedendum

16. The base circle for involute gear teeth must be:

a. Above the pitch circle
b. Under the pitch circle
c. Under the root circle
d. At the root circle

17. Stub tooth has one of the following:

a. Less depth than the typical total depth.
b. Non-standard whole depth.
c. Greater than the common entire depth.
d. Standard whole depth.

18. Gear tooth static force is caused by:

a. Misalignment in bearings.
b. Power transmitted by gears.
c. Bearing reactions.
d. Acceleration of gears.

19. Gear tooth involute profiles are frequently utilized because,

a. Face and flank are continuously curved, the pressure angle is constant, and the teeth on the involute rack are straight.
b. Face and flank form a continuous curve.
c. Pressure angle remains constant.
d. Involute rack has straight sided teeth.

20. The portion of the gear tooth between the pitch circle and dedendum circle is called:

a. Top land
b. Face
c. Flank
d. Bottom land

21. Pair of gears with full depth teeth (14.5°). Module (8). Diameter of the pitch circle for a smaller gear is ($200\ mm$). If the transmission ratio is ($3 : 2$), calculate:

a. Number of teeth per gear,
b. The upper elevation (addendum),
c. Whole depth,
d. Clearance,
e. Outer diameters pinion and gear,
f. Root diameters,
g. Year root (dedendum),
h. Diameter of base circle pinion and gear, and
i. Check of interference.

22. To move a spur steel gear ($So = 140\ MN/m^2$), a spur steel pinion ($So = 200\ MN/m^2$) is required. The pinion's diameter is to be 100 mm, and the center distance is to be (200 mm). The transmission is to be done by the pinion (5kW at 900 rpm). The teeth must have complete depths of (20°). Determine the face width and necessary module to provide the most teeth. Use the Lewis equation when designing solely for strength.

$$Ans. \{m = 2\ mm,\ b = 21.2\ mm\ (use\ 22mm)\}$$

23. For the drive of a rock crusher, two minimum-sized spur gears must be employed. The following specifications must be met by the gears: power to be transmitted (18 kW), speed of the pinion (1200 rev/min), angular velocity ratio (3.5 to 1), tooth profile (20° stub), (So) value for the pinion ($So = 100\ MN/m^2$), and (So) value for the gear ($So = 70\ MN/m^2$). Use the Lewis equation to calculate the appropriate face width and module for strength requirements only.

$Ans.\ \{m = 5, \quad b = 57\ mm\}$

24. The center distance of a pair of spur gears that transfer power from a motor to a pump impeller shaft should be kept as low as practical. To transfer (4 kW at 600 rev/min) to a cast steel gear with a transmission ratio of (2 1 4 to 1), a forged steel pinion ($So = 160\ MN/m^2$) with (20° and 24) full depth involute teeth is to be utilized. Using the Lewis equation, determine the required face width and module for strength solely.

$Ans.\ \{m = 3\ mm,\ b = 30.9\ mm\ (use\ 31mm)\}$

Chapter 13

Gears Trains

13. Gears Trains

13-1. Introduction

Gears are a type of mechanical transmission that works on the principle of engagement. They are used to transmit and convert rotary motion between shafts. Gear drives are distinguished by high efficiency for one stage $(0.97 - 0.99)$ and higher, reliability and long service life, compactness, stability of the gear ratio due to the absence of slippage. Gear drives are used in a wide range of speeds $(up\ to\ 200\ m\ /\ s)$, powers $(up\ to\ 300\ MW)$. The dimensions of the gears can be from a fraction of a millimeter to several meters.

13-2. Uses of gear trains

1. To reverse the direction of rotation,
2. To increase or decrease the speed of rotation,
3. To move rotational motion to a different axis.

To keep the rotation of two axes synchronized

13-3. Types of Gear Trains

There are four types of gears trains:

1. Simple gear train,
2. Compound gear train,
3. Reverted gear train, and
4. Epicyclical gear train.

In the first three types of gear trains, the axes of the shafts over which the gears are mounted are fixed relative to each other. But in case of epicycle gear trains, the axes of the shafts on which the gears are mounted may move relative to a fixed axis.

13-3-1. Simple gear train

A simple gear train is one that has just one gear on each shaft, as seen in Figure (13-1).

Characteristics of simple gear train

1. Gears are in the form of series,
2. Axes of gears remains fixed,
3. Each gear is mounted on different shaft.

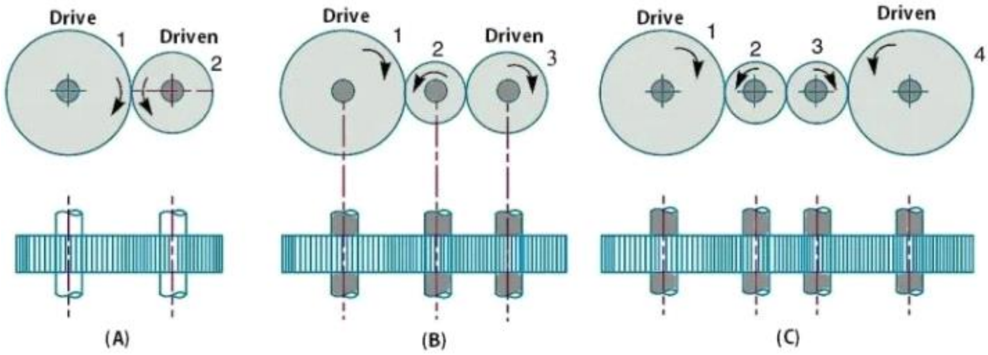


Figure 13-1: Simple gear train.

13-3-1-1. Application of a simple gear train

1. To connect gears where a large center distance is required,
2. To obtain desired direction of motion of the driven gear,
3. To obtain high speed ratio.

13-3-1-2. Speed ratio (Velocity ratio) and train value

The speed ratio (or velocity ratio) of a gear train is the ratio of the driver's speed to that of the driven or follower's speed, and the ratio of speeds of any pair of meshing gears is the inverse of their tooth count:

$$\boldsymbol{Speed\ ratio}\ (\boldsymbol{SR}) = \frac{N_1}{N_2} = \frac{t_2}{t_1} \qquad (13-1)$$

It should be remembered that the gear train's train value is defined as the ratio of the driver's speed to the speed of the driven or follower. Mathematically:

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$$Gear\ ratio\ (GR) = \frac{N_2}{N_1} = \frac{t_1}{t_2} \qquad (13-2)$$

$$Speed\ ratio\ (SR) = \frac{1}{Gear\ ratio\ (GR)} \qquad (13-3)$$

From the figure 13-1-a, the speed ratio can be found as follows:

$$\frac{N_2}{N_1} = \frac{t_1}{t_2} \qquad (13-4)$$

Were,

$$N_1 = Driver\ gear\ speed\ in\ rpm;\ t_1 = Number\ of\ teeth\ on\ driver\ gear$$

$$N_2 = Driver\ gear\ speed\ in\ rpm;\ t_1 = Number\ of\ teeth\ on\ driver\ gear$$

In the figure 13-1-b, it is a Gear ratio as in the following mathematical formula**:**

$$\frac{N_2}{N_1} = \frac{t_1}{t_2} \qquad (13-5)$$

$$\frac{N_3}{N_2} = \frac{t_2}{t_3} \qquad (13-6)$$

Now by multiplying equations (13-2 & 13-3), we have

$$\frac{N_2}{N_1} \times \frac{N_3}{N_2} = \frac{t_1}{t_2} \times \frac{t_2}{t_3}$$

$$\therefore \quad \frac{N_3}{N_1} = \frac{t_1}{t_3} \qquad (13-7) \qquad Gear\ ratio$$

$$N_1 = Driver\ gear\ speed\ in\ rpm;\ t_1 = Number\ of\ teeth\ on\ driver\ gear$$

$$N_2 = Intermediate\ gear\ speed\ in\ rpm;\ t_2 = Number\ of\ teeth\ on\ Intermediate\ gear$$

$$N_3 = Driver\ gear\ speed\ in\ rpm;\ t_3 = Number\ of\ teeth\ on\ driver\ gear$$

From the above equation it is clear that the intermediate gears have no effect on the speed ratio.

If the number of gears is four as in the figure13-1-c, then the mathematical formula for the speed ratio is as follows**:**

$$\frac{N_4}{N_3} = \frac{t_3}{t_4} \qquad (13-8)$$

Now by multiplying equations (13-2, 13-3 & 13-5), we have

$$\frac{N_2}{N_1} \times \frac{N_3}{N_2} \times \frac{N_4}{N_3} = \frac{t_1}{t_2} \times \frac{t_2}{t_3} \times \frac{t_3}{t_4}$$

$$\frac{N_4}{N_1} = \frac{t_1}{t_4} \qquad (13-9) \qquad Gear\ ratio$$

$N_1 = Driver\ gear\ speed\ in\ rpm;\ t_1 = Number\ of\ teeth\ on\ driver\ gear$

$N_2 = First\ intermediate\ gear\ speed\ in\ rpm;$

$t_2 = First\ number\ of\ teeth\ on\ Intermediate\ gear$

$N_3 = Second\ intermediate\ gear\ speed\ in\ rpm;$

$t_3 = Second\ number\ of\ teeth\ on\ Intermediate\ gear$

$N_4 = Driver\ gear\ speed\ in\ rpm;\ t_4 = Number\ of\ teeth\ on\ driver\ gear$

From the above equation it is clear that the intermediate gears have no effect on the speed ratio.

Generally, equation speed ratio in simple gear train:

$$Gear\ ratio\ (GR) = \frac{Speed\ of\ driven(follower)}{Speed\ of\ driver}$$

$$= \frac{Number\ of\ teeth\ on\ driver}{Number\ of\ teeth\ on\ driven(follower)}$$

There are three cases for the gear ratio:

1. $GR > 1$ *when the pinion is the driver*
2. $GR = 1$ *when both gears have the same size*
3. $GR < 1$ *when both gears have the same size*

13-3-1-3. Torque and efficiency

The power transmitted by torque (T) applied to a shaft rotating at (ω) is given by:

$$P = \omega.T \qquad (13-10)$$

Were,

$$Power\ (P), Watt\ (W);\ \ Torque\ (T), (N.m);\ Angular\ speed(\omega) = \frac{2\pi N}{60}\ (\frac{rad}{s})$$

In an ideal gear train, the input and output powers are the same so;

$$P = \omega_{in}.T_{in} = \omega_{out}.T_{out} \ \Rightarrow\ \frac{T_{out}}{T_{in}} = \frac{\omega_{in}}{\omega_{out}} = GR \qquad (13-11)$$

Where: $Gear\ ratio\ (GR)$

$$T_{in}: input\ torque\ (i.e.driver\ gear\ torque);\ N.m.$$

$$T_{out}: output\ torque\ (i.e.driven\ gear\ torque);\ N.m.$$

$$\omega_{in}: driver\ gear\ angular\ velocity; \frac{rad}{s}\ or\ rpm$$

$$\omega_{out}: driven\ gear\ angular\ velocity;\ rad/s\ or\ rpm$$

In an ideal gear box, the input and output powers are the same so:

$$Power\ (P) = \frac{Total\ work\ done}{Total\ tme\ taken} \qquad (13-14)$$

$$Work\ done = Force \times Distancemoved\ in\ the\ direction\ of\ the\ force$$

The efficiency is defined as:

The main function of gear train is to transmit power between two or more shafts. But, because of the friction between gears teeth some of the input power is dissipated in form of heat. Efficiency of system means how much we get from the input power. In other words, more efficient gear train means less power loss due to friction.

Mathematically, the efficiency of gear train can be given as:

$$Efficiency\ (\eta) = \frac{Power\ output}{Power\ input} \times 100\ \% \qquad (13-15)$$

$$Efficiency\ (\eta) = \frac{2\pi \times N_{out} \times T_{out}\ \times 60}{2\pi \times N_{in} \times T_{in}\ \times 60} \times 100\ \% = \frac{\omega_{out} \times T_{out0}}{\omega_{in} \times T_{in}} \times 100\ \%$$

13-3-2. Compound gear train

When a series of gears are connected in such a way that two or more gears rotates about an axis with the same angular velocity (i.e., more than one gear on a shaft), it is known as compound gear train.

Train value (TV): The train value of the compound gear train in Figure (13-2) is:

$$\frac{N_2}{N_1} \times \frac{N_4}{N_3} \times \frac{N_6}{N_5} = \frac{t_1}{t_2} \times \frac{t_3}{t_4} \times \frac{t_5}{t_6}$$

$$\therefore \quad \frac{N_6}{N_1} = \frac{t_1 \times t_3 \times t_5}{t_2 \times t_4 \times t_6} \qquad (13-16)$$

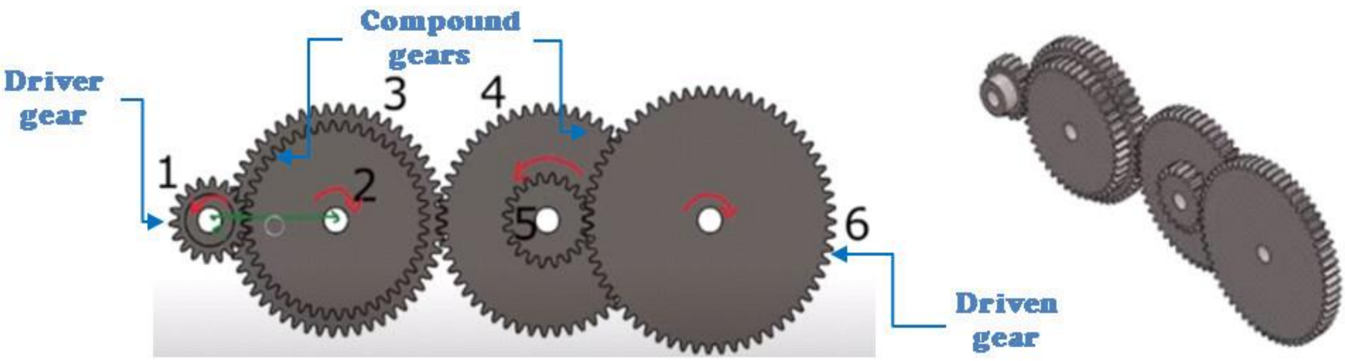

Figure 13-2: Compound gear train

13-3-3. Reverted gear train

It the axis of first and last gear of a compound gear coincide i.e., the first of driver and last driven gear co-inside, it is called of reverted gear train, application in clocks and in the simple lathes.

Train value (TV): The train value of the reverted, the motion of the first gear and last gear is like, figure (13-3) is:

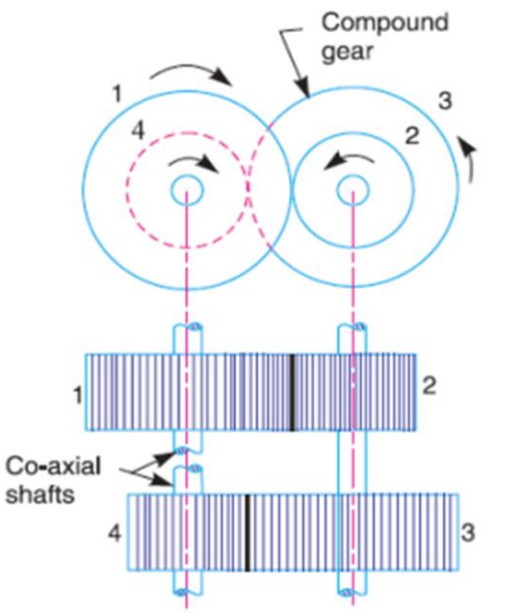

Figure 13-3: Reverted gear train

$$\frac{N_2}{N_1} \times \frac{N_4}{N_3} = \frac{t_1}{t_2} \times \frac{t_3}{t_4}$$

$$\therefore \quad \frac{N_4}{N_1} = \frac{t_1 \times t_3}{t_2 \times t_4} \qquad (13-17)$$

13-3-4. Planetary or Epicycle gear train (Epicyclical gearing)

This gear system has one or more outer gears (Planet gears) that revolve around a center shaft (Sun gear). A movable arm (carrier) that can rotate in relation to the sun gear is where the planet gear is mounted. An outer ring gear or annulus that meshes with the planet gears may also be used in epicyclic gearing systems, as seen in figure (13-4).

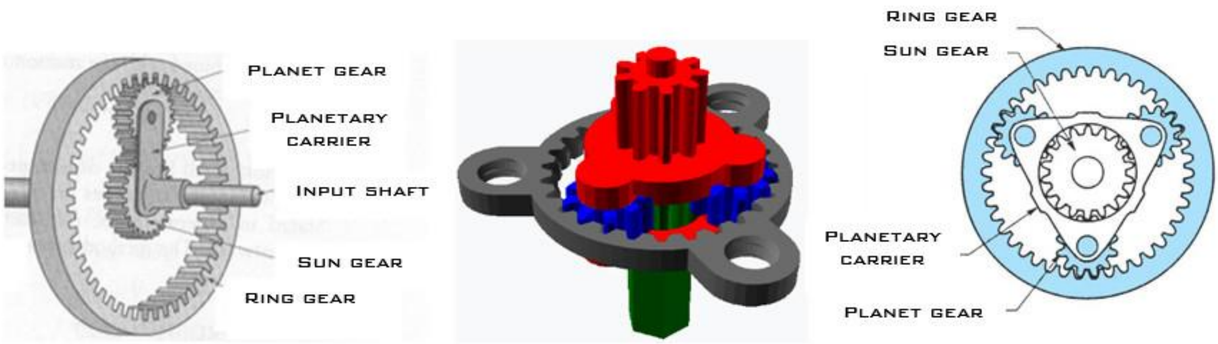

Figure 13-4: Parts of epicyclical gear train

The design of the gear teeth and the methods used to rotate the gear input can change the gear ratio in an epicyclical gearing system.

13-3-4-1. Epicyclical Gearbox Parts

The epicyclical gear's three fundamental parts are as follows:

1. **Sun:** The central gear,
2. **Planet carrier:** Holds one or more identically sized peripheral planet gears that are meshing with the sun gear,
3. **Ring Gear:** the planet gear or gears and an outer ring with teeth facing inward.

13-3-4-2. Application of Epicyclical Gear train

1. Hoists, pulley blocks, wrist clocks, differential gears in cars, back gears on lathes,. all utilize epicyclic gear systems.
2. The automatic transmission of an automobile is a good illustration of how a planetary gear system is used in everyday life,
3. Epicyclic gear systems may convey high velocity ratios with gears of moderate size in a substantially smaller volume.

13-3-4-3. Advantages of Epicyclical Gearbox

In applications needing gearing, the planetary gearbox presents an intriguing alternative to conventional gear types like helical and parallel shaft gearboxes thanks to a number of specific features:

1. increased reduction ratios،

2. Having a transmission with great torque, it is small and light.
3. the output shaft is under heavy radial loads،
4. It operates more quietly.
5. uniform weight distribution across all gears with more contact between teeth،
6. Since every gear is always in mesh, switching from one gear to another is possible without incurring any loss.

13-3-4-4. Disadvantages of planetary gear systems

1. Complexity,
2. The assembly of gears is restricted to particular tooth-to-gear ratios,
3. It's challenging to calculate efficiency,
4. To prevent further gearing, the driver and driven equipment must be lined up.

13-4 Solved Examples

Example 1: A pinion gear with ($t_p = 33\,teeth$) and a module of ($M = 7\,mm$) has a rotational speed of ($N_p = 1525\,rpm$) and drives a gear at ($N_g = 730\,rpm$). Determine:

1. The number of teeth on the gear, and;
2. The theoretical center distance.

Solution

Given,

$t_p = 33\,teeth; M = 7mm;\ N_p = 1525\,rpm;\ \ N_g = 730\,rpm; t_g = ?\ \&\ a = ?$

1. The number of teeth on the gear (Z_g)

$$\frac{N_p}{N_g} = \frac{t_g}{t_p} \quad \Longrightarrow \frac{1525}{730} = \frac{t_g}{33} \Longrightarrow t_g = \frac{1525}{730} \times 33 = 69\,teeth$$

2. The theoretical center distance (a)

$$a = r_g + r_p = \frac{D_g}{2} + \frac{D_p}{2} = \frac{MZ_g}{2} + \frac{MZ_p}{2}$$

$$\therefore\ a = \frac{7 \times 69}{2} + \frac{7 \times 33}{2} = 241.5 + 115.5 = 357\ mm$$

Example 2:

The spur gear arrangement shown in figure (13-5) has a pinion (driver Gear A) transmit power ($P = 1500\ W$) that rotates at ($N_A = 930\ rpm$), an idler Gear B and a driven gear (Gear C). The gears have a module of ($M = 7\ mm$) and a pressure angle of ($\emptyset = 30^o$). Given the number of teeth as shown in figure 13-4, determine: 1- The pitch diameters of each of the gears. 2- The torque that each shaft is required to transmit. 3- Forces and result on Gear A.

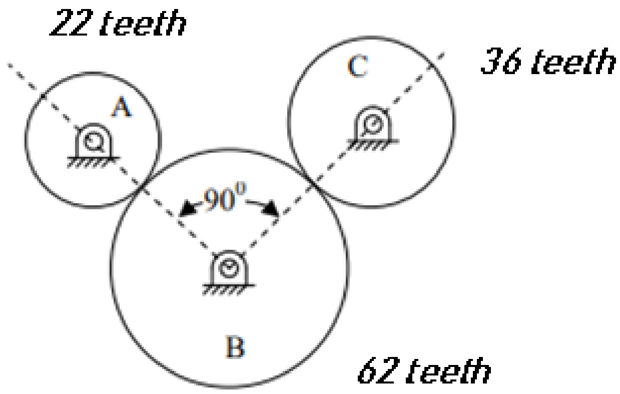

Figure 13-5: Spur Gear arrangement

Solution

Given,

$$\boldsymbol{P} = 2500\ W;\ N_A = 1500\ rpm; t_A = 22\ teeth;\ t_B = 36\ teeth;\ \ t_C = 62\ teeth;\ M = 7mm;\ \emptyset = 30^o.$$

1. The pitch diameters of each of the gears ($D_{A\,,}D_{B\,,}D_{C\,,}$)

$$D_A\ = M.t_A = 7 \times 22 = 154\ mm$$

$$D_B\ = M.t_B = 7 \times 62 = 434\ mm$$

$$D_C\ = M.t_C = 7 \times 36 = 252\ mm$$

2- The torque that each shaft is required to transmit (T_A , T_B , T_C) .

$$i = \frac{N_A}{N_B} = \frac{t_B}{t_A} \quad \& \quad T = \frac{60\ P}{2\pi N}$$

$$T_A = \frac{60\ P}{2\pi N_A} = \frac{60\ \times 2500}{2 \times 3.14 \times 930} = 25.68\ N.mm$$

$$N_B = \frac{Z_A}{Z_B}\,.N_A = \frac{22}{62}\,.930 = 330\ rpm \qquad \& \qquad T_B = 0\ (idler\ gear)$$

$$N_C = \frac{Z_B}{Z_C}\,.N_B = \frac{62}{36}\,.330 = 568.33\ rpm \quad \& \, T_C = \frac{60\ P}{2\pi N_C} = \frac{60\ \times 2500}{2 \times 3.14 \times 568.33}$$

$$= 42.03\ N.mm$$

3- Forces and result on Gear A ($F_{t.A}$, $F_{r.A}$, F_A).

$$Tangential\ force,\ \ F_{t.A} = \frac{T_A}{r_A} = \frac{25.68}{0.077} = 333.5\ N$$

$$Radial\ force,\quad F_{r.A} = F_{t.A}\tan\emptyset = 333.5 \times \tan 30 = 192.55\ N$$

$$Result\ force\ (F_A),\qquad F_A = \sqrt{(F_{t.A})^2 + (F_{r.A})^2} = \sqrt{(333.5\ )^2 + (192.55)^2} \approx 1072\ N$$

Example 3:

Determine the revolutions per minute and direction of rotation of gear (G) in the gear train shown in figure 3-6.

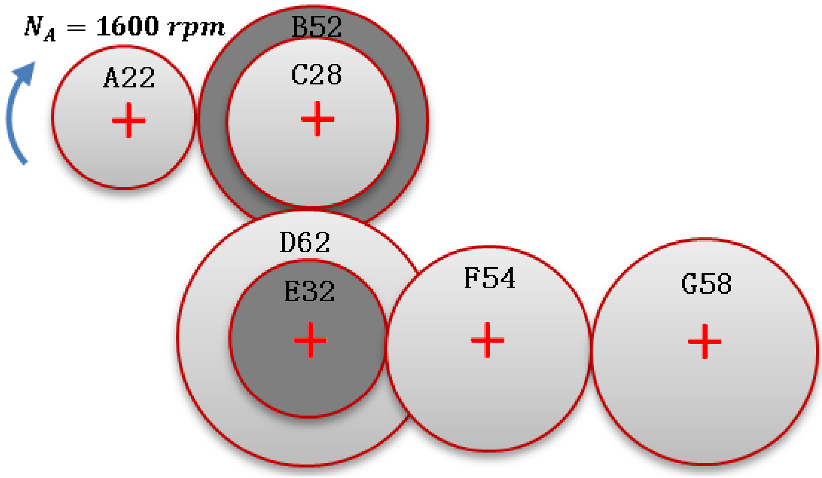

Figure 13-6: Compound Gear arrangement

<u>Solution</u>

Given,

$$N_A = 1600\,rpm;\; t_A = 22\,teeth;\; t_B = 52\,teeth;\;\; t_C = 26\,teeth;\;\; t_D = 62\,teeth;\; t_E = 32\,teeth;\;\; t_F = 54\,teeth;\; t_G = 58\,teeth.$$

$$Train\ value\ (TV) = \frac{N_G}{N_A} = \frac{t_A \times t_C \times t_E \times t_G}{t_B \times t_D \times t_F} = \frac{22\times26\times32\times58}{52\times62\times54} = +6.09797$$

$$\therefore\;\; N_G = N_A \times 6.09797 = 1600 \times 6.09797 = 9756.75\,rpm$$

$$The\ direction\ of\ rotating\ motion\ is\ clockwise$$

Example 4:

Determine the revolutions per minute and direction of rotation of gear (H) in the gear train shown in figure 13-7.

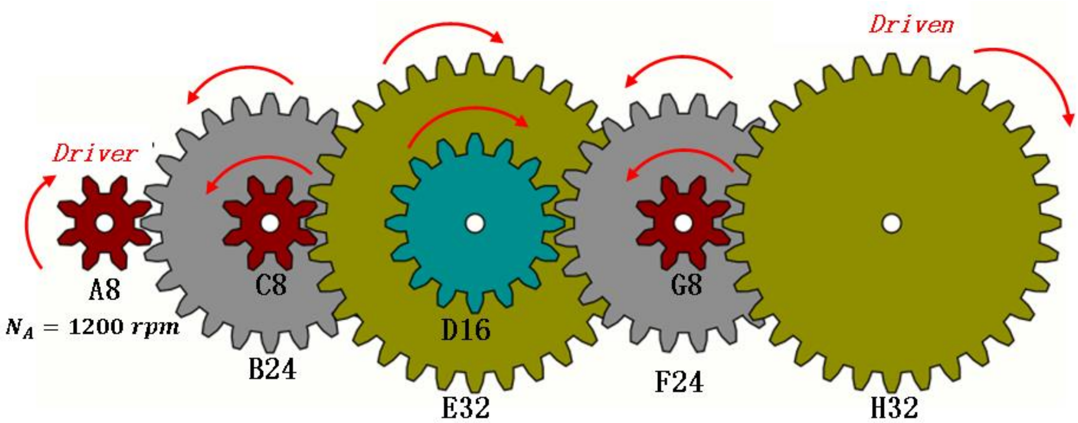

Figure 13-7: Reversed Gear arrangement.

<u>**Solution**</u>

Given,

$$N_A = 1200\,rpm;\; t_A = 8\,teeth;\; t_B = 24\,teeth;\;\; t_C = 8\,teeth;\;\; t_D = 16\,teeth;\; t_E = 32\,teeth;\;\; t_F = 24\,teeth;\;\; t_G = 8\,teeth;\;\; t_H = 32\,teeth.$$

$$Train\ value\ (TV) = \frac{N_G}{N_A} = \frac{t_A \times t_C \times t_E \times t_G}{t_B \times t_D \times t_F \times t_H} = \frac{8\times8\times32\times8}{24\times16\times24\times32} = -0.055$$

$$\therefore\;\; N_G = N_A \times 0.055 = 1200 \times 0.055 = 66.67\,rpm$$

$$The\ direction\ of\ rotating\ motion\ is\ anticlockwise$$

Example 5:

Figure 13-8 shows an Epicyclical gear train in which the wheel D is held stationary by the shaft A and the arm B is rotated at (600 rpm). The wheels E and F are fixed together and rotate freely on the pin carried by the arm. The wheel G is rigidly attached to the shaft C. Find the speed of the shaft C, stating the direction of rotation relative to that of B. The numbers of teeth are as follows: E 28, F 48, and G 38.

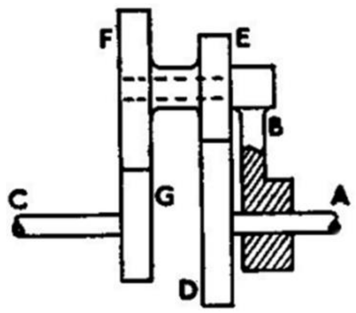

Figure 13-8: Epicyclic gear arrangement.

<u>Solution</u>

Given,

$$N_B = 600\, rpm;\ t_E = 28\, teeth;\ \ t_F = 48\, teeth;\ \ t_G = 38\, teeth.$$

$$r_D + r_E = r_G + r_F$$

$$\therefore \quad t_D + t_E = t_G + t_F\ , Assume\ pitches\ are\ equal$$

$$\therefore \quad t_D + 28 = 38 + 48$$

$$\therefore \quad t_D = 58\, teeth$$

Step	**B**	**C, D**	**F, E**	**A, D**
1	+1	+1	+1	+1
2	0	$-(\frac{t_D}{t_E}) \times (\frac{t_F}{t_G})$	$+(\frac{t_D}{t_E})$	-1
Total	+1	$1-(\frac{t_D}{t_E}) \times (\frac{t_F}{t_G})$	$1+(\frac{t_D}{t_E})$	0

$$\frac{N_G}{N_B} = \frac{1-(\frac{t_D}{t_E}) \times (\frac{t_F}{t_G})}{1} = 1 - \left(\frac{58}{28}\right) \times \left(\frac{48}{38}\right) = -1.617$$

$$\therefore \quad N_G = -1.617 \times N_B = -1.617 \times 600 = -970.2\, rpm$$

$$\therefore \quad N_C = N_G = -970.2\, rpm$$

$$Speed\ (C) in\ opposite\ direction (B).$$

13-5. Chapter Questions

1. **A fixed gear having (200 teeth) is in mesh with another gear having (50 teeth). The two gears are connected by an arm. The number of turns made by the smaller gear for one revolution of arm about the center of bigger gear is:**
   a. **4**
   b. 6
   c. 3
   d. 5

2. In which of the following type of gear train the first gear and the last gear are co-axial.
   a. Epicyclic gear train
   b. Simple gear train
   c. Compound gear train
   d. Reverted gear train

3. Which gear train is used for higher velocity ratios in a small space?
   a. Epicyclic gear train
   b. Simple gear train
   c. Compound gear train
   d. Reverted gear train

4. Which type of gear train is used in clock mechanism to join hour hand and minute hand?
   a. Epicyclic gear train
   b. Simple gear train
   c. Compound gear train
   d. Reverted gear train

5. What is the velocity ratio of two gears if the driver gear has (50 teeth) and driven gear has (20 teeth)?
   a. 2:1
   b. 4:3
   c. **5:2**
   d. 7:5

6. Which gear train contains more than one gear in a single shaft?
   a. Epicyclic gear train
   b. Simple gear train
   c. Compound gear train
   d. Reverted gear train

2. **What is the velocity ratio of a gear train if the train value is (7:2)?**
   a. 2:1
   b. 5:2
   c. 3:4
   d. 7:2

3. **What is the module of a gear if its pitch diameter is (14 mm) and it contains (7 teeth)?**
   a. 2

b. 5
c. 3
d. 4

4. **What is the angular velocity of driven gear if the driver gear has angular velocity of (60 m/s) and velocity ratio of two gears is (6:3)?**

a. 20 m/s
b. 30 m/s
c. 25 m/s
d. 35 m/s

5. **What is the speed ratio of two gears if the speed of the driver gear has (25 km/hr) and driven gear has (10 km/hr)?**

a. 2:1
b. 5:2
c. 3:4
d. 7:2

6. **What is the radius of a gear, if torque generated by the gear is (50 N.m) and the tangential force is (10 N)?**

a. 5
b. 10
c. 2
d. 25

7. **What is the torque offered by a gear, if the tangential force is (33 N) and radius of the gear is**

(2 m)?

a. 66 N.m
b. 33 N.m
c. 16.5 N.m
d. 8.6 N.m

8. A pair of gears has been designed with a velocity ratio of ($i = 3.20$). The pinion has
9. ($t_p = 20\ teeth$) and the circular pitch is ($P_c = 78.54\ mm$). Determine:

a. The number of teeth on the driven gear (t_g).
b. The module for the gears (M).
c. The theoretical center distance (a).

[Ans: $t_g = 64\ teeth$; $M = 25\ mm$; $a = 1050\ mm$]

10. The set of double-reduction gears shown in figure (13-9), is driven by a pinion (Gear A) with a module of ($M = 1.5\ mm$), which rotates at ($N_p = 600\ rpm$) and has a pitch-line velocity of ($V_A = V_B = 4.52\ ms^{-1}$). The second set of gears (Gears C and D) has a pitch-line velocity of ($V_C = V_D = 0.78\ ms^{-1}$) and a module of ($M = 2.5\ mm$). Given that the reduction ratio

between the Gears A and B is to be ($i_{AB} = 12:1$), and the reduction ratio between Gears C and D is to be ($i_{CD} = 12:1$), determine:

a. The number of teeth on each of the gears (t_g).

b. The speed of Gears B, C, and D (N_B, N_C, N_D).

c. The center distances for Gears A and B, and Gears C and D (a_{AB}, a_{CD}).

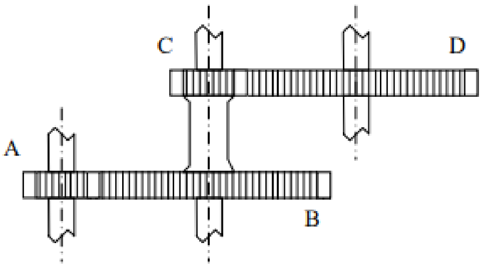

Figure 13-9: Spur Gear arrangement.

$[Ans: Z_g = 200\ teeth;\ N_B = N_C = 300\ rpm, N_D = 30\ rpm; a_{AB} = 156\ mm, a_{CD} = 275\ mm]$

11. Determine the revolutions per minute and direction of rotation of gear (G) in the gear train shown in figure 13-10.

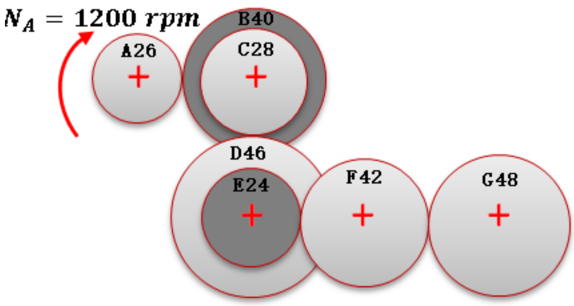

Figure 13-10: Compound Gear arrangement.

$[Ans:\ \ N_G = 237.39\ rpm]$

12. In a reverted epicyclic train, the arm (A) carries two wheels (B and C) and a compound wheel (D) and (E) as shown in Figure 3.14. The number of teeth on wheels (B, C and D) are (75, 30, and 90) respectively. Find the speed and direction of wheel (C) when wheel (B) is fixed and the arm (A) makes (+100 rpm).

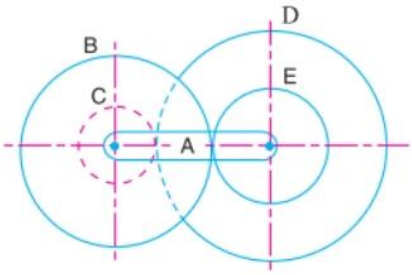

Figure 13-11: Reverted epicyclic train arrangement.

$[Ans:\ \ N_C = -400\ rpm]$

Chapter 14

Design of Simple Gears Box

14. Design of Simple Gears Box

14-1. Introduction

The word “transmission” is used for a device that is located between the clutch and the propeller shaft. It may be a gearbox, a torque converter, overdrive, fluid drive, or hydraulic drive. The purpose of the transmission is to provide high torque at the time of starting, hill climbing, accelerating, and pulling a load. When a vehicle is starting from rest, hill climbing, accelerating, and meeting other resistance, high torque is required at the driving wheels.

14-2. Types of Gearboxes

Following are the types of gearboxes used in modern vehicles:

1. Sliding mesh gearbox
2. Constant-mesh gearbox
3. Synchromesh gearbox
4. Epicyclical gearbox

14-2-1. Sliding Mesh Gearbox

It is the simplest type of gearbox. The arrangement of gears is in a neutral position. The gear housing and bearing are not shown. The clutch gear is fixed to the clutch shaft. It remains always connected to the drive gear of the counter-shaft, figure (14-1).

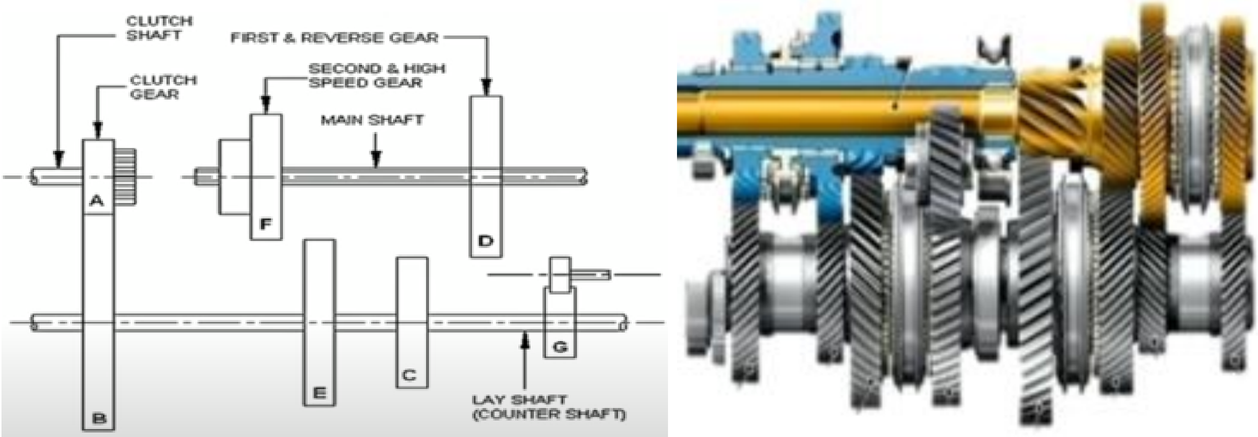

Figure 14-1: Sliding Mesh Gearbox

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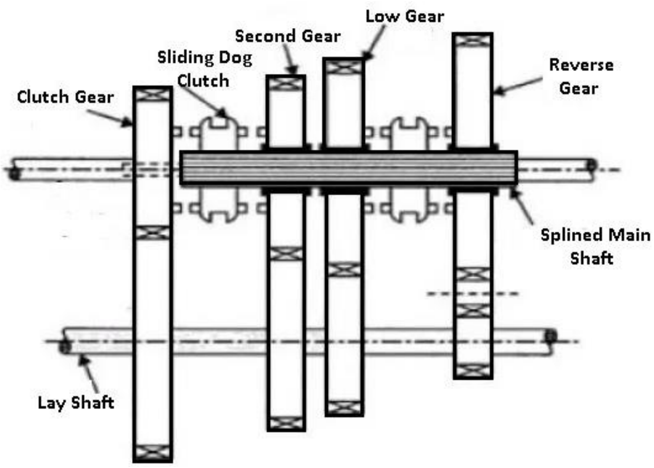
14-2-2. Constant Mesh Gearbox

In this type of gearbox, all the gears of the main shaft are in constant mesh with the corresponding gears of the countershaft. As the figure shows sliding two dog clutches are provided on the main shaft, figure (14-2).



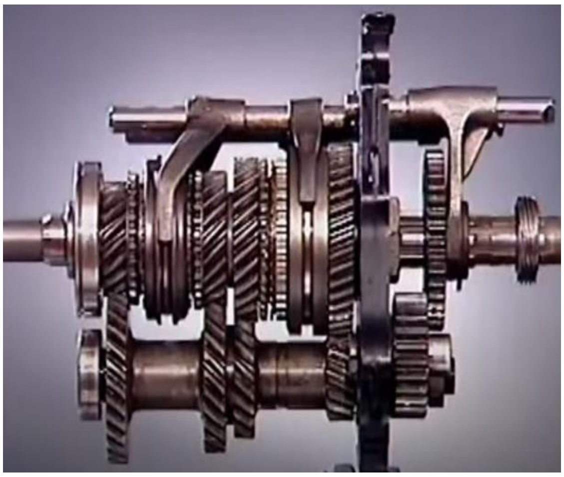
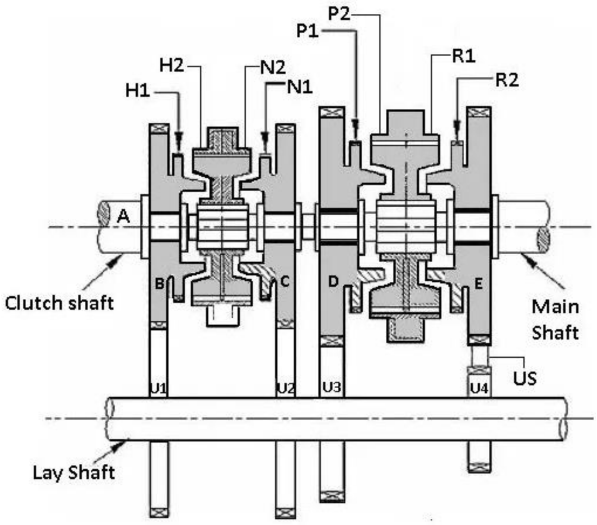


Figure 14-2: Constant Mesh Gearbox

14-2-3. Synchromesh Gearbox

Modern cars use helical gears and synchromesh devices in the gearboxes, that synchronize the rotation of gears that are about to mesh. This eliminates clashing of the gears and makes gear shifting easier, figure (14-3).



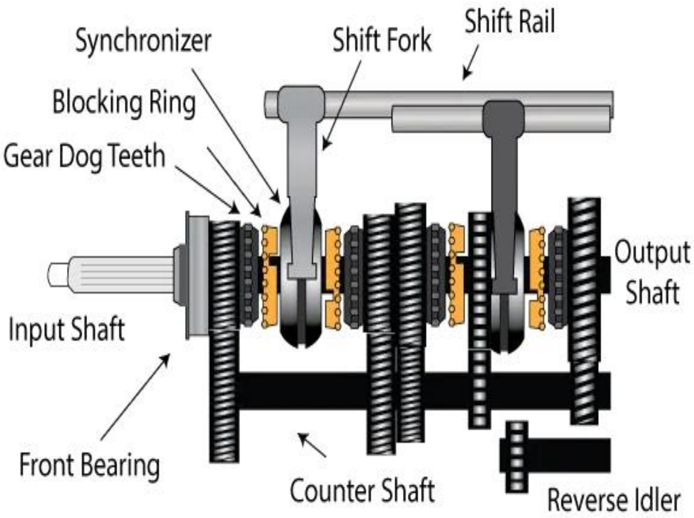


Figure 14-3: Synchromesh Gearbox

14-2-4. Epicyclic gearbox

In an ordinary gearing, the axes of the various gears are fixed, the motion of the gears being simply rotations about their own axes. In epicyclic gearing, at least one gear not only rotates, about its own axis but also rotates bodily about some other axis.

These types of gearboxes are the most widely used automatic transmission system. In an automatic transmission system, there is only an accelerator and brake will be provided. So, there will not be any clutch pedal or gear lever available on the vehicle, figure (14-4).

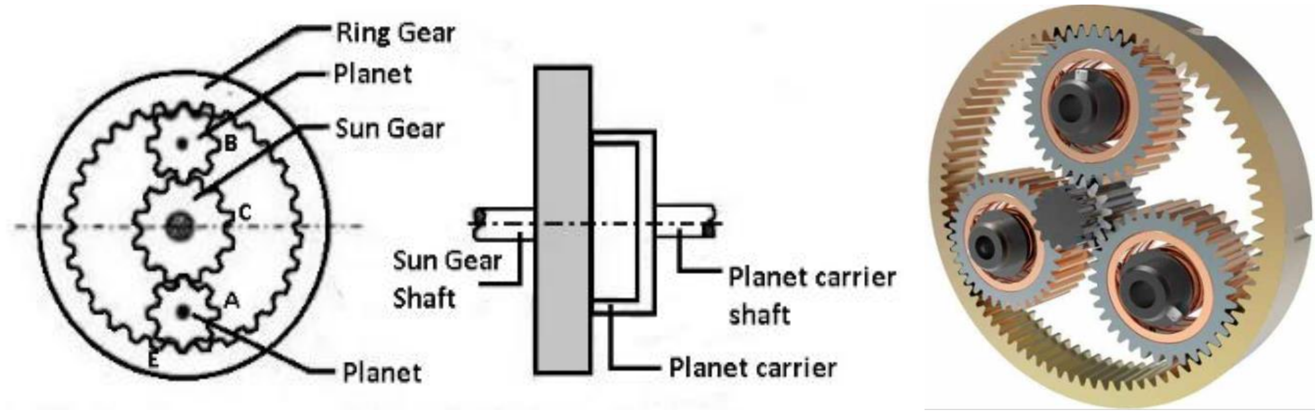


Figure 14-4: Epicyclic Gearbox

14-3. Gearbox design procedure (sliding gear type)

1. For the design of the stepper motor the following information is necessary:

a. Highest out speed
b. Lowest output speed
c. Number of steps (Z)
d. The number of stages to achieve the required number of speed steps

2. disassemble speed steps

The number of steps (Z) must be specified so that it can be divided into multiples of 2 and 3. Hence, the given values from (Z) are, table (14-1):

$$Z = 6, 8, 9, 12, 18$$

Table 14-1: Number of steps (Z)

No.	Number of Speed	First division	Second division	Third division	Notes
1	6	3(1)	2(3)	--------	$x(y), where: \ x = no\ of\ gears.$ $y = iternal\ of\ speeds$
		2(1)	3(2)	--------	
2	8	2(1)	2(2)	2(4)	
3	9	3(1)	3(3)	-------	
4	12	3(1)	2(3)	2(6)	$Always\ choose\ descending\ value\ of\ (x)\ for\ better\ result$
		2(1)	3(2)	2(6)	
		2(1)	2(2)	3(4)	
5	18	3(1)	3(3)	2(9)	
		3(1)	2(3)	3(6)	
		2(1)	3(2)	3(6)	

3. **Structural diagram that gives information about**
   a. Number of shafts in gearbox
   b. Number of gears on each shaft
   c. Arrange transmission shifts in individual groups to get the desired speed.
   d. Transfer range
   e. Group properties

There are a number of points that must be taken into consideration to complete the hierarchical diagram procedures:

a. The number of gears on the last shaft should be as small as possible.
b. The ratio of the speed reduction between the shaft and the previous shaft must be as high as possible.
c. The number of gears on any shaft shall not be more than three, and it can be four in an exceptional case.

d. Preferably ($i_{min} \times \ i_{max} = 1$) at least for the radial dimensions of the gearbox, this is possible by making the axes of the adjacent columns are synchronous, i.e., coaxial. It's possible when the cuts are the maximum speed is equal to the maximum speed increase.

4. How to draw the organization chart

a. If (n) number of transmission sets, draw vertical lines ($n + 1$) at an appropriate distance, over here the first vertical line represents the transmission from the motor shaft, and the remainder represents the gearbox transmission assembly,

b. Draw any of the horizontal lines that intersect the vertical lines at a distance of ($Log\ \emptyset$) from each other. The number of horizontal lines is equal to the number of speed steps (Z). The spacing between the horizontal lines must be equal so that the interval between shaft speeds is appropriate. In practice, the distance between adjacent horizontal lines is taken equal to ($\emptyset$) without ($Log\ \emptyset$),

c. Draw a line connecting the first shaft of the known speed and the second shaft of the calculated input speed, usually the speed between the first and second shaft is reduced by the drive belt.

d. From the second column divergent lines at the point of entry speed connect the third column. The number will be equal to the number of transfers. The maximum spacing between the lines on the three columns will be the lines according to the calculated transfer field between these two columns, ($\emptyset^1, \emptyset^2, \emptyset^3, \emptyset^4, \dots, \emptyset^n$),

e. From the third column of all groups draw divergent lines, having the maximum spacing on the fourth column by the calculated value of the group transport field.

14-4. Solved examples

Example 1:

Design a gearbox for a machine that gives (12) a speed ranging from (32) rpm to (2000) rpm. The power is provided by an electric motor of (10) kW, operating at (1500) rpm. Assume number teeth of driver equal (22)?

Solution:

$$Z = 12, N_{max} = 2000\,rpm, N_{min} = 100\,rpm, N = 3, N_m = 1511\,rpm$$

1. Step – 1: Progression / Step ratio

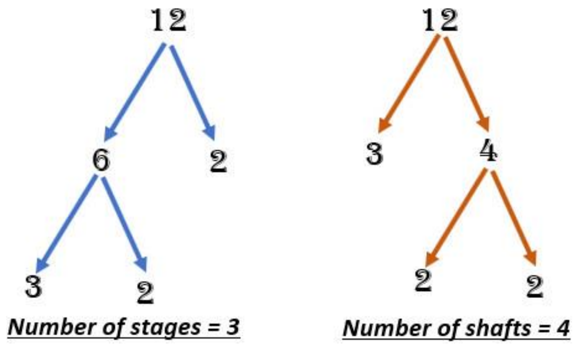
A. Arithmetic Progression: $a = (\frac{N_{max} - N_{min}}{Z-1})$

B. Geometric Progression: $\emptyset = \left(\frac{\boldsymbol{N_{max}}}{\boldsymbol{N_{min}}}\right)^{\frac{1}{Z-1}}$

2. Step – 2: Various speed

Arithmetic Progression	Geometric Progression
$\boldsymbol{a} = \left(\frac{N_{max} - N_{min}}{Z-1}\right) = \frac{2000-100}{12-1} = \mathbf{172.7272}$	$\emptyset = \left(\frac{\boldsymbol{N_{max}}}{\boldsymbol{N_{min}}}\right)^{\frac{1}{Z-1}} = \left(\frac{\mathbf{2000}}{\mathbf{100}}\right)^{\frac{1}{12-1}} = (\mathbf{20})^{\frac{1}{11}} = \mathbf{1.313}$
$N_1 = 100\,rpm$	$N_1 = 100\,rpm,$
$N_2 = N_1 + a = 273\,rpm$	$N_2 = N_1 \times \emptyset = 100 \times 1.313 = 131\,rpm,$
$N_3 = N_2 + a = 445\,rpm$	$N_3 = N_2 \times \emptyset = 131 \times 1.313 = 172\,rpm,$
$N_4 = N_3 + a = 618\,rpm$	$N_4 = N_3 \times \emptyset = 172 \times 1.313 = 226\,rpm,$
$N_5 = N_4 + a = 791\,rpm$	$N_5 = N_4 \times \emptyset = 226 \times 1.313 = 297\,rpm,$
$N_6 = N_5 + a = 946\,rpm$	$N_6 = N_5 \times \emptyset = 297 \times 1.313 = 390\,rpm,$
$N_7 = N_6 + a = 1136\,rpm$	$N_7 = N_6 \times \emptyset = 390 \times 1.313 = 512\,rpm,$
$N_8 = N_7 + a = 1309\,rpm$	$N_8 = N_7 \times \emptyset = 512 \times 1.313 = 673\,rpm,$
$N_9 = N_8 + a = 1482\,rpm$	$N_9 = N_8 \times \emptyset = 673 \times 1.313 = 883\,rpm,$
$N_{10} = N_9 + a = 1655\,rpm$	$N_{10} = N_9 \times \emptyset = 883 \times 1.313 = 1160\,rpm,$
$N_{11} = N_{10} + a = 1827\,rpm$	$N_{11} = N_{10} \times \emptyset = 1160 \times 1.313 = 1523\,rpm$
$N_{12} = N_{11} + a = 2000\,rpm$	$N_{12} = N_{11} \times \emptyset = 1560 \times 1.313 = 2000\,rpm.$

3. Step – 3: Number of stages and shafts



4. Step – 4: Structural formula

$$Z = 12\ speed = 2(1) \quad 3(2) \quad 2(6)$$

5. Step – 5: Kinematic arrangement

Figure (14-5) Kinematic arrangement gears

$$Number\ of\ shafts\ = Number\ of\ stages + 1 = 3 + 1 = 4$$

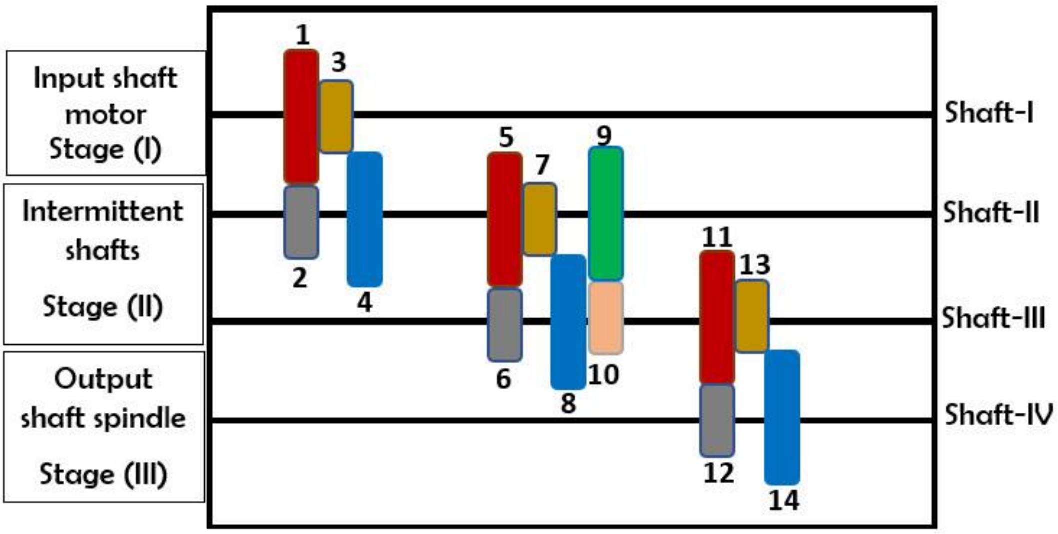


Figure 14-5: Kinematic arrangement gears

Number of pairs in stage (I)	Number of pairs in stage (II)	Number of pairs in stage (III)	Total number of gear pairs	Total number of gears
2	3	2	7	14

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$Input\ speed\ of\ gear\ box = N_i \quad \Rightarrow \quad \log N_i = log\, N_6 + log(\phi/2)$

$\therefore\ \log N_i = log(390) + log\left(\frac{1.313}{2}\right) = 2.591065 - 0.182765 = 2.4083$

$$N_i = 256.04\, rpm$$

For ease of calculation, we shall use the average of (N_6 & N_7), which is:

$$N_i = \frac{N_6 + N_7}{2} = \frac{390+512}{2} = 451\, rpm$$

6. Step – 6: Ray diagram

1. If the engine speed is reduced by a belt drive or gear train before entering the gearbox, draw vertical $(N+2)$ lines. If not, draw $(N+1)$ vertical lines,
2. Label the columns on the vertical lines. If there is a drop in engine speed, start by naming the columns from the second vertical line on the left,
3. Draw $(Z+1)$ horizontal lines,
4. Mark all calculated speeds on the last column (the line on the right),
5. Draw the third stage $(2(6) = 2\ rays, with\ 6\ speed)$.
   a. Set a point at the minimum speed (100 rpm),
   b. Go 6 steps up to (508) and mark another point (remember that two rays take two points),
   c. Calculate ($\frac{100}{0.25} = 400$),
   d. Determine the nearest lower speed to (400 rpm), which is (390 rpm),
   e. Is the obtained speed (390 rpm) higher than the speed at point two (508)? If the answer is "no", then mark the speed (390 rpm) above the second speed (3), but if the answer is "yes", choose a value higher than (0.25) indicates the point in (Shaft-III) between the two points marked on (Shaft-IV),
   f. Connect the given points with the lines (see the blue lines)،

Note: Why (0.25)? For an integrated gearbox with an appropriate radial distance between shafts, the transmission ratio between any two shafts shall be within the range:

$$0.5 \leq i \leq 4$$

6. Draw the second stage ($3(2) = \textit{three rays with two speed steps}$)،
   a. From the point on (Shaft-III), mark two more points two steps up, (remember (3) rays (3) points on the third column),
   b. Calculate ($\frac{125}{0.25} = 500$),
   c. Select a point at (500) on (Shaft-II),
   d. Connect three lines, as shown (three green lines).
7. Draw the first stage (2(1) = two rays with one speed steps)،
   a. From the point on (Shaft-II), select other points one step up،
   b. Calculate the speed of the first column, from the following relationship:

$$\frac{N_m}{i_{belt}} = \frac{1511}{1.313} = 1151\, rpm$$

   c. Select a point at (1151) on (Shaft-I),
   d. Connect two lines, as shown (two red lines).
8. Draw a line from the point on the (Shaft-I) to the shaft speed point (see the pink line). See ray diagram figure (14-6).

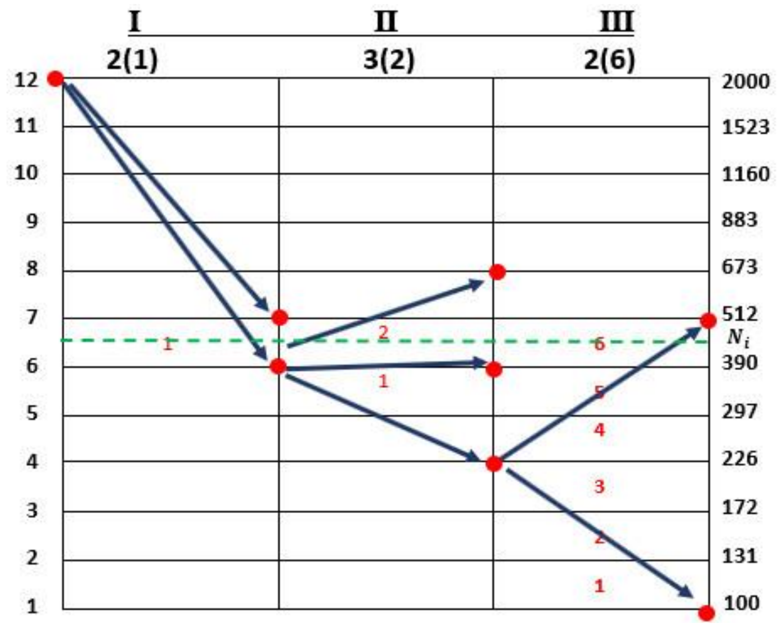

Figure 14-6: Ray diagram

7. Step – 7: Speed diagram

Similar to the radial diagram, the remaining rays are plotted according to the structural formula, as shown in figure (14-7).

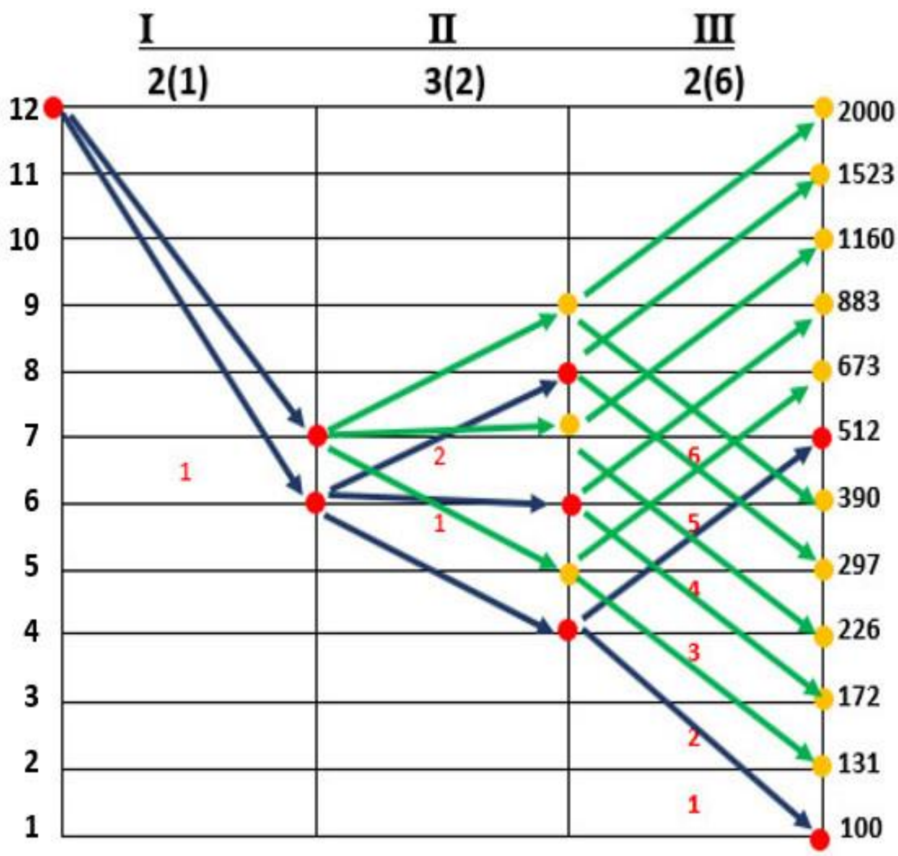

Figure 14-7: Speed diagram.

8. Step – 8: Number of teeth

For the sake of figuring out the number of teeth on each gear pair, we'll suppose the driven gear has 22 teeth. We used the following formulas:

1. For stage (I)

$$t_{1i} + t_{1o} = t_{2i} + t_{2o}$$

$$N_{1i} = N_{2i} = N_i = 446\, rpm;\ N_{1o} = N_7 = 512\, rpm\ ;\ N_{2o} = N_6 = 390\, rpm$$

$$\frac{t_{1i}}{t_{1o}} = \frac{N_{1o}}{N_{1i}} \Rightarrow \frac{t_{1i}}{22} = \frac{512}{446} \quad \Rightarrow \quad t_{1i} = 26$$

$$t_{1i} + t_{1o} = t_{2i} + t_{2o} = 22 + 26 = 48 \quad \Longrightarrow \quad t_{2i} = 48 - t_{2o} \quad - - - - - - - - (1)$$

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$$\frac{t_{2i}}{t_{2o}} = \frac{N_{2o}}{N_{2i}} \Rightarrow \frac{t_{2i}}{t_{2o}} = \frac{390}{446} \Rightarrow \frac{t_{2i}}{t_{2o}} = 0.8744 \quad --------(2)$$

$$\frac{48 - t_{2o}}{t_{2o}} = 0.8744 \Rightarrow 48 - t_{2o} = 0.8744\, t_{2o} \Rightarrow 1.8744\, t_{2o} = 48$$

$$t_{2o} = \frac{48}{1.8744} = 26 \Rightarrow t_{2i} = 48 - t_{2o} = 48 - 26 = 22$$

2. For stage (II)

$$t_{3i} + t_{3o} = t_{4i} + t_{4o} = t_{5i} + t_{5o}$$

$$N_{3i} = N_{4i} = N_{5i} = N_6 = 390\, rpm;\ N_{3o} = N_4 = 226\, rpm;\ N_{4o} = N_6 = 390\, rpm;\ N_{5o} = N_8 = 673\, rpm$$

$$\frac{t_{3i}}{t_{3o}} = \frac{N_{3o}}{N_{3i}} \Rightarrow \frac{t_{3i}}{t_{3o}} = \frac{226}{390} \Rightarrow \frac{t_{3i}}{t_{3o}} = 0.5795$$

$$\frac{t_{4i}}{t_{4o}} = \frac{N_{4o}}{N_{4i}} \Rightarrow \frac{t_{4i}}{t_{4o}} = \frac{390}{390} \Rightarrow \frac{t_{4i}}{t_{4o}} = 1$$

$$\frac{t_{5i}}{t_{5o}} = \frac{N_{5o}}{N_{5i}} \Rightarrow \frac{t_{5i}}{t_{5o}} = \frac{673}{390} \Rightarrow \frac{t_{5i}}{t_{5o}} = 1.7256$$

It will be presumed that the term's greatest multiplication factor has (22) teeth on that gear. Since (t_{5o}) has a high multiplication factor, (22) teeth are believed to exist.

3. For stage (III)

$$t_{6i} + t_{6o} = t_{7i} + t_{7o}$$

$$N_{6i} = N_{5i} = N_i = 297\, rpm;\ N_{6o} = N_2 = 131\, rpm;\ N_{7o} = N_8 = 673\, rpm$$

$$\frac{t_{6i}}{t_{6o}} = \frac{N_{6o}}{N_{6i}} \Rightarrow \frac{t_{6i}}{t_{6o}} = \frac{131}{297} \Rightarrow \frac{t_{6i}}{t_{6o}} = 0.4411$$

$$\frac{t_{7i}}{t_{7o}} = \frac{N_{7o}}{N_{7i}} \Rightarrow \frac{t_{7i}}{t_{7o}} = \frac{673}{297} \Rightarrow \frac{t_{7i}}{t_{7o}} = 2.266$$

The greatest multiplication factor in the term will be taken to have (22) teeth on that gear. The teeth are believed to be (22) teeth, because (t_{7o}) multiplication factor is high.

So that, $t_{7o} = 22$

$$\frac{t_{7i}}{t_{7o}} = 2.266 \quad \Rightarrow \quad t_{7i} = 2.266\, t_{7o} = 2.266 \times 22 = 50$$

$$t_{7i} + t_{7o} = 22 + 50 = 72$$

$$t_{6i} + t_{6o} = t_{7i} + t_{7o} \; ---------(1)$$

$$\frac{t_{6i}}{t_{6o}} = 0.4411 \; ----------(2)$$

$$0.4411\, t_{6o} + t_{6o} = 72$$

$$t_{6o} = \frac{72}{1.4411} = 50$$

$$\frac{t_{6i}}{t_{6o}} = 0.4411 \quad \Rightarrow \quad t_{6i} = 0.4411\; t_{6o} = 0.441 \times 50 = 22$$

Calculated values in table form			
Input Gear	**Number of teeth**	**Output Gears**	**Number of teeth**
$\boldsymbol{t_{1i}}$	**26**	$\boldsymbol{t_{1o}}$	**22**
$\boldsymbol{t_{2i}}$	**22**	$\boldsymbol{t_{2o}}$	**28**
$\boldsymbol{t_{3i}}$	**22**	$\boldsymbol{t_{3o}}$	**38**
$\boldsymbol{t_{4i}}$	**30**	$\boldsymbol{t_{4o}}$	**30**
$\boldsymbol{t_{5i}}$	**38**	$\boldsymbol{t_{5o}}$	**22**
$\boldsymbol{t_{6i}}$	**22**	$\boldsymbol{t_{6o}}$	**50**
$\boldsymbol{t_{7i}}$	**50**	$\boldsymbol{t_{7o}}$	**22**

Example 2:

Design a gearbox for a machine that gives (16) a speed ranging from (300) rpm to (1600) rpm?

<u>Solution:</u>

$$Z = 16, N_{max} = 1600\, rpm, N_{min} = 200\, rpm.$$

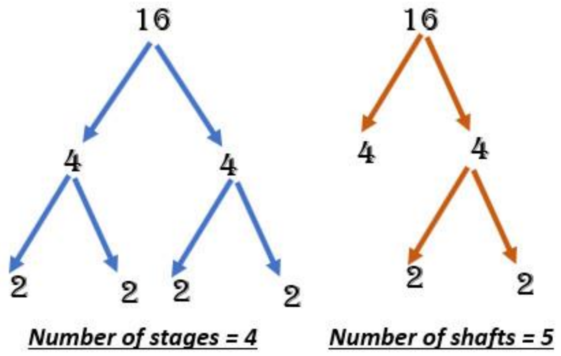
1. Step – 1: Progression / Step ratio

$$\emptyset = \left(\frac{N_{max}}{N_{min}}\right)^{\frac{1}{Z-1}} = \left(\frac{1600}{200}\right)^{\frac{1}{16-1}} = (8)^{\frac{1}{15}} = 1.1487$$

2. Step – 2: Various speed

$$N_1 = 200\, rpm,$$
$$N_2 = N_1 \times \emptyset = 200 \times 1.1487 = 230\, rpm,$$
$$N_3 = N_2 \times \emptyset = 230 \times 1.1487 = 264\, rpm,$$
$$N_4 = N_3 \times \emptyset = 264 \times 1.1487 = 303\, rpm,$$
$$N_5 = N_4 \times \emptyset = 303 \times 1.1487 = 348\, rpm,$$
$$N_6 = N_5 \times \emptyset = 348 \times 1.1487 = 400\, rpm,$$
$$N_7 = N_6 \times \emptyset = 400 \times 1.1487 = 459\, rpm,$$
$$N_8 = N_7 \times \emptyset = 459 \times 1.1487 = 528\, rpm,$$
$$N_9 = N_8 \times \emptyset = 528 \times 1.1487 = 606\, rpm,$$
$$N_{10} = N_9 \times \emptyset = 606 \times 1.1487 = 696\, rpm,$$
$$N_{11} = N_{10} \times \emptyset = 696 \times 1.1487 = 800\, rpm,$$
$$N_{12} = N_{11} \times \emptyset = 800 \times 1.1487 = 919\, rpm,$$
$$N_{13} = N_{12} \times \emptyset = 919 \times 1.1487 = 1056\, rpm,$$
$$N_{14} = N_{13} \times \emptyset = 1056 \times 1.1487 = 1213\, rpm,$$
$$N_{15} = N_{14} \times \emptyset = 1213 \times 1.1487 = 1393\, rpm,$$
$$N_{16} = N_{15} \times \emptyset = 1393 \times 1.1487 = 1600\, rpm.$$

3. Step – 3: Number of stages and shafts



4. Step – 4: Structural formula

$$Z = 16\ speed = 2(1) \quad 2(2) \quad 2(4) \quad 2(8)$$

5. Step – 5: Kinematic arrangement

Figure (14-8) Kinematic arrangement gears

$$Number\ of\ shafts\ = Number\ of\ stages + 1 = 4 + 1 = 5$$

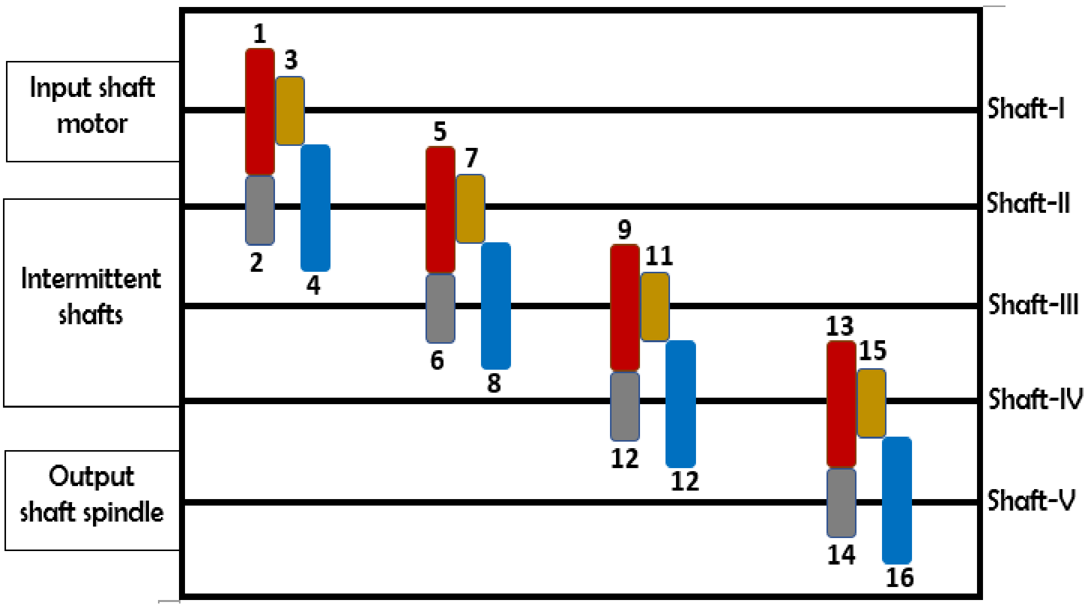


Figure 14-8: Kinematic arrangement gears

6. Step – 6: Ray diagram

See ray diagram figure (14-9).

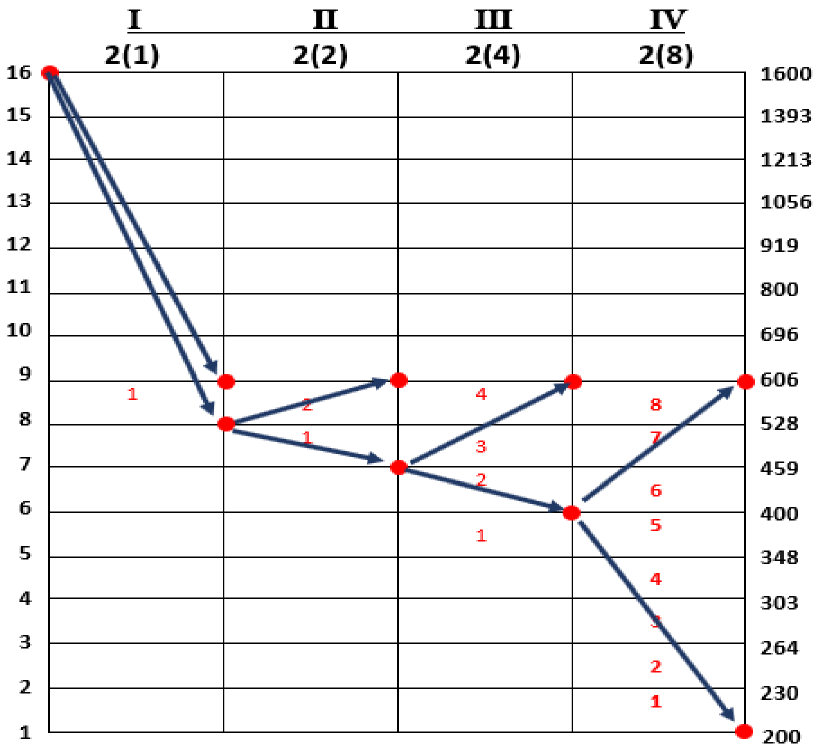


Figure 14-9: Ray diagram

7. Step – 7: Speed diagram

Similar to the radial diagram, the remaining rays are plotted according to the structural formula, as shown in figure (14-10).

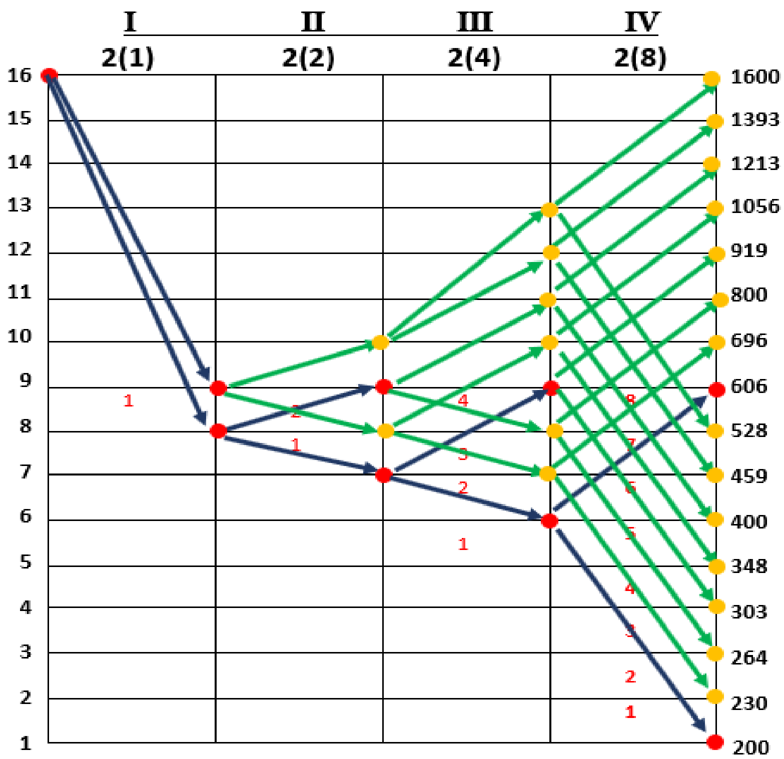


Figure 14-10: Speed diagram

14 – 4. Chapter Questions

1. What one of the following requires a gearbox?

a. To modify the vehicle's speed
b. Modifying the vehicle's torque
c. Modifying the vehicle's power
d. Modifying the vehicle's acceleration

2. Which gearbox has a constant contact between all of its gears?

a. constant-mesh transmission
b. mesh sliding gearbox
c. Synchromesh transmission
d. Epicyclic transmission

3. Which of the subsequent is not a component of automated transmission؟

a. Epicyclic transmission
b. converter of torque
c. several-plate clutch
d. mesh sliding gearbox

4. Which epicyclic gearbox arrangement will provide a forward and rapid output speed؟

a. Planet carrier drying, fixed sun gear, and ring gear propulsion
b. A stationary planet carter is driven by the sun gear and the ring gear.
c. Planet carrier dewing, operated by a sun gear, with a fixed ring gear
d. None Of the Above

5. The gearbox and transfer case are enclosed in a moat in.

a. Front wheel drive.
b. Drive in the rear whee1
c. All wheel drives
d. All of the above

5. The word "transmission" is used for a device that is located between?

a. The flywheel and clutch
b. The front whee1s
c. The rear wheels
d. The clutch and the propeller shaft

7. It remains always connected to the drive gear of the countershaft؟

a. Sliding mesh gearbox
b. Constant-mesh gearbox
c. Synchromesh gearbox
d. Epicyclical gearbox

8. In this type of gearbox, all the gears of the main shaft are in constant mesh with the corresponding gears of the countershaft?

a. Sliding mesh gearbox
b. Constant-mesh gearbox
c. Synchromesh gearbox
d. Epicyclical gearbox

9. This gearbox eliminates clashing of the gears and makes gear shifting easier?

a. Sliding mesh gearbox
b. Constant-mesh gearbox
c. Synchromesh gearbox
d. Epicyclical gearbox

10. In this gearbox, at least one gear not only rotates, about its own axis but also rotates bodily about some other axis?

a. Constant-mesh gearbox
b. Sliding mesh gearbox
c. Synchromesh gearbox
d. Epicyclical gearbox

11. These types of gearboxes are the most widely used automatic transmission system?

a. Constant-mesh gearbox
a. Epicyclical gearbox
b. Sliding mesh gearbox
c. Synchromesh gearbox

12. A turret lathe's headstock gearbox, which has nine speeds, may produce speeds between 180 and 1800 rpm. Draw the kinematic layout and speed diagram using the conventional step ratio. Additionally, determine and correct each gear's tooth count, assume number of teeth in driver ($t_5 = 20$)?

Gear	t_{i1}	t_{i2}	t_{i3}	t_{i4}	t_{i5}	t_{i6}	t_{o1}	t_{o2}	t_{o3}	t_{o4}	t_{o5}	t_{o6}
Teeth	24	44	29	39	20	48	35	48	53	30	20	63

13. Using arithmetic and geometric progression, determine 12 variable speeds for spindle speeds that vary by 20 rpm and 894 rpm, respectively. According to the R20/3 series, the geometric progression ratio. Additionally, determine and correct each gear's tooth count, assume number of teeth in driver ($t_{7o} = 18$)?

Gear	t_{i1}	t_{i2}	t_{i3}	t_{i4}	t_{i5}	t_{i6}	t_{i7}	t_{o1}	t_{o2}	t_{o3}	t_{o4}	t_{o5}	t_{o6}	t_{o7}
Teeth	22	18	18	27	36	18	51	18	22	36	27	18	51	18

13. Draw the configuration of a speed gearbox with a minimum speed of 460 revs per minute and a maximum speed of 1400 revs per minute. Create a speed diagram and kinematic arrangement that shows the total number of gear teeth, assume number of teeth in driver ($t_{1o} = 18$)?

Gear	t_{i1}	t_{i2}	t_{i3}	t_{i4}	t_{i5}	t_{o1}	t_{o2}	t_{o3}	t_{o4}	t_{o5}
	t_1	t_3	t_5	t_7	t_9	t_2	t_4	t_6	t_8	t_{10}
Teeth	22	18	18	27	36	18	51	18	22	36

Chapter 15

Worm Gears

15. Worm Gears

15-1. Introduction

A special design of the gear wheel is the so-called worm. In this case, the tooth winds around the worm shaft like the thread of a screw. The mating gear to the worm is the worm gear. Such a gearbox, consisting of worm and worm wheel, is generally referred to as a worm drive. In the mechanical design of machinery, worm gears are crucial components. Power can be transferred between shafts that are not parallel thanks to worm gear sets. The worm has a tooth with a helix form that is connected to a spur gear or helical gear. When significant gear reductions are required, worm gears are used. Worm gears frequently have reductions of (20 :1), and sometimes even (300: 1) or more.

15-2. Characteristics of a Worm Gears

1. A large speed ratio is obtained ($G = 5 - 80$),
2. A quiet and fluid operation,
3. Self-locking will happen if the lead angle is less than the equivalent friction angle,
4. High cost and low efficiency antifriction materials are required.

15-3. Distinguishes of a Worm Gears

1. Include a worm and worm wheel gear,
2. Transfer power between shafts that are not crossing and are typically at a 90-degree angle,
3. Applied when a high reduction ratio is needed,
4. Meshing combines sliding and rolling, and
5. It resembles a crossed helical gear but can bear more weight.

15-4. Types of a worms

Depending on the shape of the worm, worm drives can be classified differently, figure (15-1).

a. **Cylindrical worms**: A worm is referred to as a cylindrical worm if its exterior has a cylindrical pattern.

b. **Globoid worms** (enveloping worms): In a modified variation, the worm's outer form is an arc that partially encloses the globoid worm gear.

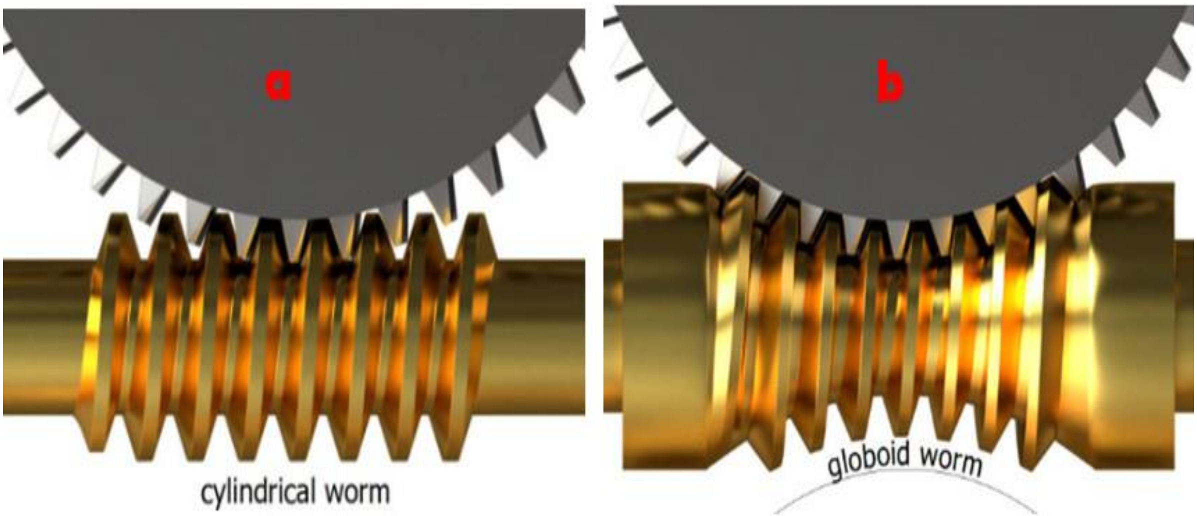


Figure 15-1: Types of worms

15-5. Types of a worm gears

Worm gear comes in three distinct varieties, figure (15-2).

A. **Non-throated** - This type uses a straight worm without a circumferential groove. Because tooth contact is provided by a single moving point, this particular form of worm gear is vulnerable to high unit load wear and tear.

B. **Single-throated** – For line contact, concave helical teeth are wrapped around the worm, allowing for higher unit loads and less severe wear,

C. **Double-throated** - Referred to as a cone of hourglass because both the worm screw and the gear itself have concave teeth، this method of increasing the contact surface enables higher unit loads with less wear and tear.

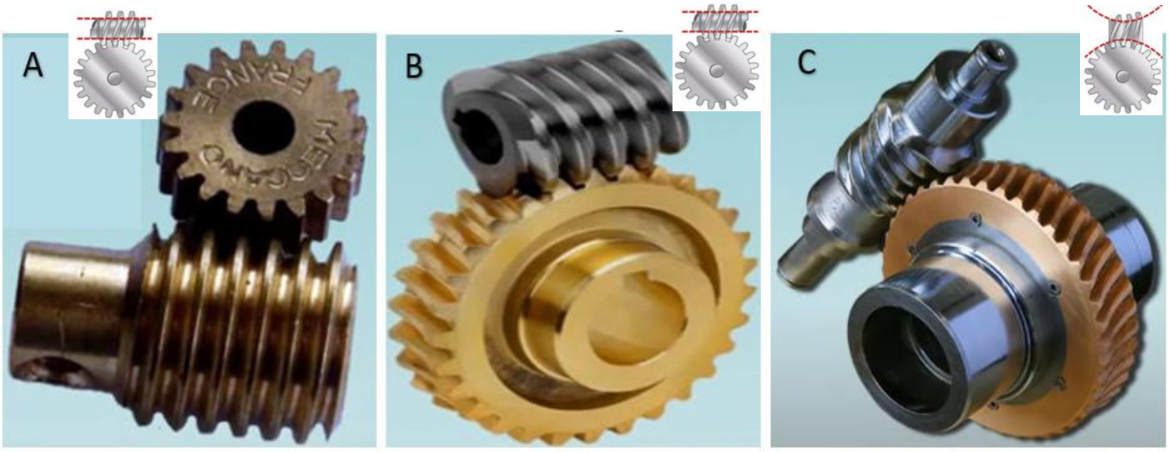


Figure 15-2: Types of worm gears

15-6. Advantages and disadvantage of a Worm Gears

1. Advantages

1. High reduction ratio,
2. Self-locking property,
3. Low weight,
4. Compact,
5. Smooth and noise less operation.

2. Disadvantages

1. High friction,
2. Low efficiency,
3. Heat generation due to sliding and rolling.

15-7. Application of a worm gears

Worm gears are commonly utilized in:

1. Presses device,

2. Lifts,
3. Conveyors,
4. Gate control mechanisms,
5. Hoisting machines,
6. Speed reducer,
7. Automobile steering mechanism,
8. Machine tools.

15-8. Worm Gear Operation

The worm receives rotational power from an electric motor or engine. The screw face presses against the teeth of the wheel while the worm turns in opposition to it. The load pushes up against the wheel, figure (15-3).

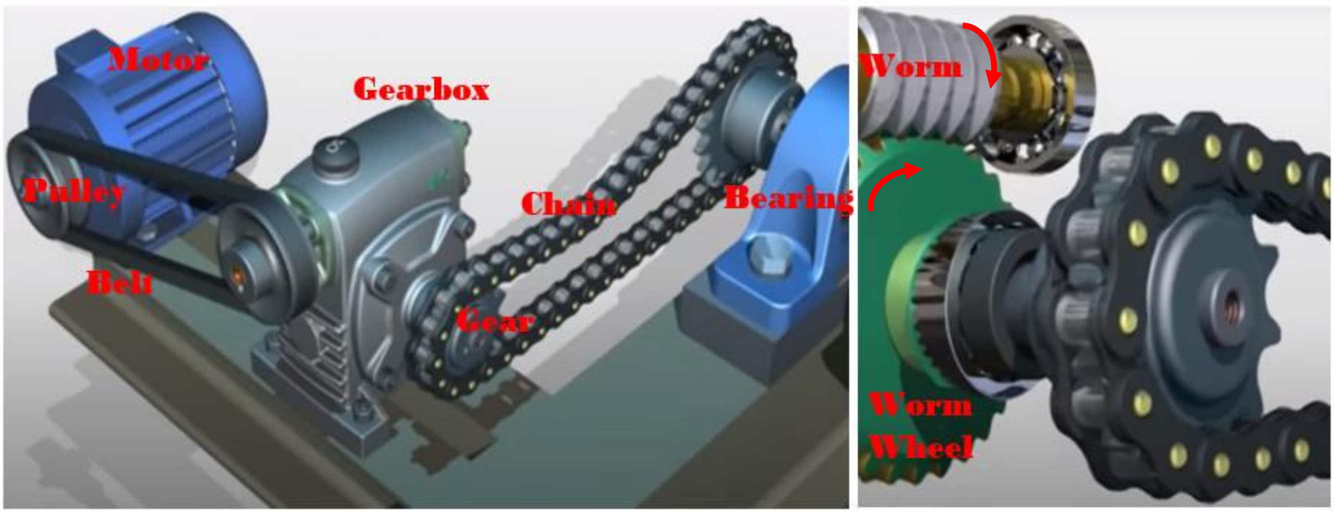


Figure 15-3: Worm Gear Operation

15-9. Mode of failure in worm gear drives

Worm gear failure can be caused by a number of factors, the most significant of which are as follows, figure (15-4):

a. Bending root failure,
b. Pitting surface failure,
c. Scoring surface failure,
d. Abrasive wear surface failure,

e. Corrosive surface failure.

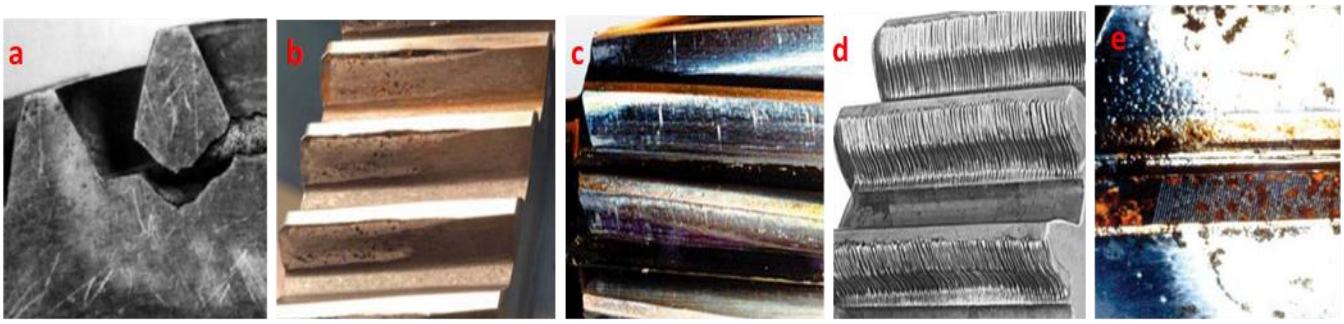


Figure 15-4: Mode of failure in worm gear drives

15-10. Worm gearing materials

1. Worm materials
    a. Cast iron
    b. Alloy steel
    c. Cast steel
    d. Carbon steel
2. Worm wheel gear materials
    a. Aluminum alloys
    b. Brass
    c. Copper
    d. Plastic

15-11. Lubrication challenges and specifications of lubricants

Worm drive designs have one significant flaw: practically all of the relative motion between the two parts' mating teeth is sliding. The lubrication is regularly scraped away, which presents a substantial issue.

A high-quality worm drive lubricant will typically have the following characteristics :

1. Low friction ،
2. Strong oxidation resistance ،
3. Good anti-wear protection, and

4. High viscosity index.

15-12. Main geometric parameters of the worm gear

Worm gears with cylindrical Archimedean worms are the most common type. The axial portion of these gears has a rectilinear trapezoidal profile, while the terminal section's helical line is represented by an Archimedean spiral.

Figure (15-3) displays the worm gears' geometric specifications.

15-12-1. Worm parameters

1. Axial pitch (p_x): The space added to the tooth thickness or the linear distance between the centers of two subsequent teeth. It is determined using the relationship shown below:

$$p_x = \pi \,.\, m \quad (mm) \qquad BB \qquad (15-1)$$

2. Lead (L): It is a distance by which a Helix advances along the axis for the one turn around, and it can be estimated using the relationship shown below:

$$L = n \,.\, p \quad (mm) \qquad (15-2)$$

3. Number of starts (n): It is a number of threads traversed on the worm for one turn around, figure (15-3).

4. Helix pitch (p_z): It is the distance that the helical duct moves towards the axis of its cylinder during one revolution, and is

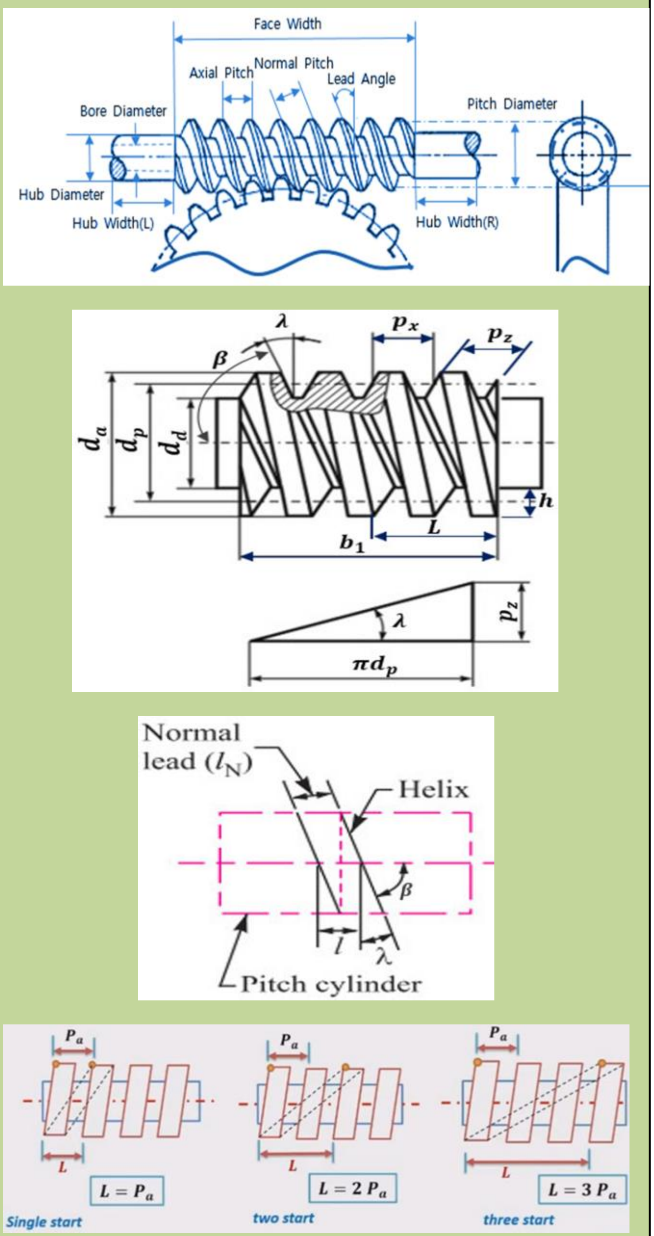


Figure 15-3: Geometric parameters of the worm

calculated by the following relationship:

$$p_z = p_x \, . \, t_w \quad (mm) \qquad (15-3)$$

Additionally, the worm gear's helical step value is often lower than the bevel gear's helical step.

5. Lead angle (λ): It is the angle at which teeth are inclined to the normal to the axis of rotation, and is calculated by the following relationship:

$$tan \, \lambda = \frac{p_z}{\pi d_p} \qquad (15-4)$$

$$Lead \ angle \ of \ worm \ (\lambda) = Helix \ angle \ of \ worm \ gear$$

6. Helix angle (β): It is the angle made by the helix or teeth with the axis of rotation, as in figure (15-3), and is calculated by the following relationship:

$$\lambda + \beta = 90^{\circ} \qquad (15-5)$$

7. Pitch circle diameter (d_p): The value of the diameter of the pitch circle (d_p), which is usually given on the technical drawing of the spiral, is calculated when designing from the following relationship:

$$d_p = m \, . \, q \quad (mm) \qquad (15-6)$$

Where (q): is number of the modulus (the axis of the worm) in the diameter of the pitch circle (d_p) in the cylinder (which is the imaginary number of teeth on the worm), which is fixed and multiplied by the value of the module and is taken from the table (15-1).

Table 15-1: Constant values (q) according to the module (m)

Module (m)	2	2.5	3	3.5	4	4.5	5	5.5	6	7	8	9	10	12	14	16	18	20	24	30
Constant (q)	13	12	12	12	11	11	10	9	9	8	8	8	8	8	9	9	8	8	8	8

8. Addendum circle diameter (d_a): It can be determined by using the relationship shown below:

$$d_a = d_p + m \quad (mm) \qquad (15-7)$$

9. Dedendum circle diameter (d_d): It can be calculated by using the following relationship:

$$d_d = d_p - 2.334\, m \quad (mm) \qquad (15-8)$$

10. The depth of the total tooth (h_w): It can be calculated using the following relationship:

$$h_w = 2.2\, m \quad (mm) \qquad (15-9)$$

15-12-2. Worm gear parameters

1. Pitch circle diameter (D_p): It is calculated when designing from the following relationship:

$$D_p = m + t_g \quad (mm) \qquad (15-10)$$

2. Addendum circle diameter (D_a): It can be determined by using the relationship shown below:

$$D_a = D_p + 2m \quad (mm) \qquad (15-11)$$

3. Dedendum circle diameter (D_d): It can be calculated by using the following relationship:

$$D_d = D_p - 2.334\, m \quad (mm) \qquad (15-12)$$

4. largest outer diameter (D_L): To calculate the largest outer diameter we use the following relationship:

$$D_L = D_a + m \quad (mm) \qquad (15-13)$$

5. External concavity radius (R_a): The teeth in the worm gear are concave so that they forget the worm's rotation in radius, according to the following relationship, which is produced when the gear body is warned on the lathe:

$$R_a = \frac{d_p}{2} \quad (mm) \qquad (15-14)$$

6. Internal concavity radius (R_d): This concavity is produced after the process of milling the teeth of the worm wheel, according to the following relationship:

$$R_d = \frac{d_p}{2} + 1.167m, \quad (mm) \qquad (15-15)$$

7. Width of the worm gear (b_2): The width of the worm gear wheel is related to the number of starts of the teeth (t_1) of the worm, as in the following equations:

$$b_2 \leq 0.75\ d_a\,, \quad (mm) \qquad (15-16)$$

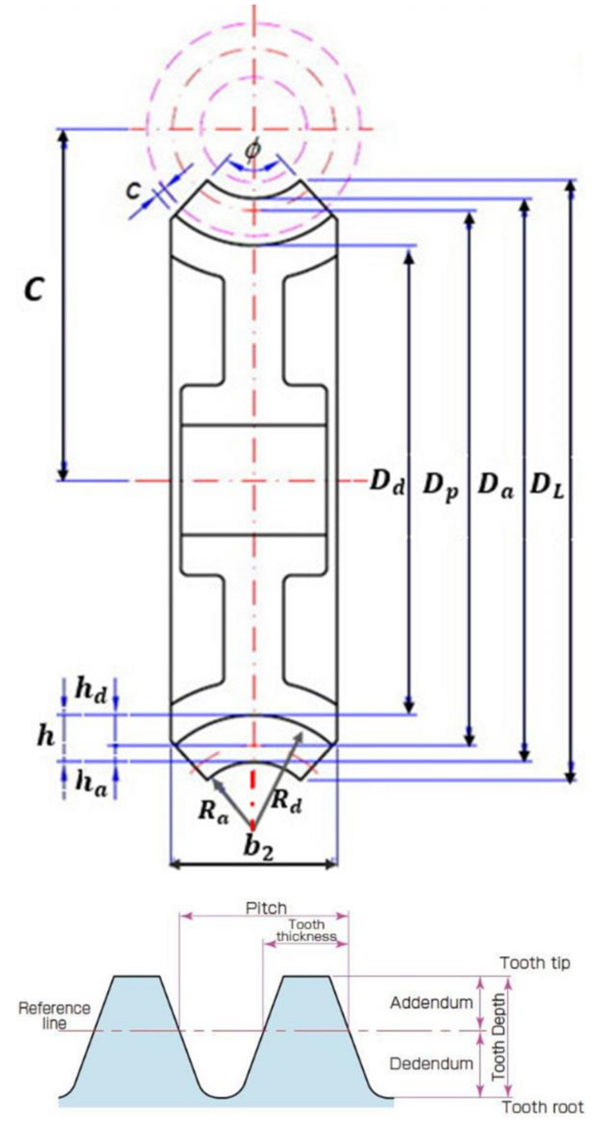


Figure 15-3: Geometric parameters of a worm gear

8. The distance between the shafts of the gears (C) It is the distance between center of worm pitch circle and center of worm gear pitch circle, and calculate from the following equation:

$$C = \frac{d_p + D_p}{2}, \quad (mm) \qquad (15-17)$$

9. Tooth Depth (h): Tooth depth is determined from the size of the module (m). Introduced here are the (ISO & JIS) (Japan industrial standards)-required tooth profiles (full depth), figure (15-3).

$$h_g = adedendum\ (h_a) + dedendum (h_d) \quad (mm) \qquad (15-18)$$

$$h_a = m, \quad (mm) \qquad (15-19)$$

$$h_d = 1.25\, m, \quad (mm) \qquad (15-20)$$

15-13. Proportions for Worm Gear

The following table shows the various proportions for worm gears in terms of circular pitch (pc) in mm.

Table 15-1: Proportions for worm gear

NO.	Particulars	Symbol	Single and double threads	Triple and quadruple threads
1	Normal pressure angle	$\emptyset$	$14.5°$	$20°$
2	Outside diameter	D_o	$D_p + 1.0135\ P_C$	$D_p + 0.8903\ P_C$
3	Throat diameter	D_T	$D_p + 0.636\ P_C$	$D_p + 0.572\ P_C$
4	Face width	b	$2.38\ P_C + 6.5\ mm$	$2.15\ P_C + 5\ mm$
5	Gear face radius	R_f	$0.882\ P_C + 14\ mm$	$0.914\ P_C + 14\ mm$
6	Gear rim radius	R_r	$2.2\ P_C + 14\ mm$	$2.1\ P_C + 14\ mm$

15-14. Efficiency of Worm Gearing

Worm gearing efficiency may be measured using the ratio of work done by the worm gear to that done by the worm. Mathematical calculations can be used to determine the effectiveness of worm gearing:

$$\eta = \frac{tan\,\lambda\,(\cos\phi - \mu\,.tan\,\lambda)}{(\cos\phi\ .tan\,\lambda) + \mu} \times 100\% \qquad (15-21)$$

$$where, \emptyset\ =\ Normal\ pressure\ angle, \mu\ =\ Coefficient\ of\ friction, and$$

$$\lambda\ =\ Lead\ angle.$$

15-15. Producer design worm gear

The representation of worm and worm wheel gears will be completed as follows:

$$t_w/t_g/q/m \qquad (15-22)$$

$For\ example,$ $(3/48/12/8\ )$: Are the worm gears represented.

$$t_w = 3\ ;\ t_g = 48\ ;\ q = 12\ ;\ m = 8$$

$$Were, \qquad t_w = Number\ of\ a\ worm\ teeth\ ;\ t_g = Number\ of\ a\ gear\ worm\ teeth\ ;$$

$$q = Diametral\ quotient\ ;\ m = Module.$$

Step 1: Calculate number of teeth on gear (t_g) from gear ratio or speed reduction ratio (G).

$$G = \frac{t_g}{t_w} = \frac{N_w}{N_g} \qquad (15-23)$$

$$N_w = Speed\ of\ a\ worm\ teeth\ (rpm);\ N_g = Speed\ of\ a\ gear\ teeth(rpm).$$

Step 2: Find minimum face width (b), from the following formula:

$$b = 2m\sqrt{q+1} \quad or \quad b = 0.73\ d_w \quad \Rightarrow\ Choose\ the\ highest\ value \qquad (15-24)$$

Step 3: Calculate bending stress

$$\sigma_b = \frac{S_{ut.}}{3} \qquad (15-25)$$

$$\sigma_b = Bending\ endutance\ stress\ (MPa);\ S_{ut.} = Ultimatetensile\ strength\ (MPa).$$

Step 4: Calculate Lewis form factor (Y)

$$Y = 0.484 - \frac{2.87}{t_g}\ , \quad for\ (20^{\circ})full-depth\ involute, \qquad (15-26)$$

$$Y = 0.39 - \frac{2.15}{t_g}\ , \quad for\ (14.5^{\circ})full-depth\ involute. \qquad (15-27)$$

Step 5: In worm gear pair, worm gear governs the design

Worm gear is designed, so there's no need to choose which is weaker.

Worm gear always has a weaker design because it controls how everything is made.

Step 6: Calculate beam strength

$$F_b = \sigma_b \,.\, b \,.\, m \,.\, Y \,.\cos\lambda \;, N \qquad (15-28)$$

$$Y = lewis\ form\ factor\ ;\ \lambda = Lead\ angle.$$

Step 7: Calculate wear strength from the following equation:

$$F_w = d_g \,.\, b \,.\, K \;, N \qquad (15-29)$$

$$F_w = Wear\ strength\ ;\ K = Load\ stress\ factor, table\ (15-1)\ \left(\frac{N}{mm^2}\right);$$

$$d_g = Pitch\ cicle\ diameter\ of\ worm\ gear.$$

Table 15-1: Load stress factor (K) values

No.	Material		Load stress factor (K), N / mm^2
	Worm	Worm gear	
1	Steel (B.H.N. 250)	Phosphor bronze	0.4150
2	Hardened steel	Cast iron	0.3450
3	Hardened steel	Phosphor bronze	0.5500
4	Hardened steel	Chilled Phosphor bronze	0.8300
5	Hardened steel	Antimony bronze	0.8300
6	Cast iron	Phosphor bronze	1.0350

Step 8: Calculate tangential load (F_t) from pitch line velocity (v).

$$(F_g)_t = \frac{P_o}{v_g} \qquad (15-30)$$

Output Power:

$$P_o = \eta . P_i \qquad (15-31)$$

$$\eta = \frac{\tan\lambda}{\tan(\phi_v + \lambda)} \qquad (15-32)$$

$$\phi_v = \tan^{-1}\left[\frac{\mu}{\cos\phi_n}\right] \qquad (15-33)$$

$Note: For\ self-loading: (\phi_v > \lambda)$

$\eta = Efficiency\ ;\ P_i = Input\ power\ ; \phi_v = Virtual\ coefficient\ of\ friction\ ;$

$\mu = Coefficientof\ friction\ ; \phi_n = Normal\ pressure\ angle.$

Step 9: Select service factor, velocity factor (K_v).

$$K_v = \frac{6}{6 + v_g} \qquad (15-34)$$

$K_a = Service\ factor\ ;\ K_v = Velocity\ factor.$

Step 10: Calculate effective load

$$F_{eff} = \frac{K_a . (F_g)_{\boldsymbol{t}}}{K_v} \qquad (15-35)$$

Step 11: Calculate module

$If\ \ (F_b < F_w)\ gear\ is\ weaker\ in\ bending$

$If\ \ (F_b > F_w)\ gear\ is\ weaker\ in\ bwear$

Based on beam strength or wear strength whichever is weaker

Step 12: Calculate dimensions of worm gear pair

$m , t_g , t_w , d_w , d_g ,$

$$p . a = \pi m \qquad (15-36)$$

$$L = (p.a) \times t_w) \qquad (15-37)$$

15-16. Solve examples

Example 1:

$$\eta = \frac{(0.4) \times (0.968 - 0.03 \times 0.4)}{(0.968 \times 0.4) + 0.03} \times 100\% = \frac{0.3824}{0.4172} \times 100\% = 91.66\,\%$$

A worm gear has the following parameters: The module ($m = 5$), the diameter of the pitch circle of the worm ($d_p = 30$), the number of worm teeth ($t_w = 3$), and the number of teeth of the worm gear ($t_g = 50$). Calculate the following:

1. Dimensions of the worm,
2. Worm gear dimensions.

<u>Solution</u>

Given: $m = 5$; $d_P = 30\,mm$; $t_w = 3$; $t_g = 50$.

1. Dimensions of the worm,

$$p_x = \pi\,.m = 3.1428 \times 5 = 15.71\,mm$$

$$p_z = p_x\,.t_w = 15.71 \times 3 = 47.14\;mm$$

$$\beta = \tan^{-1}\frac{m\,.t_w}{d_p} = \tan^{-1}\frac{5 \times 3}{30} = 26.57^{\circ}$$

$$tan\,\lambda = \frac{p_z}{\pi d_p} = \frac{47.14}{3.1428 \times 30} = 0.5 \quad \Rightarrow \quad \lambda = 26.57^{\circ}$$

$$\beta = 90 - \lambda = 90 - 26.57 = 63.43^{\circ}$$

$$d_a = d_p + m = 30 + 5 = 35\,mm$$

$$d_d = d_p - 2.334\,m = 30 - 2.334 \times 5 = 18.33\,mm$$

$$h_w = 2.2\,m = 2.2 \times 5 = 11\,mm$$

2. Worm gear dimensions.

$$D_p = m + t_g = 5 + 50 = 55\, mm$$

$$D_d = D_p - 2.334\, m = 55 - 2.334 \times 5 = 43.33\, mm$$

$$D_a = D_p + 2m = 55 + 2 \times 5 = 60\, mm$$

$$D_L = D_a + m = 60 + 5 = 65\, mm$$

$$R_a = \frac{d_p}{2} = \frac{30}{2} = 15\, mm$$

$$R_d = \frac{d_p}{2} + 1.167m = \frac{30}{2} + 1.167 \times 5 = 20.835\, mm$$

$$b_2 \leq 0.75\ d_a = 0.75 \times 35 = 26.25\, mm$$

$$C = \frac{d_p + D_p}{2} = \frac{30 + 55}{2} 42.5\, mm$$

$$h_a = m = 5\, mm$$

$$h_d = 1.25\, m = 1.25 \times 6.25$$

$$h_g = h_a + h_d = 5 + 6.25 = 11.25\, mm\, .$$

Example 2:

a triple-threaded worm with teeth measuring ($m = 8\, mm$) in modules and a pitch circle diameter of ($d_P = 60\, mm$). Determine the lead angle of the worm, the velocity ratio, the center distance, and the efficiency of the worm gearing if the worm gear has ($t_g = 24$) teeth with ($\emptyset = 14.5°$) angles and ($\mu = 0.03$). coefficients of friction.

__**Solution**__

Given : $n = 3\, ;\ m = 8\, ;\ d_P = 60\, mm\, ;\ t_g = 24\, ;\ \emptyset = 14.5°\, ;\ \mu = 0.03.$

1. Lead angle of the worm (λ)

$$tan\,\lambda \;=\; \frac{m\,.\,n}{d_W} = \frac{8 \times 3}{60} = 0.4 \;\Rightarrow\; \lambda = 21.8^{\circ}$$

2. Velocity ratio ($V.R$)

$$V.R \;= \frac{t_g}{n} = \frac{24}{3} = 8$$

3. Center distance (C)

$$D_p = m\,.\,t_g = 8 \times 24 = 192\,mm$$

$$C \;= \frac{d_{P+}D_p}{2} = \frac{60 + 192}{2} = 76\,mm$$

4. Efficiency of the worm gearing (η)

$$\eta = \frac{tan\,\lambda\,(\cos\phi - \mu\,.\,tan\,\lambda)}{(\cos\phi\;.\,tan\,\lambda) + \mu} \times 100\%$$

$$= \frac{(tan\,21.8)\;\times(\cos\phi 14.5^{\circ} - 0.03\,.\,tan\,21.8)}{(\cos 14.5^{\circ}\;.\,tan\,21.8) + 0.03} \times 100\%$$

$$\eta = \frac{(0.4)\;\times(0.968 - 0.03 \times 0.4)}{(0.968\;\times\;0.4) + 0.03} \times 100\% = \frac{0.3824}{0.4172} \times 100\% = 91.66\,\%.$$

Example 3:

A pair of worm gears is designed to as (1/45/15/12). Calculate the following:

1. Center distance,

2. Reduction of the Speed,

3. Worm dimension,

4. Worm wheel dimension.

Solution

$$Given\!:\;\; t_1 = 1\;teeth\,;t_2 = 45\;teeth\,;q = 15\;\;;m = 12\;mm$$

1. Center distance

$$C = \frac{1}{2}\, m\,(q + t_2) = \frac{1}{2}\,(12)(15 + 45) = 360\, mm$$

2. Reduction of the Speed (G)

$$G = \frac{t_2}{t_1} = \frac{45}{1} = 45$$

3. Worm dimension

$$d_1 = \mathrm{q\,.m} = 15 \times 12 = 180 \text{ mm}$$

$$d_{a_1} = \mathrm{m\,(q + 2)} = 12(15 + 2) = 204 \text{ mm}$$

$$\tan\gamma = \frac{t_1}{q} = \frac{1}{15} = 0.0667$$

$$\gamma = \tan^{-1} 0.0667 = 3.814°$$

$$d_{f_1} = \mathrm{m\,(q} + 2 - 4.4\ \cos\gamma)$$

$$d_{f_1} = 12(15 + 2 - 4.4 \times \cos 3.814°)$$

$$= 151.32 \text{ mm}$$

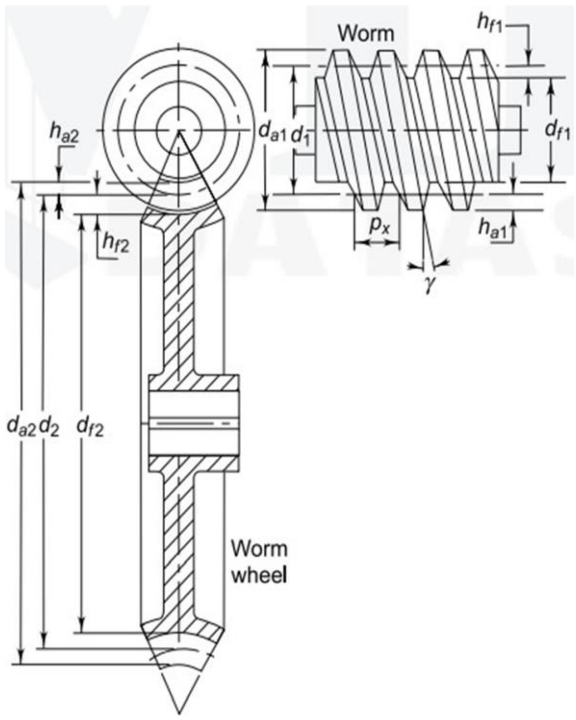


Figure 15-3: Dimension of Worm Gears

$$P_x = \pi\,.\,\mathrm{m} = 3.143 \times 12 = 37.71 \text{ mm}$$

4. Worm wheel dimension

$$d_2 = \mathrm{m}\,.\,t_2 = 12 \times 45 = 540 \text{ mm}$$

$$d_{a_2} = \mathrm{m}\,(t_2 + 4\ \cos\gamma - 2) = 12(45 + \cos 3.814° - 2) = 527.97 \text{ mm}$$

$$d_{f_1} = \mathrm{m}\,(t_2 - 0.4\ \cos\gamma - 2) = 12(45 - 0.4\, cos\, 3.814° - 2) = 559.21\, mm$$

15-18. Chapter Questions

1. In worm gear a large speed ratio is obtained.

a. $G = 5 - 50$
b. $\boldsymbol{G = 5 - 80}$
c. $G = 5 - 60$
d. $G = 5 - 70$

2. The designation (1/30/10/8) refers to a pair of worm gears. The worm and the worm gear center distance are:

a. 160 mm
b. 170 mm
c. 150 mm
d. 140 mm

3. Worm gears are frequently employed when.

a. The amount of space is limited، the velocity ratio is high، and the axes of the shafts do not intersect.
b. The amount of space is limited.
c. The axes of the shafts do not intersect.
d. The velocity ratio is high.

4. The worm gear drive must be reversible, when increase

a. Diametral quotient
b. Center distance
c. Velocity ratio to make the worm gear drive reversible
d. Number of starts

5. The primary advantage of the worm gear drive is:

a. Low power loss
b. High speed ratio
c. Simple manufacturing
d. Low price

6. Load stress factor (K) for worm, hardened steel and worm gear, cast iron is:

a. $\boldsymbol{0.345\ N\ /\ mm^2}$
b. $0.550\ N\ /\ mm^2$
c. $0.415\ N\ /\ mm^2$
d. $0.830 N\ /\ mm^2$

7. Load stress factor for worm hardened steel and worm gear, antimony bronze is:

a. $0.345\ N\ /\ mm^2$
b. $0.550\ N\ /\ mm^2$
c. $0.415\ N\ /\ mm^2$
d. $\boldsymbol{0.830 N\ /\ mm^2}$

8. Addendum circle diameter (d_a): It can be determined by using the relationship shown below:

a. $\boldsymbol{d_a = d_p + m \quad (mm)}$

b. $d_a = d_p + 2\,m \quad (mm)$

c. $d_a = d_p + 3\,m \quad (mm)$

d. $d_a = d_p + 4\,m \quad (mm)$

9. Lead (L): It is a distance by which a Helix advances along the axis for the one turn around, and it can be estimated using the relationship shown below:

a. $L = 4\,n\,.p \quad (mm)$

b. $L = 3\,n\,.p \quad (mm)$

c. $L = 2\,n\,.p \quad (mm)$

d. $\boldsymbol{L = n\,.p \quad (mm)}$

10. The following formula can be used to mathematically determine the worm gearing's efficiency:

a. $\eta = \frac{tan\,\lambda\,(\sin\phi - \mu\,.tan\,\lambda)}{si\,n\,\phi.tan\,\lambda) + \mu} \times 100\%$

b. $\boldsymbol{\eta = \frac{tan\,\lambda\,(\cos\phi - \mu\,.tan\,\lambda)}{(\cos\phi\,.tan\,\lambda) + \mu} \times 100\%}$

c. $\eta = \frac{sin\,\phi\,\lambda\,(\cos\phi - \mu\,.\,sin\,\phi\,\lambda)}{(\cos\phi\,.\,\,sin\,\phi\,\lambda) + \mu} \times 100\%$

d. $\eta = \frac{cos\,\lambda\,(\cos\phi - \mu.\,\,cos\,\lambda)}{(\cos\phi\,.\,\,cos\,\lambda) + \mu} \times 100\%$

11. The following formula can be used to mathematically determine the worm gearing's efficiency:

a. $\eta = \frac{tan\,\lambda\,(\sin\phi - \mu\,.tan\,\lambda)}{si\,n\,\phi.tan\,\lambda) + \mu} \times 100\%$

b. $\boldsymbol{\eta = \frac{tan\,\lambda\,(\cos\phi - \mu\,.tan\,\lambda)}{(\cos\phi\,.tan\,\lambda) + \mu} \times 100\%}$

c. $\eta = \frac{sin\,\phi\,\lambda\,(\cos\phi - \mu\,.\,sin\,\phi\,\lambda)}{(\cos\phi\,.\,\,sin\,\phi\,\lambda) + \mu} \times 100\%$

d. $\eta = \frac{cos\,\lambda\,(\cos\phi - \mu.\,\,cos\,\lambda)}{(\cos\phi\,.\,\,cos\,\lambda) + \mu} \times 100\%$

12. The following formula can be used to mathematically determine the worm gearing's efficiency:

a. $\boldsymbol{\eta = \frac{tan\,\lambda\,(\sin\phi - \mu\,.tan\,\lambda)}{si\,n\,\phi.tan\,\lambda) + \mu} \times 100\%}$

b. $\boldsymbol{\eta = \frac{tan\,\lambda\,(\cos\phi - \mu\,.tan\,\lambda)}{(\cos\phi\,.tan\,\lambda) + \mu} \times 100\%}$

c. $\boldsymbol{\eta = \frac{sin\,\phi\,\lambda\,(\cos\phi - \mu\,.\,sin\,\phi\,\lambda)}{(\cos\phi\,.\,\,sin\,\phi\,\lambda) + \mu} \times 100\%}$

d. $\boldsymbol{\eta = \frac{cos\,\lambda\,(\cos\phi - \mu.\,\,cos\,\lambda)}{(\cos\phi\,.\,\,cos\,\lambda) + \mu} \times 100\%}$

Chapter 16

Cams

16. Cams

16-1. Introduction

A cam is a mechanical member used to impart desired motion to follower by direct contact or the cam is a rotating part that provides reciprocating or oscillating motion to the follower by direct contact. This part is essentially used to change the movement from the rotary to linear.

It is a part of a machine, that is a revolving electric wheel or a shaft that hits a lever at several points on its circular path. It is widely used in a steam hammer as a simple tooth to transform pulses of the power to steam hammer.

The three essential components of a cam mechanism are as follows:

1. The Cam,
2. The Follower,
3. 3. The Frame.

16-2. Cam and follower mechanism

The main members of cam and follower mechanism are:

1. A driver member known as cam
2. A driven member known as follower
3. A frame which supports the cam and guides the follower

16-3. Classification of cams

Cams are classified on three main bases according to:

16-3-1. Classification of cams according to the shape

There are many types of cams according to shape, figure (16-1).

a. Wedge and flat cams,
b. Radial or disc cams,

c. Spiral cams,
d. Cylindrical cams,
e. Conjugate cams,
f. Globoidal cams,
g. Spherical cams.

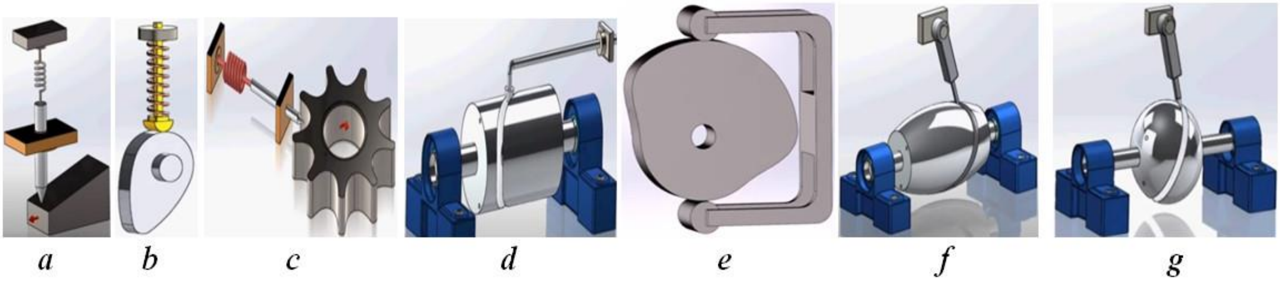


Figure 16-1: Classification of cams according to shape

16-3-2. Classification of cams according to the follower movement

There are many types of cams according to follower movement, figure (16-2).

a. Rise return rise (R – R – R),
b. Dwell rise return dwell (D – R – D),
c. Dwell rise dwell return dwell (D – R – R – R – D).

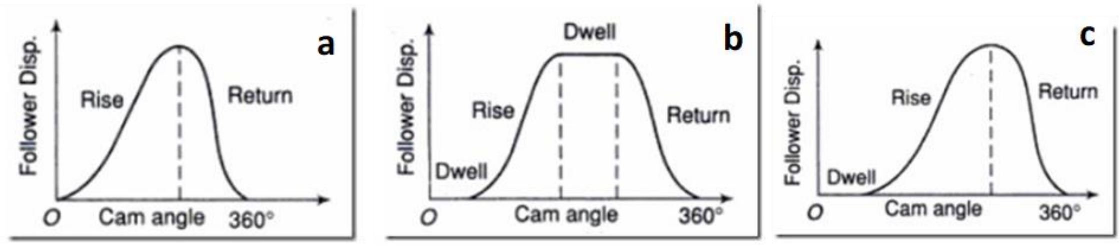


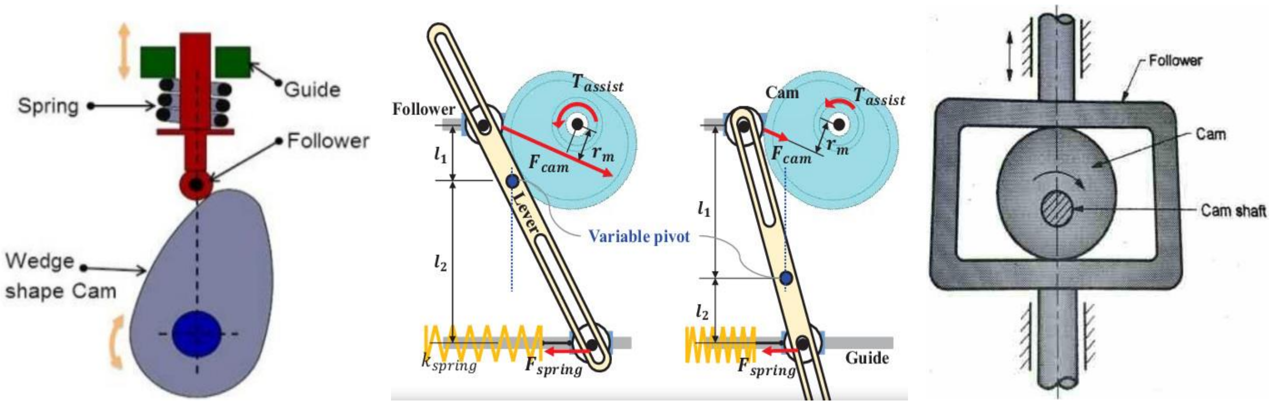
Figure 16-2: Classification of cams according to a follower movement

16-3-3. Classification of cams according to the manner of constraint of the follower

There are three types of cams according to manner of constraint of the follower, figure (16-3).

a. Preloaded spring cam,
b. Gravity cam,

c. Positive drive cam.



a *b* *c*

Figure 16-3: Classification of cams according to a manner of constraint of the follower

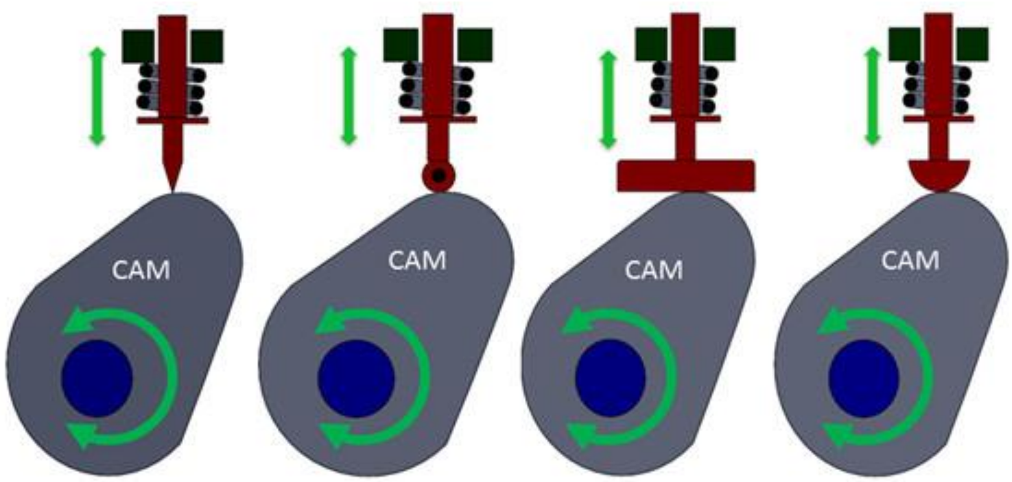
16 – 4. Classification of followers

The followers may be classified based on the following:

16-4-1. Classifications of followers according to the shape

There are three types of followers according to shape, figure (16-4).

a. Knife edge follower,
b. Roller follower,
c. Mushroom (or Spherical) follower,
d. Plate type follower.



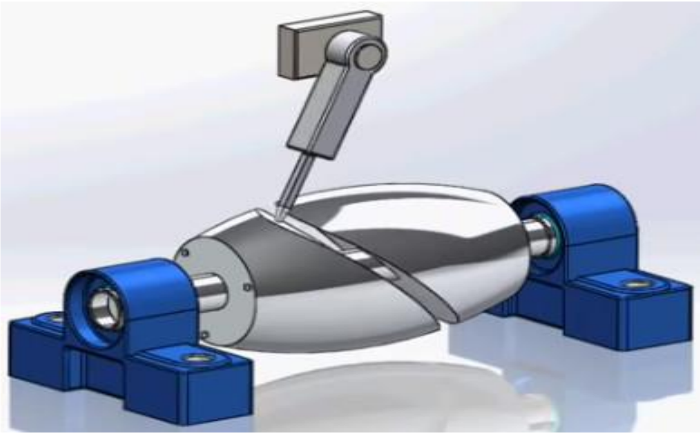
a *b* *c* *d*

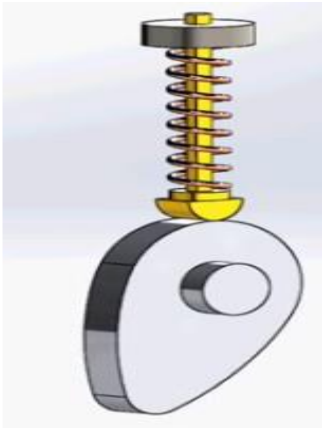
Figure 16-4: Classification of follower according to a shape

16- 4-2. Classifications of followers according to the motion

There are two types of followers according to motion of the follower, figure (16-5).

a. Translator motion follower,
b. Oscillating follower.





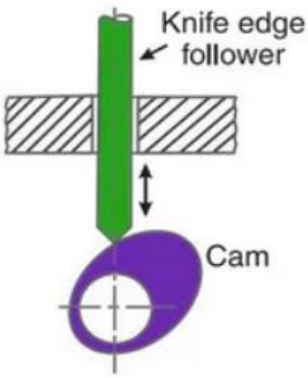
a *b*

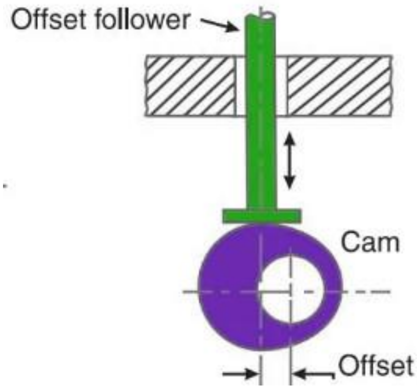
Figure 16-5: Classification of follower according to a motion of the follower

16-4–3. Classifications of followers according to line of the movement

There are two types of followers according to a line movement, figure (16-6).

a. Offset follower,
b. Radial follower.





a. Radial follower ***b. Offset follower***

Figure 16-6: Classification of follower according to a motion of the follower

16 – 5. Cam profile terms

The following points and figure (16-7) show a cam profile terms:

1. **Base Circle:** The back side of the cam lobe.
   a. This is the area of the lobe where the valve is closed,
   b. It does not provide lift,
   c. Also known as the "heel."
2. **Clearance Ramp:** The transition area between the ramp and the base circle.
   a. This part of the lobe takes up the lash in a solid valve train,
   b. Also called the "lash ramp."
3. **Opening Ramp:** The side of the lobe that raises the lifter.
4. **Lobe Centerline:** The highest point of lift on the lobe.
   a. Centerlines are listed as degrees of crankshaft rotation,
   b. The intake centerline is After Top Dead Center (ATDC),
   c. The exhaust centerline is Before Top Dead Center (BTDC).
5. **Nose:** The top part of the lobe.
   a. This is where the valve is held open as it transitions from opening to closing.
6. **Closing Ramp:** The side of the lobe that lowers the lifter.
7. **Ramps:** These control the raising and lowering of the lifter (clearance, closing, and opening).

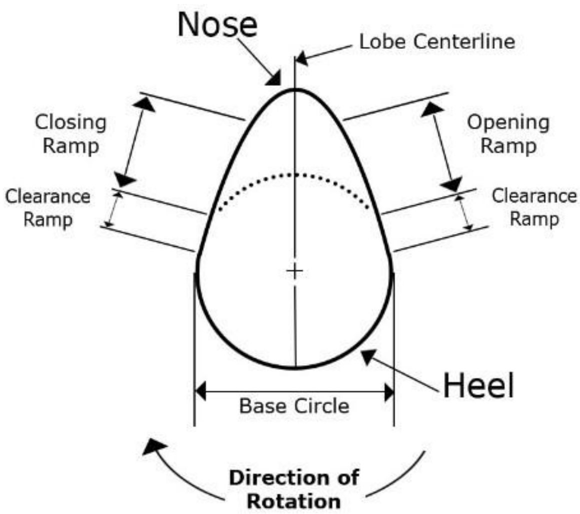


Figure 16-7: Cam profile terms

16 – 6. Materials used in cams

There are different materials that cams are manufactured from:

1. Cast irons
   a. Harden able iron,
   b. Spheroid graphite cast iron,
   c. Chilled chrome cast iron.
2. Steel camshafts
   a. Carbon steel – EN8 (BS970 080M40) /EN99(BS970 070M55),
   b. Alloyed steels – EN351 AISI 8620 and EN34,
   c. Nit riding steel – EN40B.

16-7. Methods of manufacturing cams

There are many different method of manufacturing cams:

1. Casting,
2. Forging,
3. Machining.

16-8. Applications of cams

Cams are widely used in the following applications:

1. Internal combustion engine,
2. Machine tools,
3. Printing control mechanisms,
4. Textile weaving industries,
5. Automated machines.

16-9. Cam design

Example 1:

Design profile of a Cam from the following data:

$$Base\ circle\ (\emptyset) angle = 60^o\ ,$$

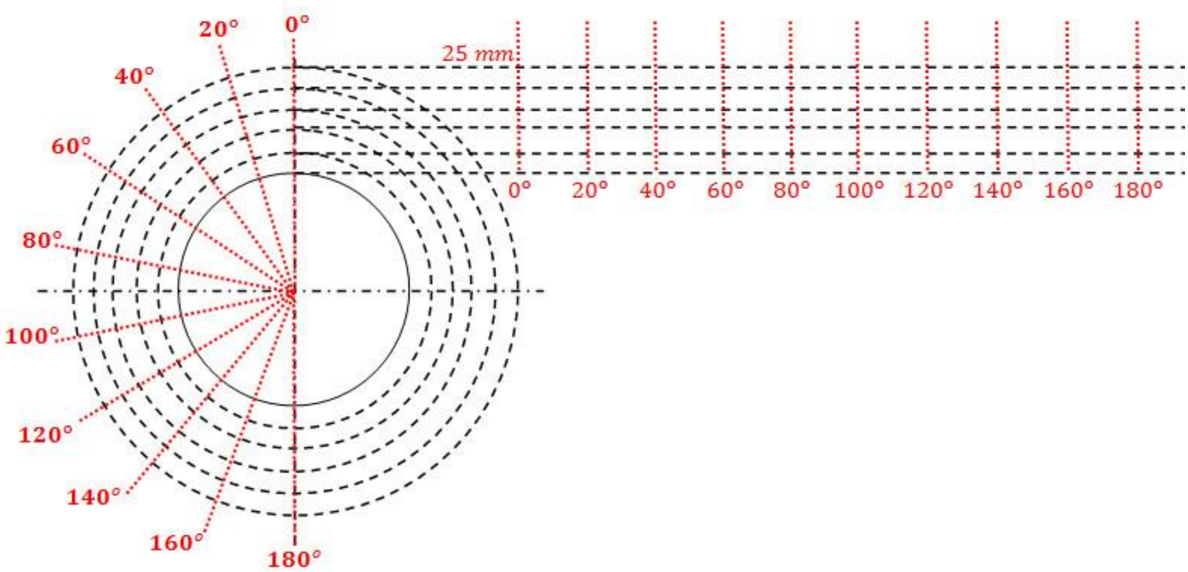
$$Rise\ \&\ \ Fall = 25\ mm\ in\ angle\ (40^o), conatant\ velocity,$$

$$Dwell\ top\ angle\ (60^o)\ and\ Dwell\ bottom\ remaindge\ (220^o),$$

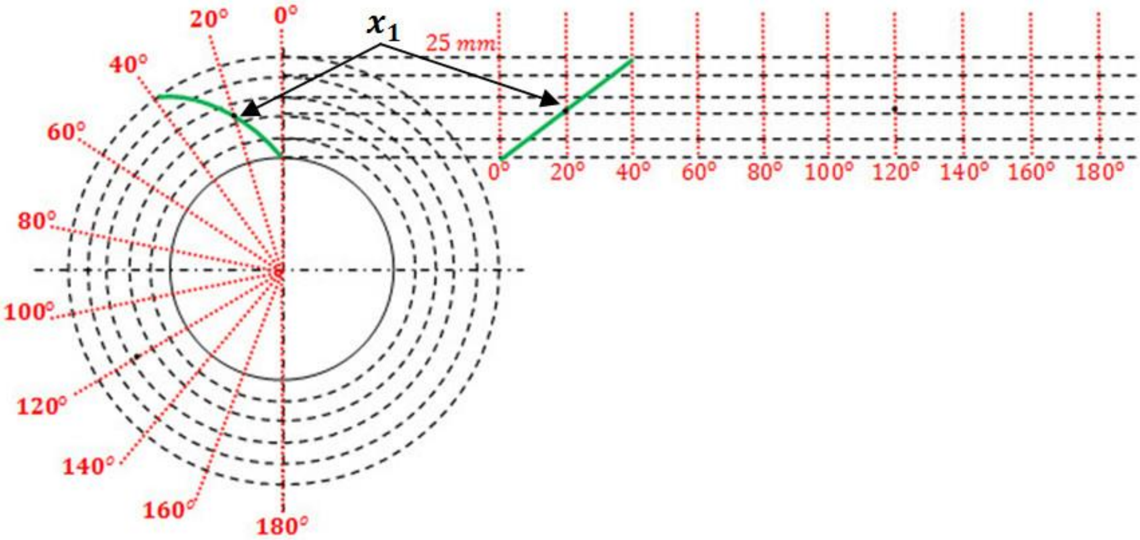
$$\omega = 80\ rpm, Calculate\ Dwell\ top\ time$$

<u>**Solution**</u>

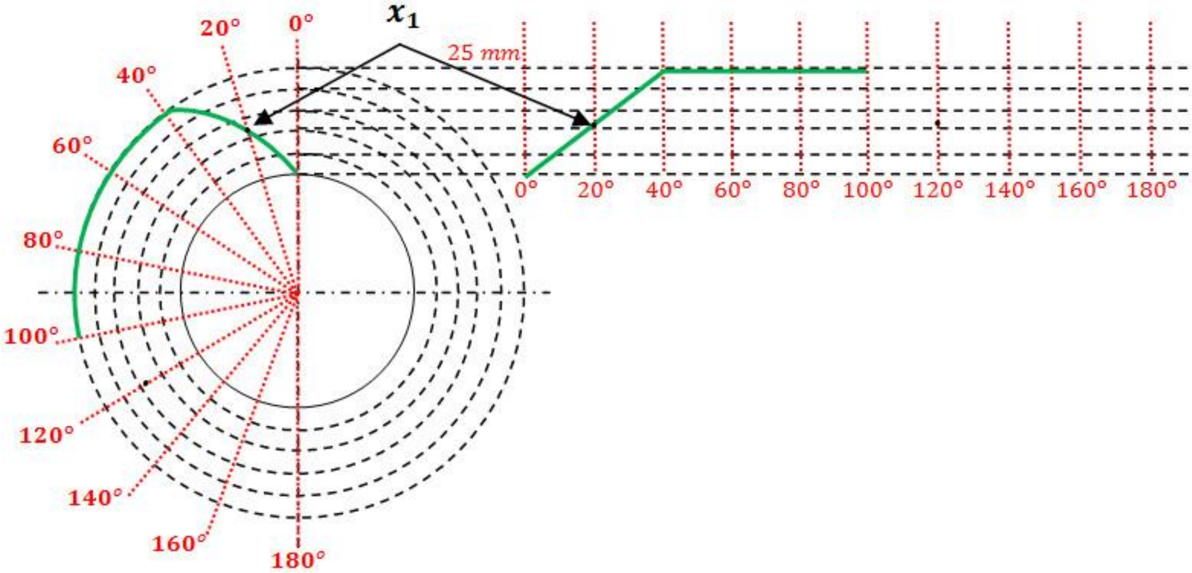
1. We draw the following figure and divide the angles and determine the heights according to the values given in the question. In the example, we have the distance between one circle and another (5 mm), and the distance between one corner and another is (20º).



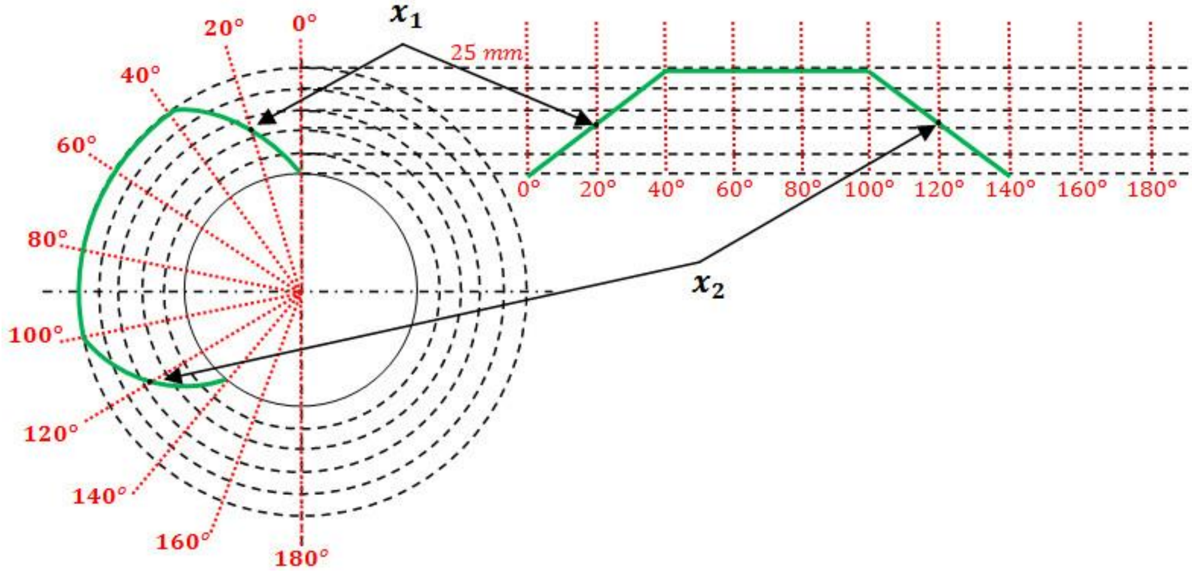
2. We draw a line (Rise) between the two points (0º, 40º), and determine the point (x_1) located on the column on the angle (20º), and with the same magnitude we determine the point (x_1) from point on line (20º) on the circles, and then we draw an arc on the circles between the two points (0º, 40º) and it passes through the point (x_1), as in figure.



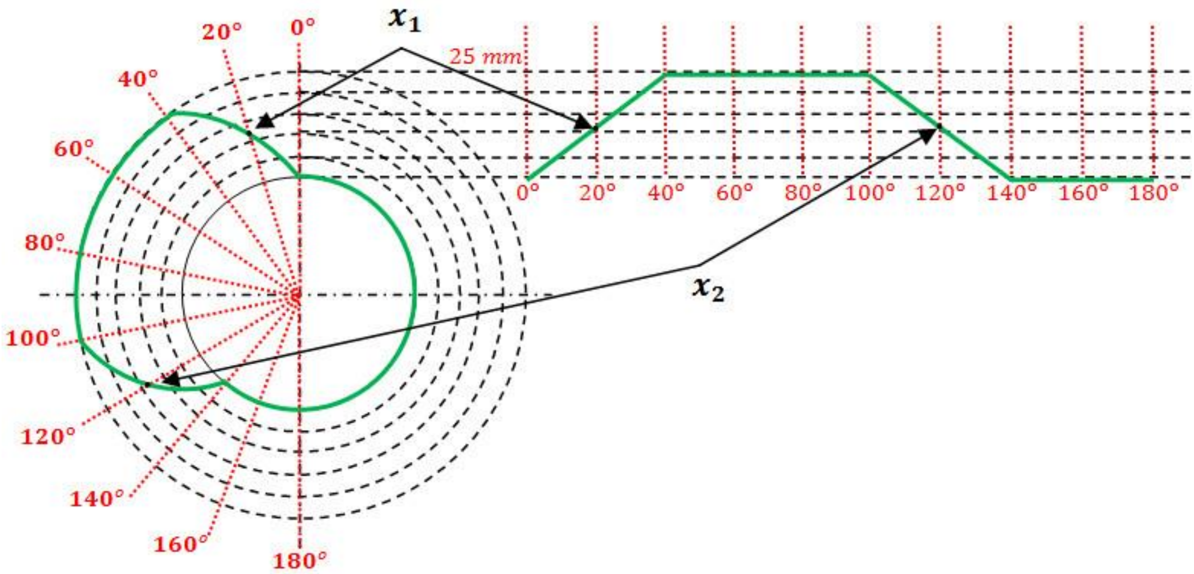
3. We draw a line and curve (Dwell top) and its value (60º) located between the two points (40º, 100º), as in figure.



4. We draw a line (Fall) between the two points (100º, 140º), and determine the point (x_2) located on the column on the angle (120º), and with the same magnitude we determine the point (x_2) from point on line (120º) on the circles, and then we draw an arc on the circles between the two points (100º, 140º) and it passes through the point (x_2), as in figure.



5. We draw a line and a curve (Dwell bottom) between the two points on (140º, 180º), and then we complete the cam circle between the two points (180º, 360º) and get the profile of the cam and the displacement magnitude, as in figure.



$$N = 800\,rpm = \frac{800}{60} = 13.33\,rps$$

$$t = \frac{1}{N} = \frac{1}{13.33} = 0.075\,Sec.$$

$$Rise\ time = \frac{40}{360}.t = \frac{40}{360} \times 0.075 = 0.0083\,Sec.$$

$$Dwell\ top\ time = \frac{60}{360}.t = \frac{60}{360} \times 0.075 = 0.0125\ Sec.$$

$$Fall\ time = \frac{40}{360}.t = \frac{40}{360} \times 0.075 = 0.0083\ Sec.$$

$$Dwell\ bottom\ time = \frac{220}{360}.t = \frac{220}{360} \times 0.075 = 0.045\ Sec.$$

$$Total\ time\ for\ one\ cycle\ of\ the\ cam$$
$$= Rise\ time + Dwell\ top\ time + Fall\ time + Dwell\ bottom\ time$$

$$Total\ time\ for\ one\ cycle\ of\ the\ cam = 0.0083 + 0.0125 + 0.0083 + 0.046$$
$$= 0.0.75\ Sec.$$

Example 2:

Design profile of a Cam in case of a knife edge follower from the following data:

$$Minimum\ radius\ of\ the\ cam = 5\ cm\ ,$$

$$Outstroke = 120°\ with\ S.H.M, \qquad Outward\ dwell = 60°,$$

$$Return\ stroke = 120°\ with\ S.H.M,$$

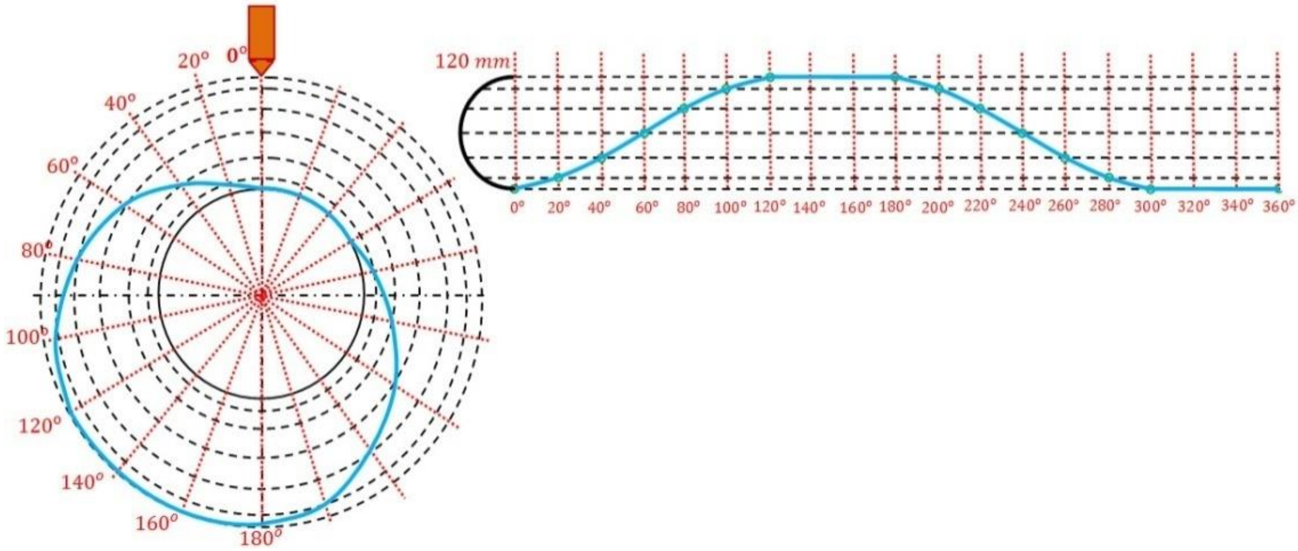
$$Dwell\ for\ the\ remaining\ period\ of\ cam\ rotation.$$

$$Stroke\ or\ life\ of\ the\ follower = 6\ cm$$

Solution

Data:

$$\theta_0 = 120°\ , \theta_{0D} = 60°, \theta_R = 120°\ , \theta_{RD} = 320° - (120° + 60° + 120°) = 60°$$

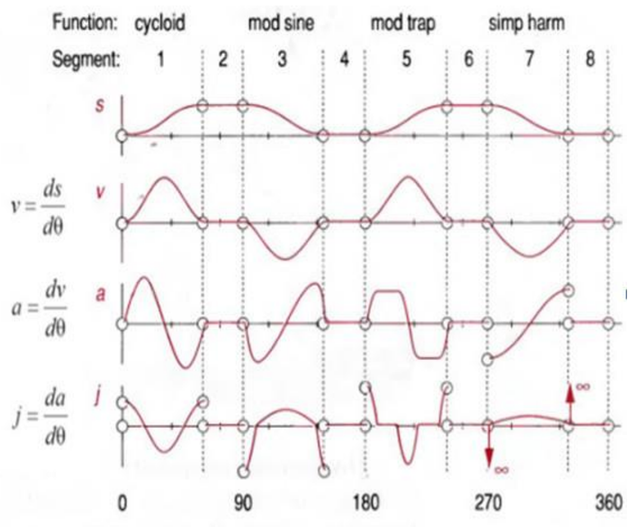


16-10. S V A J Diagrams

The first task faced by the cam designer is to select the mathematical functions to be used to define the motion of the follower. The easiest approach to this process is to "linearism" the cam, i.e., "unwraps it" from its circular shape and consider it as a function plotted on Cartesian axes. We plot the displacement function (***s***), its first derivative velocity (***v***), its second derivative acceleration (***a***), and its third derivative jerk, all on aligned axes as a function of camshaft angle e as shown in Figure (16-8). Note that we can consider the independent variable in these plots to be either time (***t***) or shaft angle (***e***), as we know the constant angular velocity (***J***) of the camshaft and can easily convert from angle to time and vice versa. Figure (16-8a) shows the specifications for a four-dwell cam that has eight segments, (*RDFDRDFD*). Figure (16-8 b) shows the (*S V A J*) curves for the whole cam over 360 degrees of camshaft rotation. A cam design begins with a definition of the required cam functions and their (*S VA J*) diagrams. Functions for the no dwell cam segments should be chosen based on their velocity, acceleration, and jerk characteristics and the relationships at the interfaces between adjacent segments including the dwells. These function characteristics can be conveniently and quickly investigated with program (*DYNACAM*) which generated the data and plots shown in figure (16-8).

Segment Number	Function Used	Start Angle	End Angle	Delta Angle
1	Cycloid rise	0	60	60
2	Dwell	60	90	30
3	ModSine fall	90	150	60
4	Dwell	150	180	30
5	ModTrap rise	180	240	60
6	Dwell	240	270	30
7	SimpHarm fall	270	330	60
8	Dwell	330	360	30

(a) Cam program specifications



(b) Plots of cam-follower's *s v a j* diagrams

Figure 16-8: S V A J Diagrams

Example 3:

Draw (SVAJ) diagram of a Cam in case of a knife edge follower from the following data:

$$Dwell\,;\ \ at\ zero\ displacement\ for\ (90°) low\ dwell$$

$$Rise\,;\ \ at\ (25\ mm) displacement\ for\ (90°)$$

$$Dwell\,;\ \ at\ (25\ mm)\ displacement\ for\ (90°) high\ dwell$$

$$Fall\,;\ \ at\ (25\ mm) displacement\ for\ (90°)$$

$$Cam\ (\omega): 2\pi \frac{rad}{sec} = 1\ \frac{rev.}{sec}$$

<u>Solution</u>

Solve the example figure (16-9).

$$\boldsymbol{y = mx + b}$$

$$\boldsymbol{s = K_v\,.\theta}$$

$$\boldsymbol{v = K_v = constant}$$

$$\boldsymbol{a = 0}$$

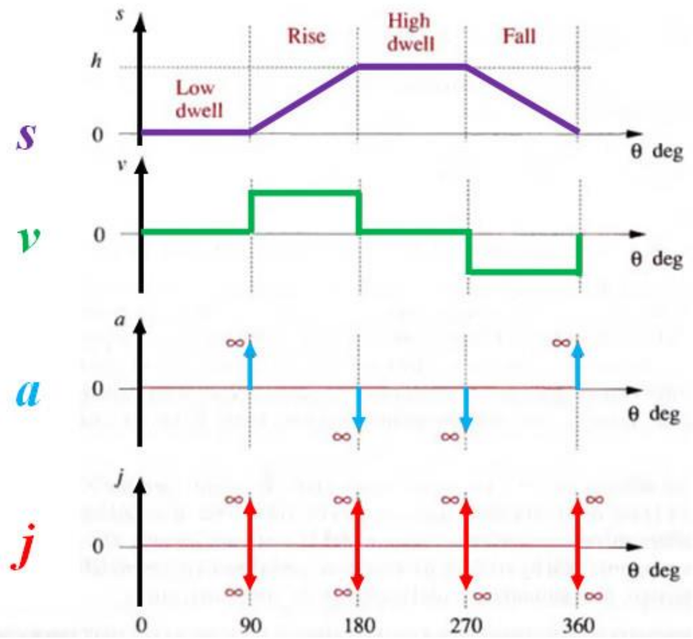


Figure 16-9: S V A J Diagrams

16-11. Chapter questions

1. **Cam follower extensively used in air - Craft engine is**
   e. **Roller follower**
   f. Flat faced follower
   g. Spherical faced follower
   h. Knife edge follower
2. **Cylindrical cam can be classified as ....**
   e. **Reciprocating**
   f. Circular
   g. Tangent
   h. None of these
3. **In Which type of cam below have not required any external force for a contact between the cam & follower**
   a. **Conjugate cam**
   b. Preloaded spring cam
   c. Both A. and B.
   d. None of these
4. **Size of cam is depending upon...........**
   a. Pitch curve
   b. Prime circle
   c. **Base circle**
   d. Pitch circle
5. **Which types of cam follower extensively used in air craft engine ....**
   a. Flat faced follower
   b. Knife edge follower
   c. **Roller follower**
   d. Spherical faced follower
6. **Which types of cam follower extensively used in automobile engine ....**
   a. Flat faced follower
   b. Knife edge follower
   c. Roller follower
   d. **Spherical faced follower**
7. **During the dwell period of the cam the follower is ....**
   a. Move straight
   b. Move uniform speed
   c. **Remain at rest**
   d. None of these
8. **Offset is provided to cam follower mechanism to ....**
   a. Accelerate
   b. Avoid jerk
   c. **Minimize the side thrust**
   d. None of these

9. **Rolling contact bearings' balls are made of**:

   a. High carbon chromium steel
   b. Case hardened steel
   c. Free cutting steel
   d. Plain carbon steel

10. **The angle between the direction of follower motion and normal to the pitch curve is known as ....**

   a. Base angle
   b. Prime angle
   c. Pitch angle
   d. Pressure angle

11. **In radial cam, follower is moves in a direction ....**

   a. parallel to the cam axis
   b. perpendicular to the cam axis.
   c. axial to the cam axis.
   d. None of these

12. **Cam and follower mechanism contribute a ....**.
   a. Screw pair
   b. Open pair
   c. Closed pair
   d. Rolling pair

13. **A cam is a ………... member used to impart desired motion to follower by direct contact**

   a. electrical
   b. **mechanical**
   c. hydraulic
   d. pneumatic

14. **Which item best describes a CAM technology?**
   a. Numerical control
   b. Documentation
   c. Drafting
   d. Geometric modeling

15. **For high-speed engines, the cam follower should move with**
   a. uniform velocity
   b. simple harmonic motion
   c. uniform acceleration and retardation
   d. cycloid motion

16. **A radial follower is one**
   a. that oscillates
   b. in which the follower translates along an axis passing through the cam centre of rotation.
   c. none of the mentioned
   d. **that reciprocates in the guides**

17. A circle drawn with center as the cam center and radius equal to the distance between the cam center and the point on the pitch curve at which the pressure angle is maximum, is called

a. base circle
b. pitch circle
c. prime circle
d. base and prime circles

19. A single-row deep groove ball bearing is subjected to a pure radial force of ($F_r = 4000\ N$) from a shaft that rotates at ($N = 000\ rpm$). The expected life of the bearing ($L_{10h} = 25000\ hour$). The minimum acceptable diameter of the shaft is ($d = 20\ mm$. Select a suitable ball bearing for this application.

20. A single-row deep groove ball bearing is subjected to a pure radial force of ($F_r = 2000\ N$) and a thrust force of ($F_a = 1000\ N$). The shaft that rotates at ($N = 700\ rpm$). The expected life of the bearing ($L_{10h} = 15000\ hour$). The minimum acceptable diameter of the shaft is $(d = 20\ mm)$. Select a suitable ball bearing for this application. Assume inner race rotating.

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