Design the simply supported beam that carries two concentrated factored loads of 214 kips each on a clear span of 12 ft as shown in Figure 1. The beam has a width of 14 in. and a 48 in. overall depth. The width of the bearing plate at each concentrated load location is 16 in. Neglect the self-weight.

Use $f_c' = 4\text{ ksi}$ and $f_y = 60\text{ ksi}$.

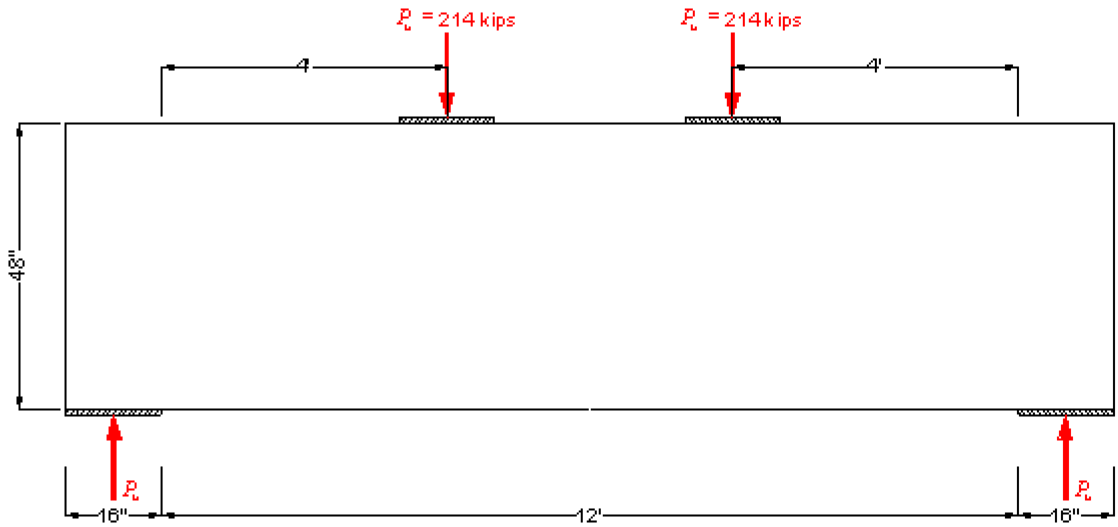

Figure 1
(Click here to view a larger image)

Check Bearing Stress at Points of Loading and Supports:

The area of bearing plate is $A_c = 16(14) = 224\text{ in}^2$. The bearing stresses at points of loading and at supports are $\dfrac{P_u}{A_c} = \dfrac{214(1000)}{224} = 955\text{ psi}$.

Since these bearing stresses are less than their corresponding limits, i.e. $0.85\phi f_c' = 0.85(0.70)(4000) = 2380\text{ psi}$ at points of loading and $0.75\phi f_c' = 0.75(0.70)(4000) = 2100\text{ psi}$ at supports, the area of bearing plates provided is adequate.

Select and Establish the Strut-and-Tie Model:

Assume that the loads are carried by a strut-and-tie model consisting of two trusses.

One truss uses a direct strut running from the load to the support. The other truss uses stirrups as vertical ties. The geometry of the assumed truss is shown in Figure 2.

The center of the horizontal ties is assumed to be located 4 in. from the bottom of the beam. Thus, $d = 48 - 4 = 44$ in. The centroid of the vertical tie (stirrups) *BC* is assumed to be at the middle of the shear span, i.e. 56/2 = 28 in. from point *A*. The location of strut *DD'* centerline can be found by determining the strut stress limit, calculating the required compressive force in. strut *DD'*, $N_{DD'}$, and imposing moment equilibrium about point *A* to obtain the strut width a as follows:

$$\phi f_{cu} = \phi\left(0.85 f'_c\right) = 0.70\left[0.85(4000)\right] = 2380 \text{ psi}$$

$$N_{DD'} = \phi f_{cu} b a = 2380(14)a/1000 = 33.32a$$

$$jd = d - \frac{a}{2} = 44 - \frac{a}{2}$$

$$33.32a\left(44 - \frac{a}{2}\right) = 214(56)$$

$$16.66a^2 - 1466.08a + 11984 = 0$$

$$a = 9.12 \text{ in.}$$

$$jd = 44 - \frac{9.12}{2} = 39.44 \text{ in.}$$

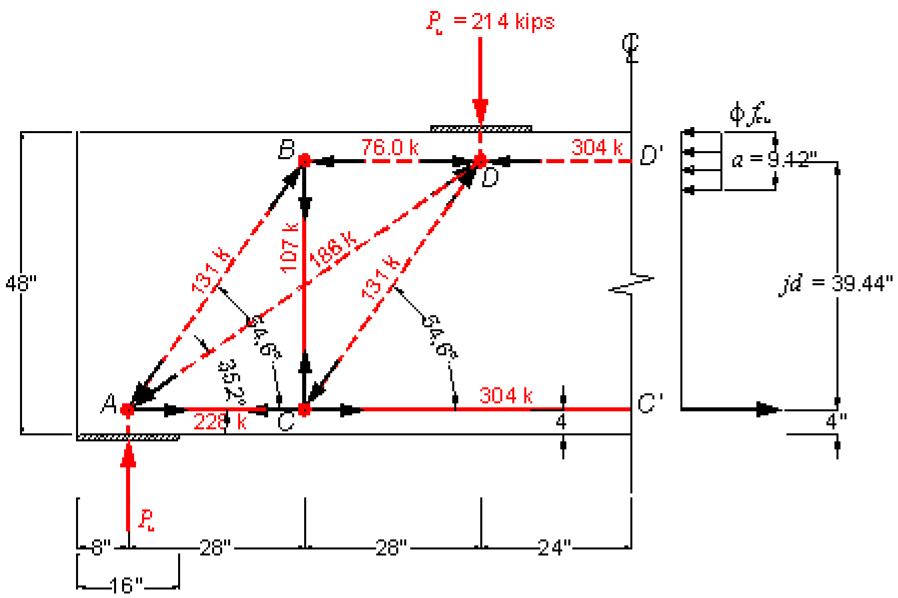

Figure 2

(Click here to view a larger image)

Thus, node strut *DD'* is located 9.12/2 = 4.56 in. from the top of the beam and $N_{DD'}$ = 33.32(9.12) = 304 kips. This fixes the geometry of the truss.

Determine the Required Truss Forces:

Since the truss shown in Figure 2 is statically indeterminate, it is necessary first to select the amount and position of the vertical tie *BC* (stirrups) and assume that the stirrups yield. The truss then becomes statically determinate and all the member forces can be found easily by statics.

Assume that 50 % of the loads, i.e. 214/2 = 107 kips, is transmitted by the stirrups at yield and the other 50 % of the loads is transmitted by the direct strut.

The required forces in all the members of the truss are given in the following table. Note that positive indicates tension, negative compression.

Member	*AB*	*AC*	*AD*	*CC'*	*CD*	*BC*	*BD*	*DD'*
Force (kips)	-131	+228	-186	+304	-131	+107	-76.0	-304
Slope (deg)	54.6	0	35.2	0	54.6	90	0	0

Select the Steel Reinforcement for the Ties:

Try to use 5 #4 two-legged stirrups at 6 in. o.c. for the vertical tie *BC*. This corresponds to a capacity of $\phi A_v f_y = 0.9(2)(5)(0.20)(60) = 108\text{ kips}$ and is very close to the assumed load. Hence provide 5 #4 two-legged stirrups at 6 in., $A_{vBC} = 2(5)(0.20) = 2\text{ in.}^2$

According the AASHTO LRFD, the minimum reinforcement for horizontal tie *CC'* and *AC* is

$$A_{s\min} = 0.03\frac{f_c'}{f_y}bh = 0.03\frac{4000}{60000}bh = 0.002bh = 0.002(14)(48) = 1.34\text{ in.}^2$$

The required area of steel reinforcement for tie *CC'* is $\frac{N_{CC'}}{\phi f_y} = \frac{304}{0.9(60)}\text{in}^2 = 5.63\text{in.}^2$ and the required area of reinforcement for tie *AC* is $A_{s\text{required}} = \frac{N_{AC}}{\phi f_y} = \frac{228}{0.9(60)}\text{in}^2 = 4.22\text{ in.}^2$ Thus, choose 2 layers of 4 #8 bars for tie *CC'*, and choose 2 layers of 3 #8 bars for tie *AC* $A_{sCC'} = 2(4)(0.79) = 6.32\text{in.}^2$

Check the Struts:

The struts will be checked by computing the strut widths and checked whether they will fit in the space available.

By neglecting the tensioning effects, the average tensile strain in tie *BC* can be estimated as $\varepsilon_s = \frac{N_{BC}}{A_{vBC}E_s} = \frac{107}{2(29000)} = 0.00184 < \frac{f_y}{E_s} = \frac{60}{29000} = 0.002.$ Similarly, the average tensile strain in tie AC can be taken as $\varepsilon_s = \dfrac{N_{AC}}{A_{AC}E_s} = \dfrac{228}{4.74(29000)} = 0.00166.$

The bottom part of strut AB is crossed by tie AC. The tensile strain perpendicular to strut AB due to tensile strain in this tie AC is $\varepsilon_1 = \varepsilon_s + (\varepsilon_s + 0.002)\cot^2\theta_s$ $= 0.00166 + (0.00166 + 0.002)\cot^2 54.6°$ = 0.00350. Thus, the stress limit at the bottom of
strut $AB$is

$$\phi f_{cu} = \phi\frac{f_c'}{0.8 + 170\varepsilon_1} = \phi\frac{f_c'}{0.8 + 170(0.00351)} = 0.72\phi f_c' = 0.72(0.70)(4000) = 2016 \text{ psi.}$$

The top part of strut AB is crossed by tie BC and the tensile strain perpendicular to strut AB due to tie BC is $\varepsilon_1 = 0.00184 + (0.00184 + 0.002)\cot^2(90 - 54.6)° = 0.00944.$ Thus, the stress limit at the top of strut AB becomes $\phi f_{cu} = \phi\dfrac{f_c'}{0.8 + 170(0.00944)} = 0.42\phi f_c' = 0.42(0.70)(4000) = 1176 \text{ psi.}$
By taking the smaller stress limit, the required width for strut AB is $\dfrac{N_{AB}}{\phi f_{cu} b} = \dfrac{131(1000)}{1176(14)} = 7.96 \text{ in.}$ Choose a width of 8 in. for strut AB width.

The bottom of strut AD is crossed by tie AC and the tensile strain perpendicular to strut AD due to tensile strain in tie AC is $\varepsilon_1 = 0.00166 + (0.00166 + 0.002)\cot^2 35.2° = 0.00901.$ Thus, the stress limit at the bottom of strut AD is $\phi f_{cu} = \phi\dfrac{f_c'}{0.8 + 170(0.00901)} = 0.43\phi f_c' = 0.43(0.70)(4000) = 1204 \text{ psi.}$ The middle part of strut $AD$is crossed by tie BC and the tensile strain perpendicular to strut AD due to tie BC is $\varepsilon_1 = 0.00184 + (0.00184 + 0.002)\cot^2(90 - 35.2)° = 0.00375.$ Thus, the stress limit at the middle of strut AD is $\phi f_{cu} = \phi\dfrac{f_c'}{0.8 + 170(0.00375)} = 0.70\phi f_c' = 0.70(0.70)(4000) = 1960 \text{ psi.}$ The required width for strut AD is $\dfrac{N_{AD}}{\phi f_{cu} b} = \dfrac{186(1000)}{1204(14)} = 11.03 \text{ in.}$ Choose a width of 11 in. for strut AD width.

The bottom part of strut CD is mostly influenced by tie BC and can be assumed to be the same as the top part of strut AB. Thus, the stress limit and the required width for strut CD are 1176 psi and 7.96 in. respectively. Choose also a width of 8 in. for strut CD.

Strut BD is mostly crossed by tie BC and the tensile strain perpendicular to strut BD due to tensile strain in tie BC is $\varepsilon_1 = \varepsilon_s = 0.00184.$ Thus, the stress limit for strut BD is $\phi f_{cu} = \phi \frac{f_c'}{0.8+170(0.00184)} = 0.90\phi f_c' > 0.85\phi f_c'$.

Take $\phi f_{cu} = 0.85\phi f_c' = 0.85(0.70)(4000) = 2380\text{ psi}$. The required width for strut BD is $\frac{N_{BD}}{\phi f_{cu} b} = \frac{76.0(1000)}{2380(14)} = 2.28\text{ in.}$ Choose a width of 3 in. for strut BD width.

The required width for short strut transmitting the applied load to node D is $\frac{214(1000)}{2380(14)} = 6.42\text{ in.}$ Choose a width equal to the bearing plate width for this strut, i.e. 16 in. The required width for short strut transmitting the force meeting at node A to the support is $\frac{214(1000)}{2100(14)} = 7.28\text{ in.}$ Choose a width of 13 in. for this short strut.

The stress demands, stress limits, and the widths of the struts are summarized in Figure 3. As shown in Figure 3, most of the strut widths fit into the outline of the beam region except that struts AB and AD near node A overlap and struts BD, AD, and CD near node D overlap. To ensure that the overlapping struts in those regions do not exceed the stress limit, the stresses due to the force resultants are checked against the corresponding stress limit. The force resultant of struts AB and AD is $\sqrt{214^2+228^2} = 313\text{ kips}$ with a slope of $\arctan\frac{214}{228} = 43.2°$ and the available width is $8\cos 43.2° + 13\sin 43.2° = 14.73\text{ in}$ (Figure 4(a)). The stress due to this force resultant is then $\frac{313(1000)}{14.73(14)} = 1518\text{ psi.}$ This force resultant zone crosses both ties AC and BC. The tensile strain perpendicular to this force resultant due to tensile strain in tie AC is $\varepsilon_1 = 0.00166 + (0.00166+0.002)\cot^2 43.2° = 0.00581$ while the tensile strain perpendicular to the force resultant due to tie BC is $\varepsilon_1 = 0.00184 + (0.00184+0.002)\cot^2(90-43.2)° = 0.00523$. By taking the larger tensile strain, it gives the lower stress limit of $\phi f_{cu} = \phi\frac{f_c'}{0.8+170(0.00581)} = 0.56\phi f_c' = 0.56(0.70)(4000) = 1568\text{ psi}$ which is greater than the stress demand, i.e. 1518 psi.

Similarly, the force resultant of struts BD, AD, and CD is $\sqrt{214^2+304^2} = 372\text{ kips}$ with a slope of $\arctan\frac{214}{304} = 35.1°$ and the available width is $9.12\cos 35.1° + 16\sin 35.1° = 16.67\text{ in.}$ (Figure 4(b)). The stress due to this force resultant is $\frac{372(1000)}{16.67(14)} = 1594\text{ psi.}$ Part of force resultant zone crosses tie BC. The tensile strain perpendicular to this force resultant due to tensile strain in tie *BC* is $\varepsilon_1 = 0.00184 + (0.00184 + 0.002)\cot^2(90 - 35.1)^\circ = 0.00374$. This gives a stress limit of

$$\phi f_{cu} = \phi \frac{f_c'}{0.8 + 170(0.00374)} = 0.70\phi f_c' = 0.70(0.70)(4000) = 1960 \text{ psi}$$

which is greater than the stress demand, i.e. 1594 psi.

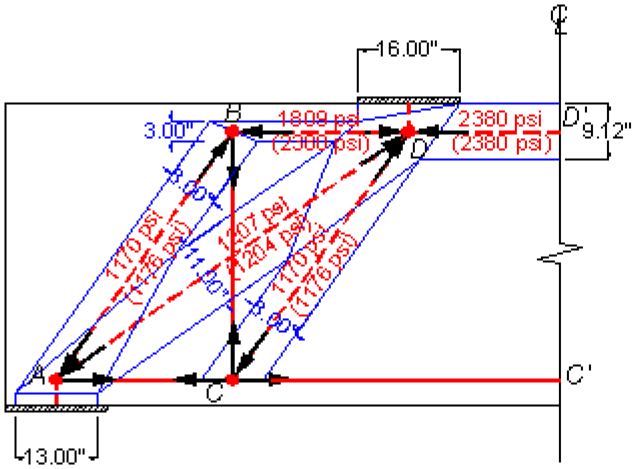

Figure 3
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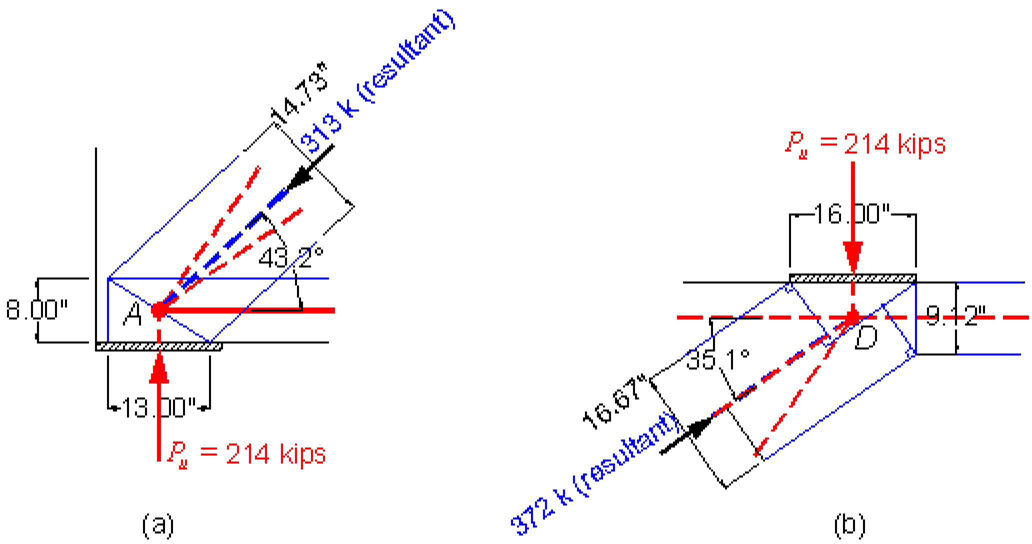

Figure 4
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Design the Nodal Zones and Check the Anchorages:

The width a of nodal zone *D* was chosen to satisfy the stress limit on that nodal zone. The stresses of the nodal zone *A* and *C* are limited to $0.75\phi f_{cu} = 0.75(0.70)(4000) = 2100$ psi and $0.65\phi f_{cu} = 0.65(0.70)(4000) = 1820\ \text{psi}$ respectively. To satisfy the stress limit of nodal zone C, the tie reinforcement must engage an effective depth of concrete at least equal to $\dfrac{N_{AC}}{0.75\phi f_{cu} b} = \dfrac{228(1000)}{(2100)(14)} = 7.76\ \text{in}.$ and to satisfy the stress limit of nodal zone A, the tie reinforcement must engage an effective depth of concrete at least equal to: These limits are easily satisfied since the nodal zone available is 8 in. The required anchorage length for tie AC is $l_{dh} = \lambda \dfrac{1200 d_b}{\sqrt{f_c'}} = 0.7 \dfrac{1200(1)}{\sqrt{4000}} = 13.28\ \text{in}.$ Since this is less than the available length, i.e. 16 – 2.5 = 13.5 in, then anchorage length is adequate.

Calculate the Minimum Reinforcement Required for Crack Control:

According AASHTO LRFD, a uniformly distributed reinforcement in vertical and horizontal directions near each face must be provided with minimum of volumetric ratio of 0.003 in each direction and the minimum bar spacing for each direction is 12 in. Try pairs of #4 bars with spacing of 9 in. for both vertical and horizontal reinforcement. The reinforcement ratio is $\dfrac{2(0.2)}{14(9)} = 0.00317.$ The ratio is greater than 0.003. Hence use pairs of #4 bars @ 9 in. o.c. in each direction.

Summary of the Design:

The reinforcement details for the deep beam designed using the strut-and-tie model according to AASHTO LRFD are shown in Figure 5.