ASIAN JOURNAL OF CIVIL ENGINEERING (BUILDING AND HOUSING) VOL. 7, NO. 5 (2006)
PAGES 461-477

DESIGN OF A CONTINUOUS DEEP BEAM USING THE STRUT AND TIE METHOD

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ABSTRACT

The strut-and-tie method can be used for the design of Disturbed regions (D-regions) of structures where the basic assumption of flexure theory, namely plane sections remaining plane before and after bending, does not hold true. Such regions occur near statical discontinuities arising from concentrated forces or reactions and near geometric discontinuities, such as abrupt changes in cross section etc. The strut-and-tie method of design is based on the assumption that the D-regions in concrete structures can be analysed and designed using hypothetical pin-jointed trusses consisting of struts and ties inter-connected at nodes. Continuous deep beams occur as transfer girders in multi-storey frames, as pile caps and as foundation wall structures etc. The usual design practice for continuous deep beams has been to employ empirical equations, which are invariably based on simple span deep beam tests. Given the unique behavioural pattern of continuous deep beams, this practice is unrealiable. Since continuous deep beams contain significant extents of D-regions and they exhibit a marked truss or tied arch action, the strut-and-tie method offers a rational basis for the analysis and design of such beams. The mechanics and behavior of continuous deep beams are briefly discussed from which a strut-and-tie model for such a beam is developed. A complete example on the analysis and design of a continuous deep beam using the strut-and-tie method is presented. The design has been carried out using the recommendations of the ACI Code 318-02.

Keywords: Struts, ties, nodes, deep beams, strut-and-tie method, anchorage

1. INTRODUCTION

Some typical examples of continuous deep beams are illustrated in Figure 1. Continuous deep beams exhibit the same general trend of increased shear strength with a decrease in shear-span/depth ratio as found in simply supported deep beams. In continuous deep beams, the locations of maximum negative moment and maximum shear coincide and the point of contraflexure may be very near the critical section for shear. Both these conditions render most of the empirical strength prediction equations for simply supported deep beams useless

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for continuous deep beams.

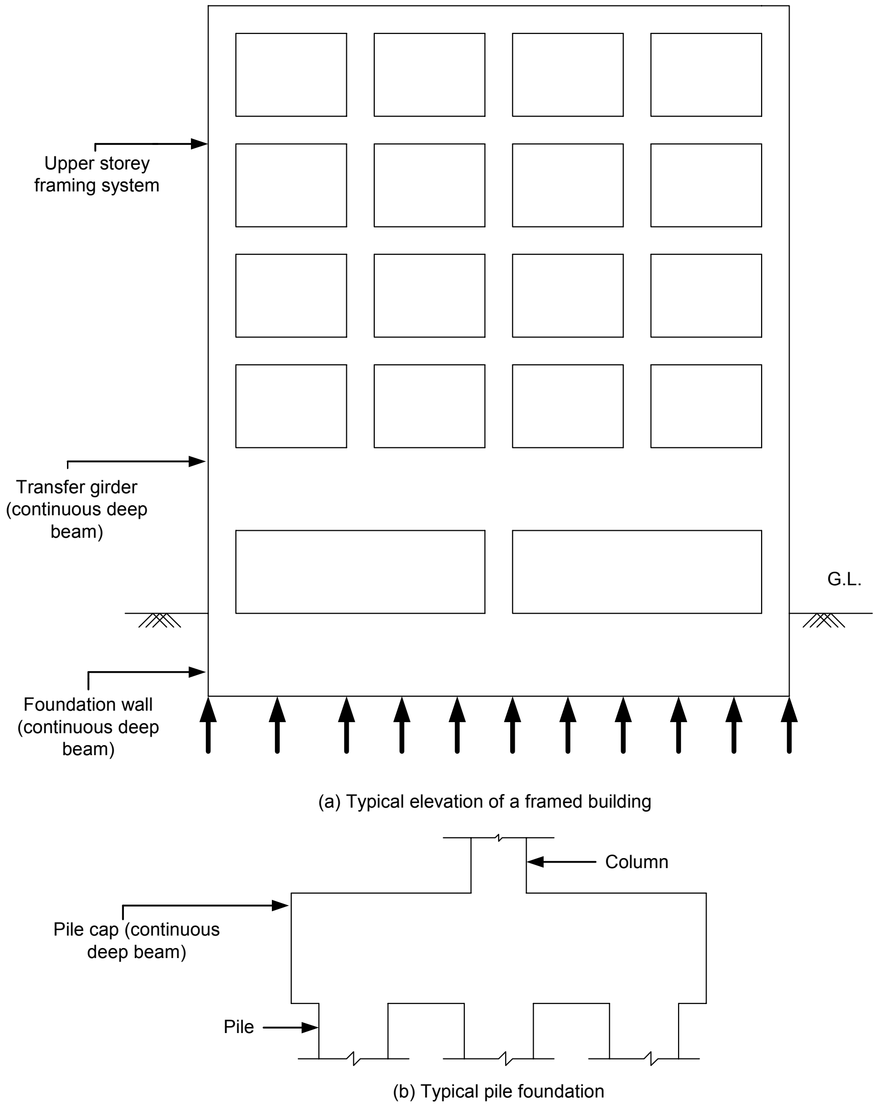
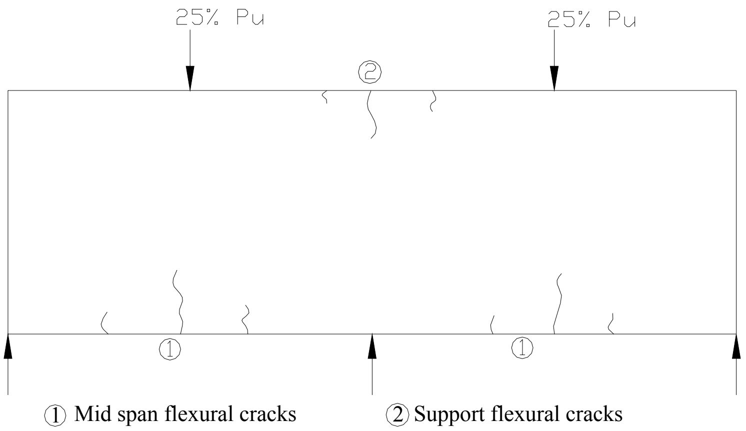
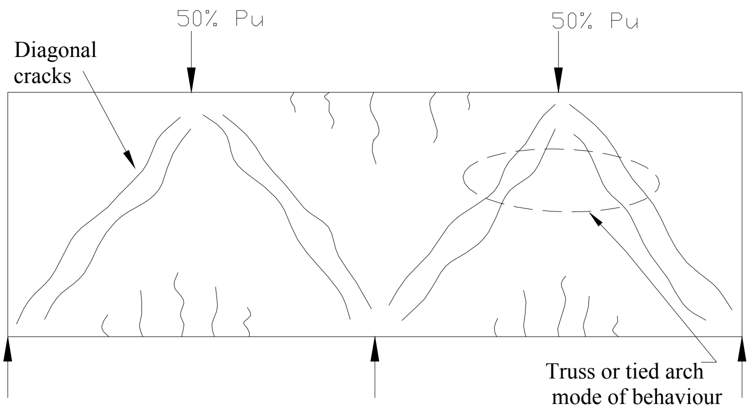


Figure 1. Examples of continuous deep beams

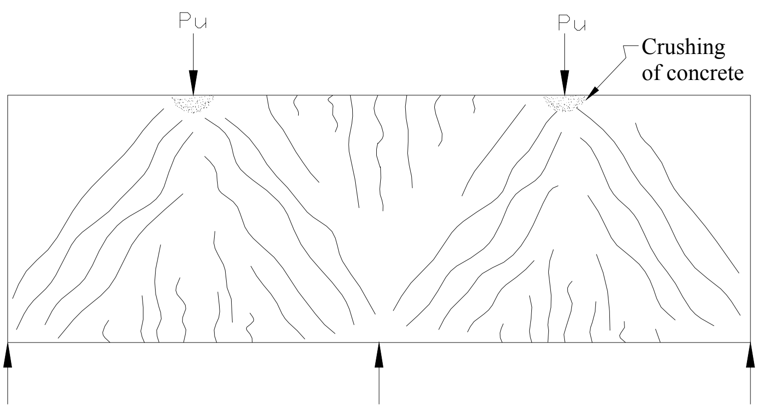
The key events in the life of a typical continuous deep beam loaded to failure are presented in Figure 2, Ref. [1].



(a) Initial flexural cracking



(b) Shear cracking

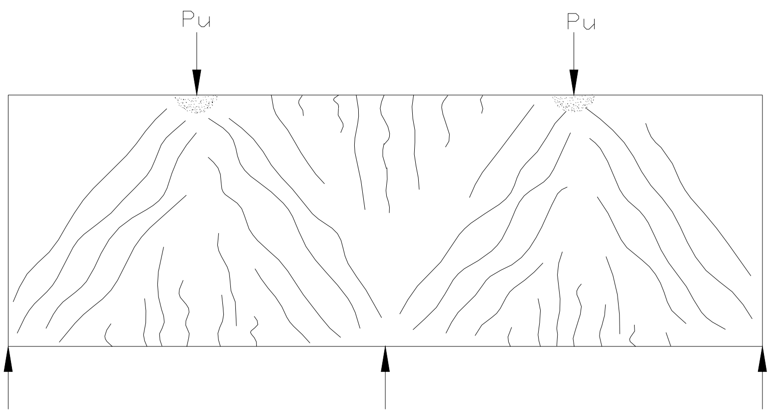


(c) Cracking at impending failure

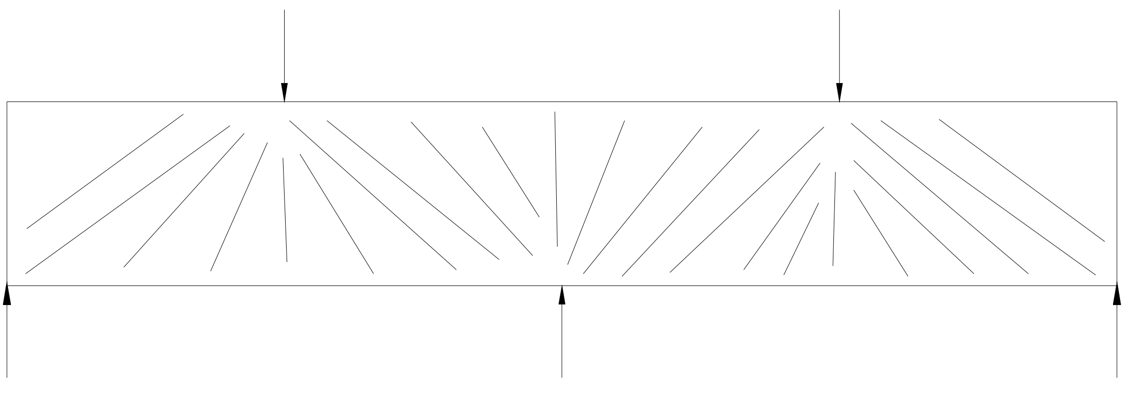
Figure 2. Typical cracking behavior of continuous deep beams

Deep beams, in general, develop little initial flexural cracking. The mid-span flexural cracks in a continuous deep beam tend to form before the negative cracks over the interior support, Figure 2(a). The first significant cracking occurs in the form of diagonal shear cracks at about 50% of the ultimate load, the cracks tending to dilineate a truss or tied arch mode of behaviour, Figure 2(b). The expansion of the diagonal cracks is accompanied by the development and growth of additional secondary flexural cracks as the reinforcement is brought to yield, Figure 2(c). The yielding of the main flexural reinforcement brings about significant deflections in the member. The strength of the member is governed by the yield of the main flexural reinforcement while the ductile behaviour is influenced by the mode of failure of concrete.

In contrast to the truss or tied arch action mode of behaviour of a continuous deep beam, continuous shallow beams transfer shear through a fairly uniform diagonal compression field with compression fans under the point loads and over the supports, Figure 3. In continuous deep beams on the other hand, most of the force is transferred to the supports through distinct direct compression struts, which basically are zones of predominantly uniaxial compression.



(a) Force transfer in continuous deep beam



(b) Force transfer in continuous shallow beam

Figure 3. Force transfer in a continuous deep beam and in a continuous shallow beam

As a consequence of the truss or tied arch mode of behaviour, the main flexural reinforcement in continuous deep beams carries significant tension along its full length, as illustrated in Figure 4, Ref. [1]. Hence, the development and anchorage of the main reinforcement is critical in the case of such members.

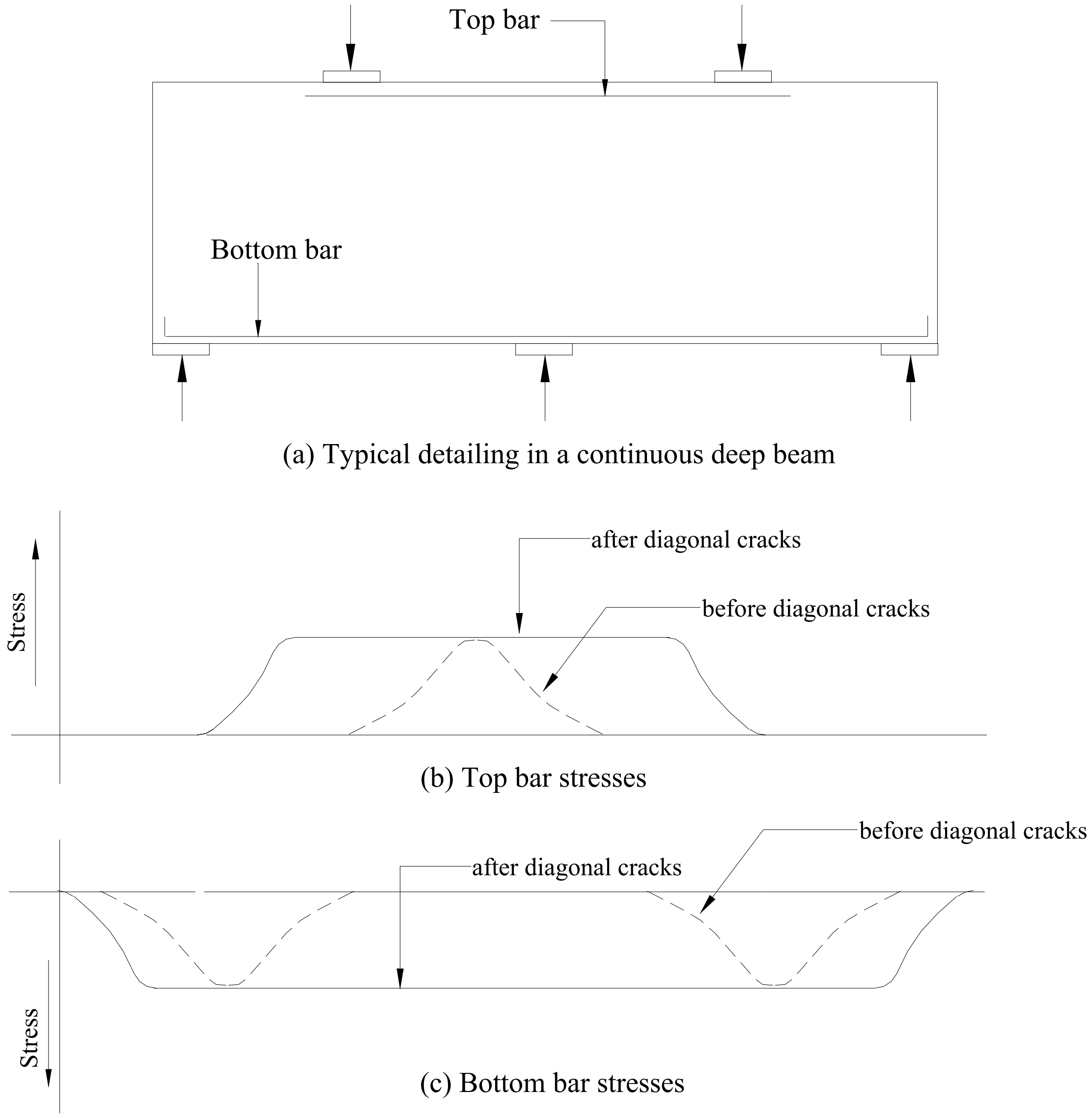


Figure 4. Stress distribution in main reinforcement [1]

By intuitively considering the mode of load transfer to the supports in the case of a continuous deep beam, a truss model consisting of a network of struts and ties intersecting at nodes, can be built up. The deep beam under consideration can be assumed to be made up of a primary negative moment truss and a primary positive moment truss as presented in Figure

5(a) and (b). Both these trusses superimposed upon each other give a strut-and-tie model for the continuous deep beam, Figure 5(c).

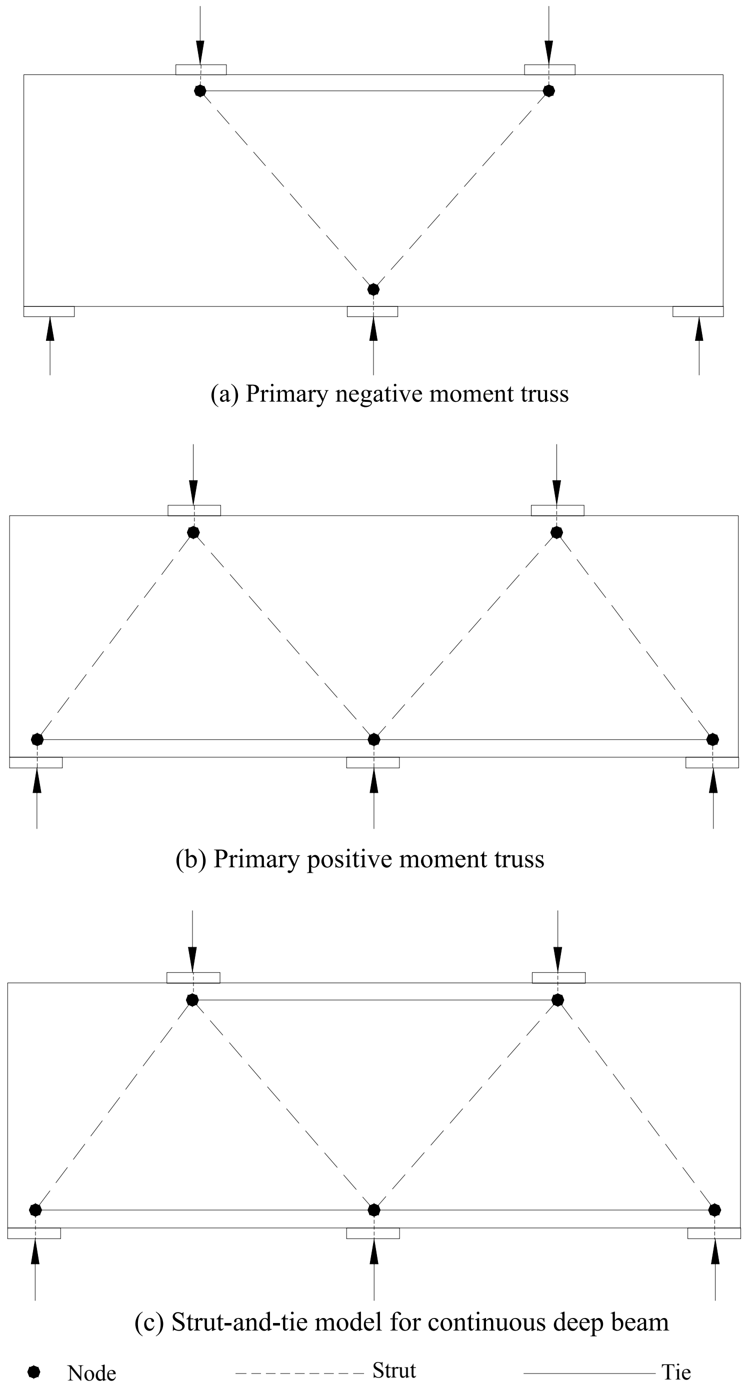


Figure 5. Development of a strut-and-tie model for a continuous deep beam

Multiple strut-and-tie models can be developed for a single load case on a structure. Some options for the beam being designed are presented in Figure 6. The moot question at this stage is how the optimal model should be selected. Usually, that model is the best in which the loads follow the path with the least force and the least deformation [2].

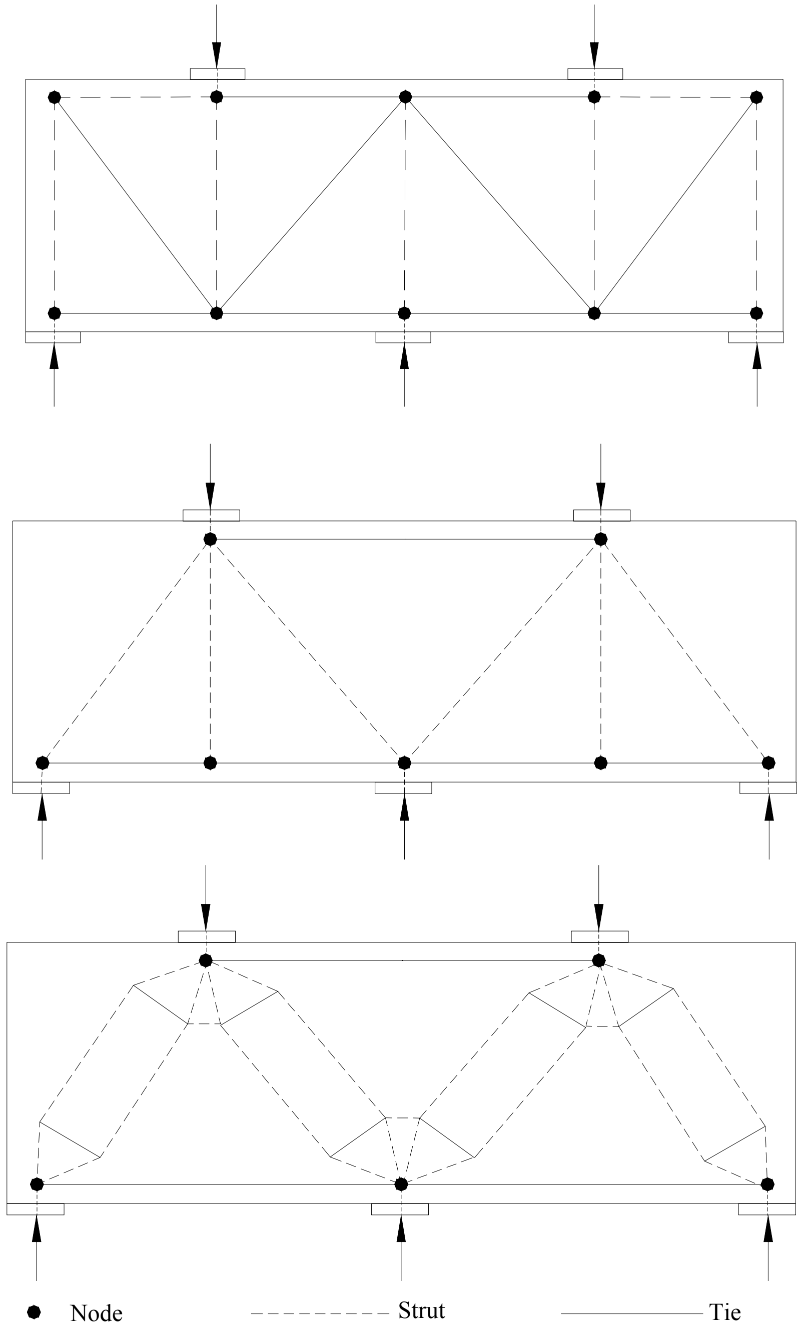


Figure 6. Strut-and-tie models for a continuous deep beam

At the same time, since ties are more deformable than concrete struts, a model with the least number and the shortest ties is likely the best. This requirement can be quantified as $\sum F_i l_i \varepsilon_{mi}$ = minimum [2], where F_i is the force in the strut or tie, l_i is the length of the member 'i' and ε_{mi} is the mean strain in member 'i'. In addition to the above requirements, the selected strut-and-tie model should be such that the angle between the axes of the struts and ties acting on a node should be as large as possible to mitigate cracking and to avoid incompatibilities due to shortening of the struts and lengthening of the ties occurring otherwise in almost the same directions. The ACI Code [3] recommends that the angle between the axes of a strut and a tie entering a single node shall not be less than 25º. Based on the above recommendations, the strut-and-tie model of Figure 5(c) is selected for modeling the continuous deep beam.

2. LOADS AND MATERIAL PROPERTIES

The loads, spans and dimensions of the deep beam selected for analysis are presented in Figure 7.

Design vertical load = 1500 kN and 2000 kN at mid-spans of both spans, Figure 7
Characteristic cube compressive strength of concrete (assumed) = f_{ck} = 30 MPa
Take cylinder compressive strength = $f'_c = 0.80\ f_{ck}$ = 24 MPa
Yield strength (0.2% proof stress) of reinforcement bars (assumed) = f_y = 415 Mpa

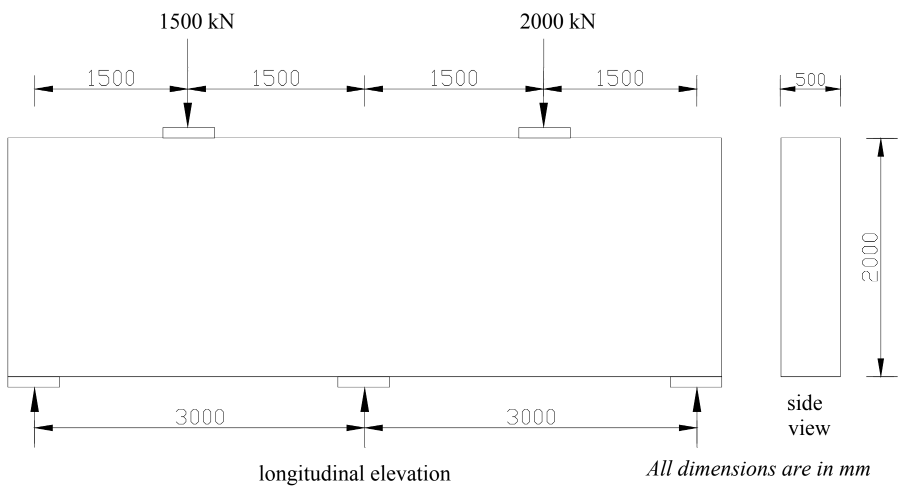


Figure 7. Loads, span and dimensions of the continuous deep beam

3. DETERMINATION OF TRUSS FORCES

The strut-and-tie model selected for the continuous deep beam together with the loads, is shown in Figure 8. The forces in the members of the truss are determined from equilibrium conditions. The location and orientation of the struts and ties is defined by the position of the nodes. The horizontal position of the nodes A, B, C, D and E can be assumed to lie on the line of action of the respective applied loads and the support reactions. The vertical position of the nodes is fixed next.

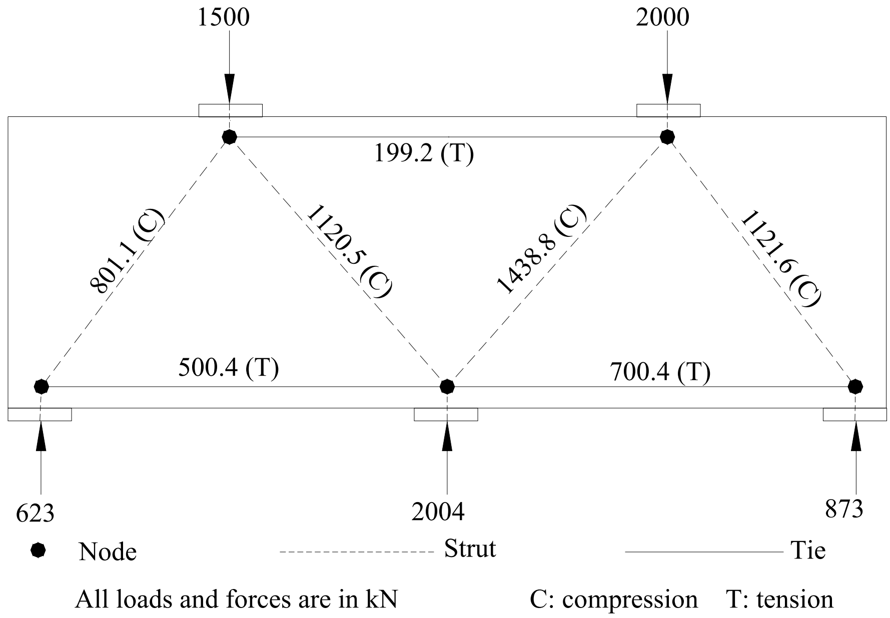


Figure 8. Loading and analysis of the strut-and-tie model for the continuous deep beam

In order to exploit the full load carrying capacity of the beam it is imperative that nodes A, E and D lie as close as possible to the bottom face of the beam. Similarly, the nodes B and C should lie as close as possible to the top face of the beam. It will be assumed that the centerline of the tie BC and that of the ties AE and DE is lying at a distance of 75 mm from the top and bottom faces of the beam respectively. This will make available a width of 75×2=150 mm for the ties BC, AE and DE. This tie width will allow provision of at least two layers of reinforcing bars in the ties if required, and also provide sufficient concrete cover to the tie reinforcement. The assumed tie width will be later checked for adequacy with respect to the calculated tie force and the permissible stress in concrete in the node anchoring the tie.

The support reactions together with the forces set up in the members of the truss under the externally applied loads are also summarized in Figure 8.

4. DESIGN OF BEARING PLATES

The bearing plates are to be provided at the loading points and at the supports. The reactions are determined as 623 and 873kN at the exterior supports and 2004kN at the interior support, Figure 8.

The sizes of the bearing plates are to be determined next.

The bearing plates at the points of application of the loads will be resting above the underlying C-C-T (Compression-Compression-Tension) nodes of the strut-and-tie model. The bearing stresses exerted by the bearing plates on the faces of the underlying nodes should be less than the permissible bearing stresses for these nodes.

Similarly, the bearing plates at the support locations are below the overlying C-C-T nodes of the strut-and-tie model and the bearing stresses at the faces of these nodes should be less than the permissible bearing stresses for these nodes.

Assume the size of all the bearing plates as 600 x 500 mm each.

Since the interior support carries the maximum reaction, the adequacy of the assumed size of the bearing plates is checked for this support and if found safe, the same size of the bearing plates is provided at the two exterior supports.

Hence, the bearing stress at the interior support is $= \dfrac{2004 \times 10^3}{600 \times 500} = 6.68$ MPa

As per Clause A.5.2 eq. (A-8) of [3] the effective compressive strength of a C-C-T node $= f_{cu} = 0.85\beta_n f_c^{'}$. As per Clause A.5.2.3 of [3] for a C-C-T node anchoring two or more ties (T), $\beta_n = 0.60$.

Hence, $f_{cu} = 0.85 \times 0.60 \times 24 = 12.24$ MPa

The allowable bearing stress $= \Phi f_{cu}$, where Φ is the strength reduction factor, which for strut-and-tie models, as per Clause 9.3.2.6 of [3], = 0.75

Hence, the allowable bearing stress $= \Phi f_{cu} = 0.75 \times 12.24 = 9.18 > 6.68$ MPa, ok.

Hence, the selected size of the bearing plates is adequate.

Provide bearing plates of size 600×500 mm at all the supports and at the loading points.

5. DESIGN OF TIES

The tie capacity is furnished by steel reinforcement and concrete is not assumed to carry any tensile loads.

The area of reinforcement required for a typical tie is equal to $A_{st} = \dfrac{F_t}{\sigma_Y}$, where F_t is the tensile force in the tie and σ_y is the permissible tensile stress in the steel reinforcement and is equal to Φf_y. The strength reduction factor [3], Φ, for the reinforcement yield stress f_y, is taken as 0.75 as per recommendations of Clause 9.3.2.6 [3].

Therefore, the area of reinforcement required for tie BC $= \dfrac{F_{BC}}{\sigma_Y} = \dfrac{199.2 \times 10^3}{0.75 \times 415} = 640$ mm^2

Provide 4 nos. of 16 mm diameter bars for the tie BC. Area of steel provided = 804mm^2> 640 mm^2, ok.

The area of reinforcement required for tie AE $= \dfrac{F_{AE}}{\sigma_Y} = \dfrac{500.4 \times 10^3}{0.75 \times 415} = 1607.71$ mm^2

Provide 9 nos. of 16 mm diameter bars for the tie AE. Area of steel provided = 1809mm^2> 1607.71 mm^2, ok.

The area of reinforcement required for tie DE = $\frac{F_{DE}}{\sigma_Y} = \frac{700.4 \times 10^3}{0.75 \times 415} = 2250.28$ mm^2

Provide 12 nos. of 16 mm diameter bars for the tie DE. Area of steel provided = 2412 mm^2 > 2250.28 mm^2, ok.

To ensure continuity of reinforcement in the bottom tie at the node E, the reinforcement in the tie AE is changed to 12 nos. of 16 mm diameter bars.

As per Clause 11.9.5 [3], the minimum required area of tensile reinforcement in any tie =

$$0.04\left(\frac{f_c^{'}}{f_y}\right)bd = 0.04\left(\frac{24}{415}\right)500 \times 1925 = 2226.50 \text{ mm}^2$$

The minimum amount of reinforcement is required to prevent the possibility of sudden failure under the action of flexural moment.

The area of reinforcement provided in the ties BC (804 mm^2) and AE (1809 mm^2) is less than the minimum. Hence, provide 12 nos. of 16 mm diameter bars in each of the ties BC and AE. Area of steel provided = 2412 mm^2 > 2226.50 mm^2. ok.

6. CHECK ON NODAL ZONES AND ANCHORAGES

The nodal zones of interest are at nodes A, B, C, D, and E. The dimensions of the nodal zones have to be such that the stresses acting on the faces of the nodal zones are within permissible limits. Since the main reinforcement in continuous deep beams carries significant tension along its full length, as illustrated in Figure 4 of [1], adequate rebar anchorage is critical to the performance of the member and anchorage check therefore, assumes added importance.

Nodes A and D are CCT (Compression-Compression-Tension) nodes. If any of the forces acting on a node is tensile, the required minimum width of a side of the nodal zone containing such a node is calculated from the width of a hypothetical bearing plate anchoring one end of the tie which is assumed to exert a uniform bearing pressure on the back side of the nodal zone, Figure 9. The width of the hypothetical bearing plate in turn is equal to the required width of the tie anchored in the node.

As per Clause A.5.2 eq. A-8 [3], the effective compressive stress at the face of a node= $f_{cu} = 0.85\beta_n f'_c$

The value of the parameter β_n is specified in Clauses A.5.2.1, A.5.2.2 and A.5.2.3 of [3].

For the CCT node A, which anchors one tie (the 'T' in CCT), Clause A.5.2.2 of [3] is applicable wherein $\beta_n = 0.80$.

Therefore, $f_{cu} = 0.85 \times 0.80 \times 24 = 16.32$ MPa

The allowable bearing stress is equal to Φf_{cu} where Φ is the strength reduction factor [3]. As per Clause 9.3.2.6 of [3], for strut-and-tie models, $\Phi = 0.75$

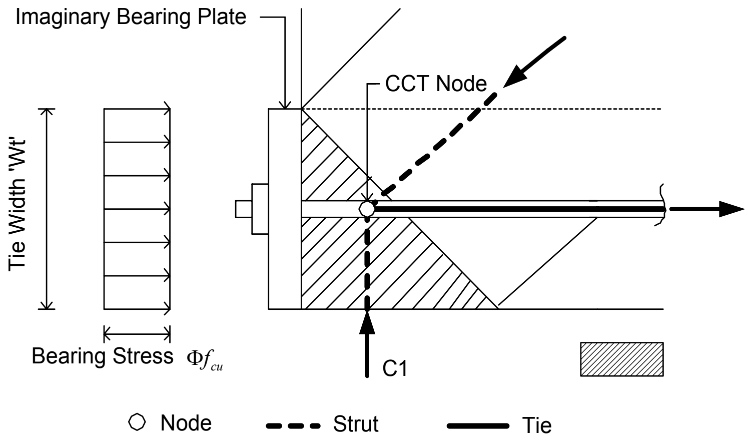


Figure 9. Tie width in a CCT node

Hence, Φf_{cu} = 0.75 x 16.32 = 12.24 MPa

Therefore, the required width of the tie AE to be anchored in the CCT node A = $\dfrac{F_{AD}}{\Phi f_{cu} b}$ =

$\dfrac{500.4 \times 10^3}{12.24 \times 500}$ = 81.76 mm

Available tie width = 75 × 2 = 150 > 81.76 mm, ok.

Similarly, the required width of the tie DE to be anchored in the CCT node D = $\dfrac{F_{AD}}{\Phi f_{cu} b}$ =

$\dfrac{700.4 \times 10^3}{12.24 \times 500}$ = 114.44 mm

Available tie width = 75 × 2 =150 > 114.44 mm, ok.

Since adequate tie widths are available, the stresses at nodes A and D are assumed to be within permissible limits.

To provide positive anchorage to tie AE, weld the 12 numbers of 16 mm diameter bars constituting tie AE to a steel angle ISA 100 x 100 x 12 @ 17.7 kg/m located at the bottom left end of the continuous beam. Similarly, to provide positive anchorage to the 12 numbers of 16 mm diameter rebars constituting tie DE, they may be welded to a steel angle ISA 100 × 100 × 12 @ 17.7 kg/m located at the far bottom right end of the continuous beam.

The reinforcement in the two ties AE and DE is continuous through the node E and hence, there are no anchorage requirements as such at node E.

Node E anchors two ties, AE and DE. For computing the effective compressive stress at the face of such a node, the parameter β_n is specified in Clause A.5.2.3 of [3] as 0.60.

Hence, f_{cu} = 0.85 x 0.60 x 24 = 12.24 MPa

The allowable bearing stress = Φf_{cu} = 0.75 x 12.24 = 9.18 MPa.

Therefore, the required width of the tie DE to be anchored at node E = $\frac{F_{DE}}{\Phi f_{cu} b}$ = $\frac{700.4 \times 10^3}{9.18 \times 500}$ = 152.59 mm.

Available tie width = 75 x 2 = 150 mm ≈ 152.59 mm, ok.

Hence, the stresses at node E are within permissible limits.

Like Node A, **Nodes B and C** are CCT nodes.

For a CCT node which anchors one tie (the 'T' in CCT), Clause A.5.2.2 of [3] is applicable wherein β_n = 0.80.

Therefore, f_{cu} = 0.85 × 0.80 × 24 = 16.32 MPa

The allowable bearing stress is equal to Φf_{cu} where Φ is the strength reduction factor [3]. As per Clause 9.3.2.6 of [3], for strut-and-tie models, Φ = 0.75

Hence, Φf_{cu} = 0.75 × 16.32 = 12.24 MPa

Therefore, the required width of the tie BC to be anchored in the CCT nodes B and C = $\frac{F_{AD}}{\Phi f_{cu} b} = \frac{199.20 \times 10^3}{12.24 \times 500}$ = 32.54 mm

Available tie width = 75 × 2 = 150 mm > 32.54 mm, ok.

Hence, the stresses at nodes B and C are within permissible limits.

The rebars in the tie BC terminate at nodes B and C. Instead of providing positive anchorage for the rebars in the tie BC, as has been done for the rebars in the ties AE and DE, it is proposed to examine the required development length for the rebars and check the same against the available development length in the member. Thus, the requirement of development length for the rebars at B and C has to be ascertained.

The required anchorage length for the reinforcement in the tie BC at the nodes B and C, as per Clause 12.2.2 [3] is $l_d = (\frac{f_y \alpha \beta \lambda}{25\sqrt{f_c'}}) d_b$, where α, the 'reinforcement location factor', is taken as 1.0 as per Clause 12.2.4 [3]; β the 'coating factor' for uncoated reinforcement, as is the present case, is taken as 1.0 as per Clause 12.2.4 [3]; λ the light weight aggregate concrete factor, being taken as that corresponding to normal weight concrete viz. 1.0 as per Clause 12.2.4 [3]; d_b being the nominal diameter of the reinforcing bar.

Hence, $l_d = \frac{415 \times 1.0 \times 1.0 \times 1.0 \times 16}{25 \times \sqrt{24}}$ = 54.21 mm. Since sufficient clearance is available, provide an anchorage length of 75 mm to the rebars in the tie BC beyond the points B and C that are assumed as the anchor points for the rebars.

7. CHECK ON STRUTS

The check on struts involves determination of strut widths required to shoulder the computed strut forces and to determine whether the required strut widths fit within the geometry of the structure.

The effective compressive strength of the concrete in all the struts is limited to Φf_{cu} where $f_{cu} = 0.85\beta_s f_c^{'}$; the parameter β_s being taken equal to 0.75 as per Clause A.3.2.2 of [3]. For this value of β_s it will be necessary to provide reinforcement suitably proportioned to resist the transverse tensile force resulting from the spreading of the compression force in the concrete struts .

Therefore, $\Phi\ f_{cu} = 0.75 \times 0.85 \times 0.75 \times 24 = 11.47$ MPa.

Hence, the required width for strut AB = $\dfrac{F_{AB}}{\Phi f_{cu} b} = \dfrac{801.1 \times 10^3}{11.47 \times 500} = 139.68$ mm. Choose a width of 150 mm for strut AB.

The required width for strut BE = $\dfrac{F_{BE}}{\Phi f_{cu} b} = \dfrac{1120.5 \times 10^3}{11.47 \times 500} = 195.37$ mm. Choose a width of 200 mm for strut BE.

The required width for strut CE = $\dfrac{F_{CE}}{\Phi f_{cu} b} = \dfrac{1438.8 \times 10^3}{11.47 \times 500} = 250.88$ mm. Choose a width of 260 mm for strut EC.

The required width for strut CD = $\dfrac{F_{CD}}{\Phi f_{cu} b} = \dfrac{1121.6 \times 10^3}{11.47 \times 500} = 195.57$ mm. Choose a width of 200 mm for strut CD.

As can be seen in Figure 10, all the strut widths fit within the geometry of the beam and thus the proposed strut-and-tie model is acceptable.

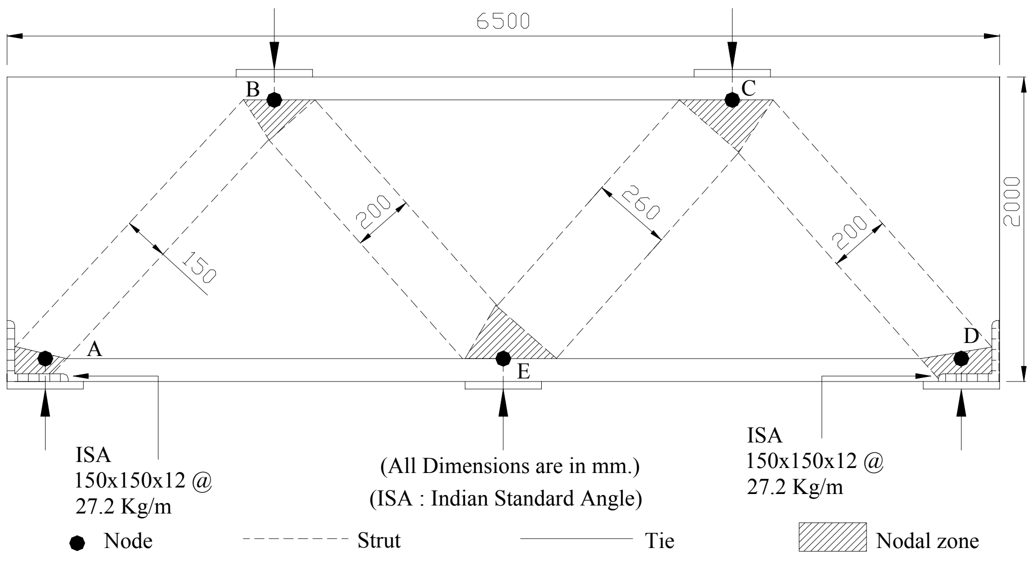


Figure 10. Computed strut widths

8. CRACK CONTROL REINFORCEMENT

The crack control reinforcement is provided in the form of vertically and horizontally oriented stirrup reinforcement in the beam.

As per Clause 11.8.4 of [3], the area of the vertical stirrups should not be less than:

$A_v = 0.0025\ b\ s$; s being the stirrup spacing

Similarly, as per Clause 11.8.5 of [3], the area of the horizontal stirrups should not be less than:

$A_h = 0.0015\ b\ s_2$; s_2 being the stirrup spacing

It is also stipulated that both s and s_2 cannot exceed $\frac{d}{5}$ or 300 mm.

For vertical crack control reinforcement, provide 2-legged 16 mm diameter stirrups at 300 mm c/c.

$$\left(\frac{A_v}{bs}\right) \text{ provided} = \frac{2\times 201}{500\times 300} = 0.0026 > 0.0025, \text{ ok.}$$

For horizontal crack control reinforcement, provide 2-legged 12 mm diameter stirrups at 275 mm c/c.

$$\left(\frac{A_h}{bs_2}\right) \text{ provided} = \frac{2\times 113}{500\times 275} = 0.0016 > 0.0015, \text{ ok.}$$

Since β_s has been assumed as 0.75 for computing the strength of the struts, minimum reinforcement to resist the splitting force and to restrain crack widths in the struts has to be provided to satisfy the following requirement of Clause A.3.3.1 eq. (A-4) of the Code:

$$\sum \frac{A_{st}}{bs_i} \sin\gamma_i \geq 0.0030$$

where s_i is the rebar spacing and γ_i is the angle between the axis of the minimum reinforcement and the axis of the struts.

The horizontally oriented crack control reinforcement provided above makes an angle of 50.96° and the vertically oriented crack control reinforcement provided above makes an angle of (90° - 50.96°) = 39.04° with the axis of the struts and these will serve to reinforce the struts against the bursting forces.

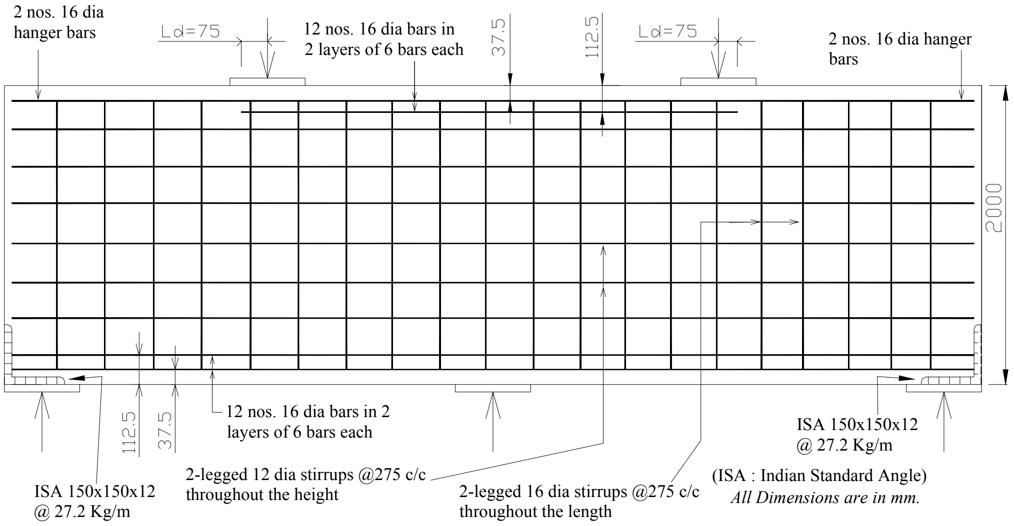
Hence,

$$\sum \frac{A_{st}}{bs_i} \sin\gamma_i = \frac{2\times 201}{500\times 300}\sin 39.04^\circ + \frac{2\times 113}{500\times 275}\sin 50.96^\circ = 0.0029 < 0.0030, \text{ not ok.}$$

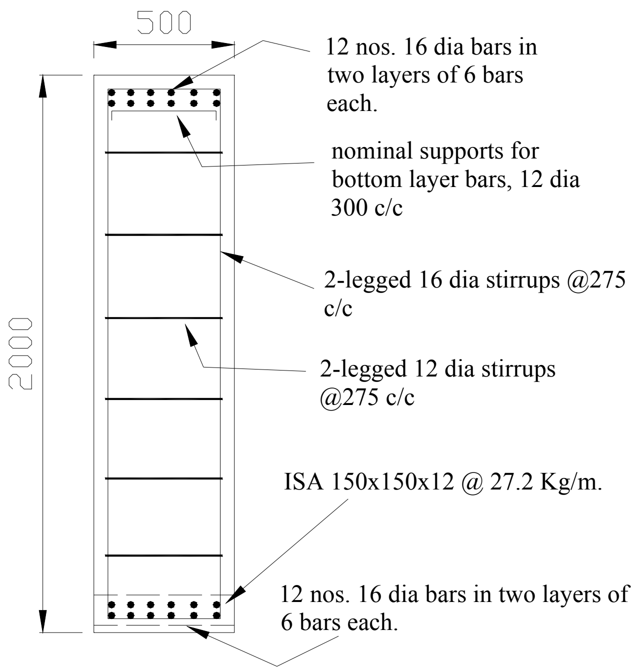
Revise the spacing of the vertical crack control reinforcement to 275 mm c/c.

$$\sum \frac{A_{st}}{bs_i} \sin\gamma_i = \frac{2\times 201}{500\times 275}\sin 39.04^\circ + \frac{2\times 113}{500\times 275}\sin 50.96^\circ = 0.0031 > 0.0030, \text{ ok.}$$

The detailing of the reinforcement in the beam is shown in Figure 11.



Elevation



Section

Figure 11. Detailing of reinforcement

Acknowledgement: Generous assistance has been derived from ACI SP-208 [4] while preparing this manuscript, which is gratefully acknowledged.

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